Socially Optimal Correlated Equilibrium in Class-Anonymous Offloading Game with Computing Access Points

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***Abstract*—We consider a multi-user mobile offloading network with multiple computing access points (CAPs). Each user has one task to be processed, and may choose to reduce the cost of processing the task by offloading it to a CAP or to a remote cloud server. Each user belongs to one of a fixed number of classes, which determines the distribution of their task parameters. We aim to produce an offloading decision that minimizes the expected social cost of the system, while giving selfish users an incentive to follow that decision. Towards that goal, we show that our system can be formulated as a class-anonymous game, and we derive the reduced form of this game to prove that a socially optimal correlated equilibrium (CE) can be computed in polynomial time and space with respect to the number of users. Like the Nash Equilibrium, the CE maintains the necessary conditions for stability in a system with rational and selfish users, while being much easier to compute for non-potential finite games. Simulation results demonstrate the superior results of our solution when compared with random mapping and an alternate means of computing a CE.**

I. Introduction

With the continually increasing complexity of mobile applications, there is a growing demand for accessible computational resources external to the mobile device. Mobile Edge Computing (MEC) reduces the communication latency in accessing these computational resources by positioning them at the edge of the mobile network [1], [2], [3]. One common MEC approach is to install servers into the wireless access point or cellular base station, also known as a *computing access point* (CAP). A CAP is thus responsible for providing the users with the communication resources necessary to access remote computational resources in the cloud, as well as its own computing capability for possible task offloading.

In a three-tier offloading system with a CAP, studied in [4], [5], and [6], mobile tasks may be processed at the local mobile device, at the CAP, or at a remote cloud. These works study the associated offloading problem—how to distribute individual user tasks among these heterogeneous offloading sites. This offloading problem is inevitably coupled with a resource allocation one, as both the computational and communication resources are limited and should be distributed judiciously. Furthermore, a rational solution should account for strategic behaviour among the users that may result if individual users have agency over their offloading decisions. By analysing the offloading system as a strategic game, [6] used the existence of a potential function to compute a pure-strategy Nash Equilbrium (PSNE); a similar method was applied in [7] and [8]. However, most strategic games do not have a potential function, and in such cases do not necessarily possess a PSNE [9]. This limits the applicability of those previous works to a set of special cases. And while all finite games (games with a finite set of strategy profiles) possess mixed-strategy Nash Equilibria (MSNE) [9], computation of such equilibria is a PPAD-complete problem that is likely hard to solve if $\mathbf{P} \neq \mathbf{NP}$ [10]. A preferable solution would be a more general equilibrium condition that is more easily computed in a wider range of systems, while maintaining the advantages of NE for systems with selfish users.

In this work, we consider a three-tier offloading system with multiple CAPs, each having their own available communication and computational resources. Each user has a single task, which can be processed at the user's local device, at a remote cloud, or at one of the CAPs. Individual users seek to minimize their expected cost, which is an unique convex function of energy consumption and completion time. Meanwhile, the system seeks to minimize the social cost, defined as a linear function of individual costs.

The contributions of this work are as follows:

- We model the above system as a strategic game, where individual users can strategically choose their offloading decisions based on the actions of other users. Because the PSNE and MSNE may be impossible or difficult to compute, we instead choose to obtain a *correlated equilibrium* (CE) of the system. A CE is an equilibrium that ensures that no user has an incentive to deviate from a given strategy (assigned by an external agent), assuming that other users in the system are also acting according to their assigned strategies [11]. The CE's advantages over the NE are that one can be computed by known methods in all finite games, it can achieve a lower social cost over the NE, and it still satisfies equilibrium requirements for rational, selfish users [12].
- While finite games generally have a convex set of CE's, our objective is to find the CE that minimizes the social cost (i.e. the socially optimal CE). We consider a general

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class-anonymous (CA) game, where the users within a class are indistinguishable from each other in terms of their task parameters and resource utilization. We prove by deriving a reduced form of the game that as long as the number of classes in the system is fixed, an optimal CE can be found in polynomial time with respect to the number of users. We then present a solution for doing so through linear programming (LP).

- Simulation results show that our system performance is superior to both a random mapping of the tasks as well as a regret-based learning method used to compute a non-optimal CE.

Our work is organized as follows. Section II reviews game-theoretic approaches in mobile offloading systems. Section III formulates the system model. Section IV defines the offloading game, shows that it is an example of a CA game, discusses the CE, and provides a proof that a socially optimal CE can be found for CA games in polynomial time. Section V uses those results to develop our solution method. Section VI presents our simulation results. Finally, Section VII concludes the work.

II. Related Works

Game-theoretic solutions to the mobile network offloading problem were presented previously in [6], [7], [8], [13], [14], [15], [16], [17], [18], and [19]. Two works that stand apart are [18] and [19], which consider Poisson generation of user tasks and a queueing system for offloading, while all of the remaining works listed consider systems with a single atomic task per user, as does our work. Of those works, all except [6] consider two-tier offloading systems, while our system and that of [6] allow for three tiers of offloading.

Of the two tier models, [13] and [14] consider systems with communication through one or multiple orthogonal wireless channels respectively, which are shared between the users in an interference model. In contrast, [7] considers a single wireless channel using time-division multiplexing, [15] considers multiple wireless channels via different access points, with the available bandwidth distributed equally between offloading users, [16] considers task transmission through carrier sense multiple access (CSMA) over multiple wireless channels, and [8] and [17] consider multiple shared wireless channels with a generic rate sharing mechanism. Our system follows the communication model of [15], but does not require equal distribution of bandwidth between the users. Furthermore, all of these works except [7] assume the users optimize for a weighted sum of energy and time consumption, while [7] exclusively optimizes for energy and adds time requirements as constraints. Our system allows the users to optimize for any nondecreasing convex function of energy and time consumption. More importantly, all of the above works except [17] present methods that rely on the existence of a potential function in their system to compute a PSNE. We note that the potential function is hard to identify in general and may not exist in practical systems, thus limiting the applicability of this approach. Only [17] presents a system that is not a potential game, but their solution method uses the fact that a subgame within their system is a potential game. Furthermore, none of their solution methods necessarily produce a socially optimal NE. Our approach, by using the CE, does not require the existence of a potential function, and we find the socially optimal CE.

Similar to our system, [6] considers a three-tier system with local, CAP, and cloud processing available, with CAP bandwidth and processor rate allocation to each user. However, our model includes multiple CAPs. More importantly, [6] again seeks a PSNE based on the potential function approach. As such, it specifically optimizes for a weighted sum of energy consumption and total system run time, in order to ensure the existence of a potential function in the game. As explained above, our system in this work uses the CE and allows a more general optimization objective.

III. System Model

Consider a cloud access network consisting of one remote cloud server, M CAPs denoted by the set $\mathcal{M} \in \{1,\ldots,M\}$, and N mobile users denoted by the set $\mathcal{N} \in \{1,\ldots,N\}$. Each user has a single task to be processed at time zero. This model can be extended to the scenario where the users have multiple tasks, if the tasks are processed in a round-robin manner among the users.

A. Offloading Decision

Each user may process their task locally, or offload it to one of the M CAPs. The tasks offloaded to the CAP may be processed there or may be further offloaded to the remote cloud server. Denote the offloading decision for each user as the binary vector

$$\mathbf{x}_i = [x_{i0}, x_{i1}, \ldots, x_{iM}, x_{i(M+1)}, \ldots, x_{i(2M)}]^T, \qquad (1)$$

where $x_{ij} = 1$ for $j = 0$ if user i's task is to be processed locally, $x_{ij} = 1$ for $1 \le j \le M$ if the task is to be offloaded and processed at CAP j, and $x_{ij} = 1$ for $j > M$ if the task is to be offloaded at CAP $j - M$ and processed at the cloud. The tasks are atomic and thus cannot be split between sites:

$$\sum_{j=0}^{2M} x_{ij} = 1, \quad \forall i \in \mathcal{N}. \qquad (2)$$

Each user may also have a possibly empty set $\mathcal{L}_i \in \{0,\ldots,2M\}$ of forbidden offloading decisions. These placement constraints are expressed as

$$x_{il} = 0, \quad \forall l \in \mathcal{L}_i, \forall i \in \mathcal{N}. \qquad (3)$$

B. Local Processing and Task Parameters

The input data size, output data size, and required processing cycles of user i's task are denoted by $D_{\text{in}}(i), D_{\text{out}}(i)$, and $Y(i)$ respectively. The processing time required by the local device is T_i^L, and the energy consumption is E_i^L, both of which we assume are user-specific deterministic functions of $D_{\text{in}}(i), D_{\text{out}}(i)$, and $Y(i)$.

C. CAP Processing

CAP processing requires energy and time consumption for both the uplink and downlink transmission of the task. The energy consumption for CAP processing from user i at CAP j can be expressed as

$$E_{ij}^A = E_{ij}^t + E_{ij}^r, \qquad (4)$$

where E^t_{ij} and E^r_{ij} represent the uplink and downlink transmission energy costs respectively, and are user-specific CAP-specific deterministic functions of $D_{\text{in}}(i), D_{\text{out}}(i)$ and $Y(i)$.

The time requirement depends on the bandwidth and processing rate allocated to the user. Each CAP has its own wireless channel for offloading. The uplink and downlink transmission times for user i at CAP j are respectively

$$T^t_{ij} = \frac{D_{\text{in}}(i)}{\eta^u_{ij} c^u_{ij}}, \quad T^r_{ij} = \frac{D_{\text{out}}(i)}{\eta^d_{ij} c^d_{ij}}, \tag{5}$$

where c^u_{ij} and c^d_{ij} are the assigned uplink and downlink bandwidths respectively, and η^u_{ij} and η^d_{ij} are the spectral efficiencies of the uplink and downlink transmissions. The bandwidths c^u_{ij} and c^d_{ij} are constrained by the bandwidth capacities C^{UL}_j, C^{DL}_j, and C^{Total}_j:

$$\sum_{i=1}^{N} c^u_{ij} \leq C^{\text{UL}}_j, \sum_{i=1}^{N} c^d_{ij} \leq C^{\text{DL}}_j, \quad \forall j \in \mathcal{M}, \tag{6}$$

$$\sum_{i=1}^{N} (c^u_{ij} + c^d_{ij}) \leq C^{\text{Total}}_j, \quad \forall j \in \mathcal{M}. \tag{7}$$

The processing time of task i at CAP j is

$$T^a_{ij} = \frac{Y(i)}{f^a_{ij}}, \tag{8}$$

where f^a_{ij} is the assigned processing rate for the task, constrained by the total processing rate available at the CAP f^A_j:

$$\sum_{i=1}^{N} f^a_{ij} \leq f^A_j, \quad \forall j \in \mathcal{M}. \tag{9}$$

D. Cloud Processing

In cloud processing, uplink/downlink transmission times and energy costs are identical to the CAP processing case. There is however a further transmission between the CAP and the cloud $T^{ac}_{ij} = (D_{\text{in}}(i) + D_{\text{out}}(i))/r_{ac}$, where r_{ac} is a predetermined wired transmission rate between the CAP and the cloud. The computation time is $T^c_i = Y(i)/f^C_i$, where f^C_i is a predetermined processing rate assigned to each user at the cloud. The total cloud processing time is then

$$T^C_{ij} = T^t_{ij} + T^r_{ij} + T^{ac}_{ij} + T^c_i. \tag{10}$$

For energy consumption, we add to the transmission requirements a cloud utility cost C^C_i, weighted for each user by β_i:

$$E^C_{ij} = E^t_{ij} + E^r_{ij} + \beta_i C^C_i. \tag{11}$$

E. User Classes

In practical systems, users often share similar characteristics. For example, in the Internet-of-Things (IoT) environment, many sensors are of the same type. These similar users and devices can be treated similarly in resource assignment. We will see that this can substantially reduce the computational complexity of the optimal CE.

To formalize this, we consider a partition of $\mathcal{N}$ into K classes, $\mathcal{K} = \{\mathcal{K}_1, \ldots, \mathcal{K}_K\}$. Among the users within a class $\mathcal{K}_k$ all parameters $D_{\text{in}}(i), D_{\text{out}}(i)$, $Y(i)$, η^u_{ij}, and η^d_{ij} are random but independent and identically distributed, with known distributions, and all users have the same function for determining T^L_i, E^L_i, E^t_{ij}, and E^r_{ij}, and these functions are known. The system responds accordingly to the users within a class by assigning identical cloud processing rates f^C_i and cloud utility costs and weights C^C_i and β_i, and allocating equal bandwidth and CAP processing rate to each user within the same offloading site, thus ensuring symmetric expected energy and time consumption between the users within each offloading site. The amount of each resource allocated does not vary based on the identity of the users at each site.

Given these assumptions, the users no longer need to know the exact value of their system parameters—knowledge of their class membership and their associated distributions is sufficient for rational decision making. We assume the system knows each user's class membership and the associated distributions, but not individual parameter values.

With the introduction of classes, we can now consider the aggregate of offloading decisions by the users within classes. This is denoted by a $(2M + 1)$-tuple of positive integers,

$$\bar{\mathbf{x}}_k = [\bar{x}_{k0}, \bar{x}_{k1}, \ldots, \bar{x}_{kM}, \bar{x}_{k(M+1)}, \ldots, \bar{x}_{k(2M)}]^T, \tag{12}$$

where

$$\bar{x}_{kj} = \sum_{i \in \mathcal{K}_k} x_{ij}, \quad \forall j \in \{0, \ldots, 2M\}. \tag{13}$$

Individual task parameters can now be expressed as expectations conditional on class:

$$D_\pi(k) = \mathbb{E}[D_\pi(i) | i \in \mathcal{K}_k], \pi \in \{\text{in, out}\}, \tag{14}$$

$$Y(k) = \mathbb{E}[Y(i) | i \in \mathcal{K}_k]. \tag{15}$$

Similarly,

$$\{E^L_k, T^L_k, T^C_k\} = \mathbb{E}[\{E^L_i, T^L_i, T^C_i\} | i \in \mathcal{K}_k], \tag{16}$$

$$E^\pi_{kj} = \mathbb{E}[E^\pi_{ij} | i \in \mathcal{K}_k], \pi \in \{t, r, A, C\}, \tag{17}$$

$$T^\pi_{kj} = \mathbb{E}[T^\pi_{ij} | i \in \mathcal{K}_k], \pi \in \{ac, A, C\}. \tag{18}$$

We express resource allocation in terms of classes as follows, the couplet representing CAP and cloud processing respectively:

$$\begin{aligned} \mathbf{c}^\pi_{kj} &= (c^{\pi^A}_{kj}, c^{\pi^C}_{kj}), \quad \pi \in \{u, d\}, \\ &= \left(\sum_{i \in \mathcal{K}_k} c^\pi_{ij} x_{ij}, \sum_{i \in \mathcal{K}_k} c^\pi_{ij} x_{i(j+M)} \right), \end{aligned} \tag{19}$$

$$f^a_{kj} = \sum_{i \in \mathcal{K}_k} f^a_{ij} x_{ij}. \tag{20}$$

Using the fact that resource allocation must be equal for each user in the class, we re-express terms in (5) and (8) as expectations,

$$\mathbf{T}^\pi_{kj} = (T^{\pi^A}_{kj}, T^{\pi^C}_{kj}), \quad \pi \in \{t, r\}, \tag{21}$$

$$T^{t^A}_{kj} = \frac{D_{\text{in}}(k) \eta^{u*}_{kj} \bar{x}_{kj}}{c^{u^A}_{kj}}, T^{t^C}_{kj} = \frac{D_{\text{in}}(k) \eta^{u*}_{kj} \bar{x}_{k(j+M)}}{c^{u^C}_{kj}}, \tag{22}$$

$$T^{r^A}_{kj} = \frac{D_{\text{out}}(k) \eta^{d*}_{kj} \bar{x}_{kj}}{c^{d^A}_{kj}}, T^{r^C}_{kj} = \frac{D_{\text{out}}(k) \eta^{d*}_{kj} \bar{x}_{k(j+M)}}{c^{d^C}_{kj}}, \tag{23}$$

$$T^a_{kj} = \frac{Y(k) \bar{x}_{kj}}{f^a_{kj}}, \tag{24}$$

where

$$\eta^{\pi*}_{kj} = \mathbb{E}[1/\eta^\pi_{ij} | i \in \mathcal{K}_k], \quad \pi \in \{u, d\}. \tag{25}$$

And we rewrite constraints (6), (7), and (9) as

$$\sum_{k=1}^{K} c^{u^A}_{kj} + c^{u^C}_{kj} \leq C^{\text{UL}}_j, \sum_{k=1}^{K} c^{d^A}_{kj} + c^{d^C}_{kj} \leq C^{\text{DL}}_j, \tag{26}$$

$$\sum_{k=1}^{K} c^{u^A}_{kj} + c^{u^C}_{kj} + c^{d^A}_{kj} + c^{d^C}_{kj} \leq C^{\text{Total}}_j, \tag{27}$$

$$\sum_{k=1}^{K} f_{kj}^{a} \leq f_j^A, \quad \forall j \in \mathcal{M}. \tag{28}$$

The expected processing times T_{kj}^A and T_{kj}^C can now be expressed as

$$T_{kj}^A = T_{kj}^{t^A} + T_{kj}^{r^A} + T_{kj}^{a}, \tag{29}$$

$$T_{kj}^C = T_{kj}^{t^C} + T_{kj}^{r^C} + T_{kj}^{ac} + T_{k}^{c}, \tag{30}$$

and the total expected class energy and time consumption as

$$\bar{E}_k = E_k^L x_{k0} + \sum_{j=1}^{M} E_{kj}^A x_{kj} + \sum_{j=1}^{M} E_{kj}^C x_{k(j+M)}, \tag{31}$$

$$\bar{T}_k = T_k^L x_{k0} + \sum_{j=1}^{M} T_{kj}^A x_{kj} + \sum_{j=1}^{M} T_{kj}^C x_{k(j+M)}. \tag{32}$$

F. Individual and Social Cost Functions

Each user aims to minimize their individual cost, which can be any nondecreasing convex function of energy and time consumption within the system, and is dependent on the offloading decision of the other users.

The energy and time consumption for each user can be expressed as

$$E_i = E_i^L x_{i0} + \sum_{j=1}^{M} E_{ij}^A x_{ij} + \sum_{j=1}^{M} E_{ij}^C x_{i(j+M)}, \tag{33}$$

$$T_i = T_i^L x_{i0} + \sum_{j=1}^{M} T_{ij}^A x_{ij} + \sum_{j=1}^{M} T_{ij}^C x_{i(j+M)}. \tag{34}$$

Our individual cost function can now be expressed as

$$u_i(\mathbf{x}_1, \ldots, \mathbf{x}_N) = f_i(E_1, \ldots, E_N, T_1, \ldots, T_N). \tag{35}$$

While in many cases individual users will only optimize for their own energy/time consumption, our system allows for the more general case.

Given the class-based symmetry between the users within class, we assume that the users do not distinguish between other users within a single class to compute their individual cost (but they may still however distinguish themselves apart from the rest of their class). Furthermore, we assume the users minimize over their expected individual cost given uncertain task parameters. Thus, the cost function u_i can now be expressed in terms of user i's offloading decision $\mathbf{x}_i$, and the class-based offloading assignments $\bar{\mathbf{x}}_1$ to $\bar{\mathbf{x}}_K$. Let $k(i)$ be the class index of user i. Then, with slight abuse of notation, we re-express u_i as follows:

$$\begin{aligned} u_i(\mathbf{x}_1, \ldots, \mathbf{x}_N) &= u_i(\mathbf{x}_i, \bar{\mathbf{x}}_1, \ldots, \bar{\mathbf{x}}_{k(i)} - \mathbf{x}_i, \ldots, \bar{\mathbf{x}}_K) \\ &= f_i(E_i, \bar{E}_1, \ldots, \bar{E}_{k(i)} - E_i, \ldots, \bar{E}_K, \\ &\qquad T_i, \bar{T}_1, \ldots, \bar{T}_{k(i)} - T_i, \ldots, \bar{T}_K). \end{aligned} \tag{36}$$

We note that $\bar{E}_k$'s and $\bar{T}_k$'s are equal for every strategy profile whose $\bar{\mathbf{x}}_k$'s are equal, given the system response to the class-based structure of the users.

The social cost is a weighted linear sum of individual objectives,

$$U(\mathbf{x}_1, \ldots, \mathbf{x}_N) = \sum_{i=1}^{N} w_i u_i, \tag{37}$$

where $w_i > 0$ is the weight for user i.

We aim to design an offloading system that minimizes the social cost. However, the users in a practical system often have agency over their offloading decisions $\mathbf{x}_i$, and they rationally minimize their individual objective functions. Thus, this offloading system can be analyzed as a finite game. This game is not guaranteed to have a PSNE, and the computation of an MSNE is usually too complex, as previously explained. We therefore use the CE to derive a solution to the offloading problem, so that rational users will not deviate from it.

IV. Class-Anonymous Offloading Game

In this section, we first describe how our offloading system can be analyzed as, a *class-anonymous* (CA) game. We then present the CE for this game. Finally, we show that polynomial time and space computation of the optimal CE is possible, by expressing the game in a reduced form.

A. Offloading Game

Our system can be analyzed as a finite game G with structure

$$G = (\mathcal{N}, \{X_i\}_{i \in \mathcal{N}}, \{u_i\}_{i \in \mathcal{N}}), \tag{38}$$

where $\mathcal{N}$ is the player set consisting of N system users, $X_i \subseteq \mathcal{X}$ is the strategy set of each user i, and u_i is their corresponding cost function the user seeks to minimize. The strategy set X_i consists of all possible offloading decisions available to the user—from Section III-A, this can be represented as the set $\{\mathbf{x}_i\}$ subject to (2)-(3). We let $\mathcal{S} = \prod_{i=1}^{N} X_i$ denote the set of strategy profiles. The individual costs u_i is defined in (36).

Because of the class-based structure of the system, offloading game (38) falls within the general category of CA games.

Definition 1 (Class-Anonymous Game)**.** *Consider a finite game G defined in (38). G is a* class-anonymous *game of order K if there exists a partition $\mathcal{K}$ of $\mathcal{N}$ into K classes, $\mathcal{K} = \{\mathcal{K}_1, \ldots, \mathcal{K}_K\}$, such that for any two users m and n in the same class, if user m adopts strategy $\mathbf{x}_m$ and user n $\mathbf{x}_n$, then the cost experience by every other user is the same if m and n switch strategies and no other changes are made.*

We note that the special case of the CA game where $K = 1$ is termed an anonymous game [20]. An optimal CE solution was given in [21] for this case, but it is not directly applicable to a CA game with general K.

B. Correlated Equilibria

Given an arbitrary finite game G defined in (38), a CE is defined as follows [11]:

Definition 2 (Correlated Equilibrium)**.** *Let σ be a distribution on $\mathcal{S}$, and let $s = \{\mathbf{x}_1, \ldots, \mathbf{x}_N\} \in \mathcal{S}$ be a strategy profile drawn from σ. Then, σ is a correlated equilibrium if for all players $i \in \mathcal{N}$, the expected cost of playing strategy $\mathbf{x}_i$ is no larger than that of any other strategy $\mathbf{x}_i' \in X_i$:*

$$\mathbb{E}_{s \sim \sigma}[u_i(\mathbf{x}_i, \mathbf{x}_{-i}) | \mathbf{x}_i] \leq \mathbb{E}_{s \sim \sigma}[u_i(\mathbf{x}_i', \mathbf{x}_{-i}) | \mathbf{x}_i], \quad \forall i \in \mathcal{N}, \mathbf{x}_i, \mathbf{x}_i' \in X_i. \tag{39}$$

Note that the users do not know the strategies of the other users, only a respective conditional probability distribution. The above condition can be expressed as the following set of equations:

$$\sum_{s \in \mathcal{S} | \mathbf{x}_i} P(s)[u_i(\mathbf{x}_i', \mathbf{x}_{-i}) - u_i(\mathbf{x}_i, \mathbf{x}_{-i})] \geq 0, \tag{40}$$

$$\forall i \in \mathcal{N}, \mathbf{x}_i, \mathbf{x}_i' \in X_i,$$

where $P(s)$ denotes the probability of drawing strategy profile s from σ. Note that the set $\{P(s)\}_{s\in\mathcal{S}}$ forms the distribution σ, and that the CE conditions are all linear with respect to $P(s)$. Thus, one means of computing the CE is solving a linear feasibility program over variables $P(s)$ with (40) as constraints. There are $\sum_{i=1}^{N}\binom{|X_i|}{2}$ such constraints, which is polynomial in N and $|X_i|$ [21]. Additionally, the following constraints must be added, since σ must be a distribution:

$$P(s) \geq 0, \quad \forall s \in \mathcal{S}, \tag{41}$$

$$\sum_{s\in\mathcal{S}} P(s) = 1. \tag{42}$$

Furthermore, from [11] we know that *every finite game has a CE*. Thus, a solution to the feasibility program is guaranteed to exist.

Since any distribution over $\mathcal{S}$ that satisfies (40)-(42) is a CE, we can find an optimal CE by minimizing in expectation a social cost function U over the constraints. This CE is more easily computed than an MSNE, and satisfies rationality conditions for selfish users, as the users have no incentive to deviate from their assigned strategy so long as they expect other users to do the same. We remark that the CE relies on an assumption that the users do not know the strategies of other users and thus cannot respond accordingly, unlike in an MSNE. However, the users in a mobile offloading network are unlikely to have or be able to obtain such information, rendering the CE sufficiently stable.

C. Reduced Form of the Class-Anonymous Game

Even though (40) gives a polynomial number of constraints in N and $|X_i|$, we still have $|\mathcal{S}| = O(|\mathcal{X}|^N)$, which implies an exponential time and space requirement to compute and store $P(s)$. Thus, we need a means to represent the strategy space of the game in a reduced form, despite the size of its normal form. In the following, we adopt the reduced form definition from [21] to show that the optimal CE for the proposed CA offloading game can be found in polynomial time and space complexity.

Definition 3 (p-equivalence)**.** *Consider a finite game G defined in (38). Let $Q_i = \{q_i^1, \ldots, q_i^{r_i}\}, i \in \mathcal{N}$ be a partition of $\mathcal{S}_{-i}$ into r_i sets. For a player i, two strategy profiles $(\mathbf{x}_i, \mathbf{x}_{-i})$ and $(\mathbf{x}_i', \mathbf{x}_{-i}') \in \mathcal{S}$ are p-equivalent if $\mathbf{x}_i = \mathbf{x}_i'$ and $\mathbf{x}_{-i}$ and $\mathbf{x}_{-i}'$ belong to the same partition set in Q_i.*

Definition 4 (Reduced Form)**.** *The set of partitions $\mathcal{Q} = \bigcup_{i=1}^{N} Q_i$ is a reduced form of G of size $|\mathcal{Q}|$ if $u_i(\mathbf{x}_i, \mathbf{x}_{-i}) = u_i(\mathbf{x}_i', \mathbf{x}_{-i}')$ for any p-equivalent pair of $(\mathbf{x}_i, \mathbf{x}_{-i})$ and $(\mathbf{x}_i', \mathbf{x}_{-i}')$.*

With a reduced form of a game, the following theorem from [21] provides sufficient conditions for polynomial time computation of an optimal CE:

Definition 5 (Separation Problem)**.** *Let $\mathcal{Q}$ be the reduced form of a game. The separation problem for $\mathcal{Q}$ is as follows: given a mapping $y_i : (\mathbf{x}, j) \to \mathbb{Q}, \forall i \in \mathcal{N}, \mathbf{x} \in X_i, j \in \{1, \ldots, r_i\}$, decide whether there exists a strategy profile $s \in \mathcal{S}$ such that*

$$\sum_{(i,\mathbf{x},j):s\in\mathcal{S}_i(\mathbf{x},j)} y_i(\mathbf{x}, j) < 0, \tag{43}$$

where $\mathcal{S}_i(\mathbf{x}, j)$ is the set of all strategy profiles $(\mathbf{x}_i, \mathbf{x}_{-i}) \in \mathcal{S}$ such that $\mathbf{x}_i = \mathbf{x}$ and $\mathbf{x}_{-i} \in q_i^j$.

Theorem 1. *Let $\mathcal{Q}$ be the reduced form of a game. If the separation problem for $\mathcal{Q}$ has a polynomial time solution, a CE that optimizes for a linear combination of individual player cost (i.e. the optimal CE) can be computed in polynomial time in the size of $\mathcal{Q}$ [21].*

We now derive the reduced form of the CA game. Let $\mathbf{x}_{-i} \in \mathcal{S}_{-i} = \prod_{i'\neq i} X_{i'}$. Let $\bar{\mathbf{X}}_{-i}$ represent the matrix form of $(\bar{\mathbf{x}}_1, \ldots, \bar{\mathbf{x}}_{k(i)} - \mathbf{x}_i, \ldots, \bar{\mathbf{x}}_K)$. Let r_i be the total number of possible $\bar{\mathbf{X}}_{-i}$'s. Let $Q_i = \{q_i^1, \ldots, q_i^{r_i}\}$ be a partition of all possible $\mathbf{x}_{-i}$'s according to their associated $\bar{\mathbf{X}}_{-i}$ class assignments. From the form of the individual cost function in (36), any two p-equivalent strategy profiles will experience the same individual cost u_i. Thus, from Definition 4, $\mathcal{Q}_K = \bigcup_{i=1}^{N} Q_i$ is a reduced form of G. The individual cost function can now be expressed in terms of the reduced form, again with slight abuse of notation,

$$u_i(\mathbf{x}_i, \mathbf{x}_{-i}) = u_i(\mathbf{x}_i, j) = f_i(E_i, T_i, \mathbf{E}_{-i}(j), \mathbf{T}_{-i}(j)), \tag{44}$$

where j indicates the set $q_i^j \in Q_i$ that $\mathbf{x}_{-i}$ belongs to, and $\mathbf{E}_{-i}(j)$ and $\mathbf{T}_{-i}(j)$ represent their associated energy and time costs.

Lemma 1. *The size of $\mathcal{Q}_K$, $|\mathcal{Q}_K| = O(N(N+X)^{KX})$, where $X = |\mathcal{X}|$, which is polynomial with respect to N.*

This result can be obtained through the fact that $|\mathcal{Q}_K| = \sum_{i=1}^{N} r_i$, and that r_i is equal to the number of distinct assignments of the $|\mathcal{K}_k|$ players in class k into the X strategies, which equals $O((N+X)^{KX})$. The detailed proof is omitted due to space limitation.

Using these properties, we can now prove the following:

Theorem 2. *The separation problem (see Definition 5) for $\mathcal{Q}_K$ has a polynomial time solution.*

This solution uses the class-based assignments q_i^j to construct a graph of strategy profiles, using $y_i(\mathbf{x}, j)$ as edge weights. The separation problem can then be shown to be equivalent to a minimum cost flow problem which admits a polynomial time solution. The detailed proof is omitted due to space limitation.

From this result, the following is a direct consequence of Theorem 1:

Theorem 3. *Class-anonymous games of order K have a polynomial time and space solution for the optimal CE.*

V. Optimal Correlated Equilibrium Solution

In this section, we show that the optimal CE of a CA game can be computed by linear programming. We use those results to obtained the associated offloading decisions for our MEC system.

To find the optimal CE, we must optimize the social cost in (37) in expectation over a distribution over $\mathcal{S}$, and represent that distribution over the reduced form of the game to ensure polynomial time and space complexity. Consider the objective function

$$\sum_{i=1}^{N} w_i \left(\sum_{j=1}^{r_i} \sum_{\mathbf{x}\in\mathcal{X}} P_i(\mathbf{x}, j) u_i(\mathbf{x}, j) \right), \tag{45}$$

where the objective variables $P_i(\mathbf{x}, j)$ form a distribution over $\mathcal{S}_i(\mathbf{x}, j)$ (defined in Definition 5), for each user $i \in \mathcal{N}$. We presume $u_i(\mathbf{x}, j)$ is computed in advance $\forall i \in \mathcal{N}, \mathbf{x} \in \mathcal{X}_i, j \in \{1, \ldots, r_i\}$. Given the size of r_i (see Lemma 1), there are polynomial number of such calculations, and an equal number of objective variables $P_i(\mathbf{x}, j)$, implying a polynomial space representation.

The constraints (40)-(42) directly extend as follows:

$$\sum_{j=1}^{r_i} P_i(\mathbf{x}, j)[u_i(\mathbf{x}', j) - u_i(\mathbf{x}, j)] \geq 0, \quad (46)$$
$$\forall i \in \mathcal{N}, \mathbf{x}, \mathbf{x}' \in \mathcal{X}$$
$$P_i(\mathbf{x}, j) \geq 0, \quad \forall \mathbf{x} \in \mathcal{X}, j \in \{1, \ldots, r_i\} \quad (47)$$
$$\sum_{j=1}^{r_i} \sum_{\mathbf{x} \in \mathcal{X}} P_i(\mathbf{x}, j) = 1, \quad \forall i \in \mathcal{N}. \quad (48)$$

While $\{P_i\}$ is not itself a CE (which is a distribution over $\mathcal{S}$), it may extend to a CE as follows [21]:

Lemma 2. *Let $\sigma = \{P(s)\}_{s \in \mathcal{S}}$ be a CE of G. Then, if*

$$P_i(\mathbf{x}, j) = \sum_{s \in \mathcal{S}_i(\mathbf{x}, j)} P(s), \quad \forall i, \mathbf{x}, j, \quad (49)$$

then $\{P_i\}_{i \in \mathcal{N}}$ satisfies constraints (46)-(48) and the objective function (45) is equal to

$$\sum_{i=1}^{N} w_i \sum_{s \in \mathcal{S}} \bar{u}_i(s) P(s), \quad (50)$$

where $\bar{u}_i(s) = u_i(\mathbf{x}, j)$ such that $s \in \mathcal{S}_i(\mathbf{x}, j)$.

Note that (50) is the expected value of social cost of the CE as defined in (37).

Definition 6. *Let σ' be a distribution on $\mathcal{S}$. Then, $\{P_i\}_{i \in \mathcal{N}}$ extends to σ' if $\{P_i\}$ and σ' satisfies (49). σ' would then be an extension of $\{P_i\}$.*

Lemma 3. *If $\{P_i\}$ extends to some distribution σ' on $\mathcal{S}$ and satisfies (46)-(48), then σ' is a CE with expected social cost equal to (50).*

However, as demonstrated by [21], a feasible solution to the LP above does not necessarily extend to a distribution on $\mathcal{S}$—the above problem is thus a relaxed version of the optimal CE problem. We therefore need additional constraints to ensure a CE solution.

Let the set of $X \times K$ matrices $\Phi = \{\bar{\mathbf{X}} = [\bar{\mathbf{x}}_\mathbf{1}, \ldots, \bar{\mathbf{x}}_\mathbf{K}]\}$ represent the set of all possible class-based assignments of the users in $\mathcal{X}$. The size of Φ is $O((N+X)^{KX})$ like r_i, for analogous reasons. Note that $\bar{\mathbf{X}}_{\mathbf{x},k}$ equals the total number of users in class k adopting strategy $\mathbf{x}$. Let the binary $X \times K$ matrix $\mathbf{Z}^{\mathbf{x},k}$ be defined such that

$$\mathbf{Z}^{\mathbf{x},k}_{\mathbf{m},n} = \begin{cases} 1, & \mathbf{m} = \mathbf{x} \text{ and } n = k \\ 0, & \text{otherwise} \end{cases}.$$

Note that if $\bar{\mathbf{X}}_{\mathbf{x},k} > 0$, then $\bar{\mathbf{X}} - \mathbf{Z}^{\mathbf{x},k}$ corresponds to a possible $\bar{\mathbf{X}}_{-i}, \forall i \in \mathcal{K}_k$. Let $J_i(\bar{\mathbf{X}} - \mathbf{Z}^{\mathbf{x},k}) = j$ such that the class-based assignments in q_i^j correspond to the assignments in $\bar{\mathbf{X}}_{-i} = \bar{\mathbf{X}} - \mathbf{Z}^{\mathbf{x},k}$

We add a polynomial number of decision variables $p(\bar{\mathbf{X}})$, representing a distribution over $\bar{\mathbf{X}} \in \Phi$. We now add the following constraints to our LP:

$$p(\bar{\mathbf{X}}) \geq 0, \quad \forall \bar{\mathbf{X}} \in \Phi, \quad (51)$$
$$\sum_{\mathbf{x}: \bar{\mathbf{X}}_{\mathbf{x},k(i)} > 0} P_i(\mathbf{x}, J_i(\bar{\mathbf{X}} - \mathbf{Z}^{\mathbf{x},k(i)})) = p(\bar{\mathbf{X}}), \quad (52)$$
$$\forall i \in \mathcal{N}, \bar{\mathbf{X}} \in \Phi,$$
$$\sum_{i \in \mathcal{K}_k} P_i(\mathbf{x}, J_i(\bar{\mathbf{X}} - \mathbf{Z}^{\mathbf{x},k})) = \bar{\mathbf{X}}_{\mathbf{x},k} p(\bar{\mathbf{X}}), \quad (53)$$
$$\forall \mathcal{K}_k \in \mathcal{K}, \bar{\mathbf{X}} \in \Phi, \mathbf{x} \in \mathcal{X},$$
$$\sum_{\bar{\mathbf{X}} \in \Phi} p(\bar{\mathbf{X}}) = 1. \quad (54)$$

Finally, we add the placement constraints:

$$P_i(\mathbf{x}, j) = 0, \quad \forall i \in \mathcal{N}, \mathbf{x} \in \mathcal{L}_i. \quad (55)$$

The linear program for computing the CE is now as follows:

$$\min_{P_i, p} \sum_{i=1}^{N} w_i \left(\sum_{j=1}^{r_i} \sum_{\mathbf{x} \in \mathcal{X}} P_i(\mathbf{x}, j) u_i(\mathbf{x}, j) \right), \quad (56)$$

s.t. (46)-(47), (51)-(55) ((48) is now redundant).

The following lemma suggests that no optimality is lost in considering problem (56) to find the optimal CE.:

Lemma 4. *Let $\sigma = \{P(s)\}_{s \in \mathcal{S}}$ be a CE of G. Let $\mathcal{S}(\bar{\mathbf{X}})$ be the set of all strategy profiles in $\mathcal{S}$ where the users are assigned according to $\bar{\mathbf{X}}$. Then, if (49) is satisfied, and*

$$p(\bar{\mathbf{X}}) = \sum_{s \in \mathcal{S}(\bar{\mathbf{X}})} P(s), \quad (57)$$

then $\{P_i\}$ and $\{p\}$ satisfy constraints (51)-(54)—thus, all CE are within the feasible set of (56).

Proof. Since $\{\mathcal{S}(\bar{\mathbf{X}})\}_{\bar{\mathbf{X}} \in \Phi}$ is by definition a partition of $\mathcal{S}$, (51) and (54) hold from the fact that $P(s)$ forms a distribution over $\mathcal{S}$. (52) and (53) are both direct results from the law of total probability. □

From this we now show how a CE can be achieved from the distributions $\{p\}$ and $\{P_i\}$.

Theorem 4. *For any class-anonymous game of order K, a feasible solution to (56) extends to a CE σ, and a strategy profile from σ can be sampled in polynomial time.*

Proof. To show this result, we develop an polynomial time algorithm that produces randomly a strategy profile $s \in \mathcal{S}$, whose resultant distribution $\sigma = \{P(s)\}$ satisfies the results in (56)— that is:

$$\mathbf{Pr}[s \in \mathcal{S}(\bar{\mathbf{X}})] = \sum_{s \in \mathcal{S}} P(s) = p(\bar{\mathbf{X}}), \quad \forall \bar{\mathbf{X}} \in \Phi, \quad (58)$$
$$\mathbf{Pr}[s \in \mathcal{S}_i(\mathbf{x}, j)] = \sum_{s \in \mathcal{S}_i(\mathbf{x}, j)} P(s) = P_i(\mathbf{x}, j), \quad (59)$$
$$\forall i \in \mathcal{N}, \mathbf{x} \in X_i, j \in \{1, \ldots, r_i\}.$$

The implicit distribution that results from this algorithm, σ, satisfies (49) and thus is an extension of $\{P_i\}$ by Definition 6. Since $\{P_i\}$ satisfies (46)-(48), Lemma 3 ensures that σ is a CE, which must be optimal as per Lemma 2.

To generate a CE with such properties, we first sample $\bar{\mathbf{X}}$ from Φ via the distribution $\{p(\bar{\mathbf{X}})\}$. From this, we construct K complete bipartite graphs $(V_1, V_2, E)_k$, one for each class in $\mathcal{K}$, where V_1 consists of one vertex for each user i in class

$\mathcal{K}_k$, and V_2 contains $\bar{\mathbf{X}}_{\mathbf{x},k}$ vertices for each strategy $\mathbf{x}$ in $\mathcal{X}$. We assign to each edge (V_1, V_2) a weight corresponding to their respective user i and strategy $\mathbf{x}$, generating a fractional perfect matching:

$$h_k(i,\mathbf{x}) = \frac{P_i(\mathbf{x}, J_i(\bar{\mathbf{X}} - \mathbf{Z}^{\mathbf{x},k}))}{\bar{\mathbf{X}}_{\mathbf{x},k}p(\bar{\mathbf{X}})}. \tag{60}$$

From (52), the sum of the weights incident to any user i is 1. Similarly, from (53), the sum of the weights for all vertices in V_2 is 1. Thus, the weights of the graph can be expressed as a doubly stochastic matrix, which under Birkhoff's theorem, can be decomposed into a convex combination of $O(V^2)$ perfect matchings in polynomial time [24]. This decomposition then represents a probability distribution over the perfect matchings, each of which is a possible assignment of the users in the class to the available strategies. When one such matching is sampled over all classes, the result is a strategy profile in $\mathcal{S}$.

The resultant implicit distribution σ clearly satisfies (58) from the initial sampling. To show that (59) is satisfied, we note that the solution method results in the following probability of drawing a strategy profile $s \in \mathcal{S}_i(\mathbf{x}, j)$:

$$\mathbf{Pr}[s \in \mathcal{S}_i(\mathbf{x},j)] = \mathbf{Pr}[s \in \bar{\mathbf{X}}^*]\bar{\mathbf{X}}^*_{\mathbf{x},k}h_k(i,\mathbf{x}), \tag{61}$$

$$= p(\bar{\mathbf{X}}^*)\bar{\mathbf{X}}^*_{\mathbf{x},k}\frac{P_i(\mathbf{x},j)}{\bar{\mathbf{X}}^*_{\mathbf{x},k}p(\bar{\mathbf{X}}^*)}, \tag{62}$$

$$= P_i(\mathbf{x},j), \quad \forall i \in \mathcal{N}, j \in \{1,\ldots,r_i\}, \mathbf{x} \in \mathcal{X}, \tag{63}$$

where $\bar{\mathbf{X}}^*$ is the single value of $\bar{\mathbf{X}} \in \Phi$ such that $\mathcal{S}(\bar{\mathbf{X}}) \cap \mathcal{S}_i(\mathbf{x},j) \neq \emptyset$. □

Following from these results, our solution method for computing the optimal CE is as follows:

1) A central controller in the system obtains the cost function for each user in the system, and determines a respective weighting factor for the social objective. Class information is also obtained, either directly from the users or inferred through a clustering method.
2) The controller determines the distributions of all partitions in the reduced form $\mathcal{Q}$ of the system. Using that, it determine the resource allocation to compute all individual utilities $u_i(\mathbf{x},j), \mathbf{x} \in X_i$, for the users along with the corresponding resource allocation values.
3) The controller solves the LP (56), and uses the result to select probabilistically a single strategy profile using the bipartite graph matching described under Theorem 4. This is sent to the users, who then offload their task based on their given assignment, and by the properties of the CE, have no incentive to deviate from their assignment.

Regarding the resource allocation, we note that any resource allocation scheme is allowed so long as the CA property of the system is maintained. This is the case as long as the resource allocation values are invariant for any set of strategy profiles with identical class-based user assignments:

$$\mathbf{c}^{\pi}_{kj} = \mathbf{c}^{\pi}_{kj}(\bar{\mathbf{X}}), f^a_{kj} = f^a_{kj}(\bar{\mathbf{X}}), \quad \pi \in \{u,d\}. \tag{64}$$

As long as a resource allocation scheme satisfies this condition, our solution method produces an optimal CE of the system. For our simulations, we use an optimization method to compute the resource allocation similar to that in [6], but modified to maintain the CA property. The detailed approach is omitted due to space limitation.

VI. Simulation Results

In this section, we evaluate the performance of the proposed optimal CE via simulation using real-world system parameters. We present performance comparison with two alternatives: *Random Mapping*, where offloading decisions are chosen randomly for each user; and *Learned CE*, where a non-optimal CE is computed using the regret-based learning method detailed in [25].

We utilize the x264 CBR encoding application, which requires 1900 cycles/byte [26]. We set by default the number of users per class to 7, number of CAPs to 2, $\beta = 1.7 \times 10^{-7}$ J/bit. The bandwidths at each CAP are $C^j_{\text{UL}} = C^j_{\text{DL}} = 20$ MHz, and $C^j_{\text{Total}} = 40MHz$ for each CAP. We use an iPhone X mobile device with a CPU speed of 2.39×10^9 cycles/s, leading to a local computation time of 9.93×10^{-8} s/bit [27], and adopt a CPU rate of 5×10^9 cycles/s at the CAPs, and 7.5×10^9 cycles/s at the cloud. The transmission and receiving energy per bit at each mobile device are both 1.42×10^{-7} J/bit as indicated in Table 2 in [26]. For offloading a task to the cloud, the transmission rate is $R_{ac} = 15$ Mpbs. Also, we set the cloud utility cost C_{c_i} to be the same as that of the input data size $D_{\text{in}}(i)$. In each class, the input and output data are normally distributed with a specified mean and variance. Spectral efficiencies are uniformly distributed with fixed minimum and maximum values. We scaled the cloud utility cost C_{c_i} to the expected input data size $D_{\text{in}}(i)$. These parameters are all listed for each class in Table I.

We consider a system of three classes, with seven users per class, and two CAPs by default. Each user shares the same individual cost function, defined as a weighted sum of energy and time consumption:

$$u_i(\mathbf{x}_1,\ldots,\mathbf{x}_N) = E_i + \alpha T_i, \tag{65}$$

where α has unit J/s, so that u_i is measured in the unit of J. We set $\alpha = 0.5$ J/s by default. Our social cost function is a simple sum of all user cost, with every user's weight $w_i = 1$.

Tables II, III, and IV show the expected social cost, averaged over 40 trials, sweeping through the number of users per class (from 6 to 8), α (from 0.1 to 0.9), and the number of CAPs (1 to 3) respectively. In each of these tables, the proposed optimal CE achieves the best results, followed by the non-optimal CE, and finally by random mapping. Tables II and III show an approximately linearly increasing relationship between the expected cost and the number of users and α, both of which are to be expected from the system model. Table IV shows a decreasing relationship between the expected cost and the number of CAPs, which is due to the availability of additional offloading sites.

Figure 1 shows the costs experience for each particular class in the system against the number of users. We observe that Class A has the highest cost, followed by Class B and Class C. This is in accordance with Class A having the largest data size and lowest spectral efficiency, Class B with improved spectral efficiency, and Class C a smaller data size.

TABLE I: Class Parameter Values

Parameter	Class A	Class B	Class C
$\mathbb{E}[D_{\text{in}}(i)]$	48 MB	48 MB	24 MB
$\text{Var}[D_{\text{in}}(i)]$	6×10^7	6×10^7	3×10^7
$\mathbb{E}[D_{\text{out}}(i)]$	6 MB	6 MB	3 MB
$\text{Var}[D_{\text{out}}(i)]$	1×10^7	1×10^7	0.5×10^7
Min η_{ij}^u, η_{ij}^d (b/s/Hz)	1	2	2
Max η_{ij}^u, η_{ij}^d (b/s/Hz)	2	4	4

TABLE II: Cost against number of users per class

	Number of Users		
Expected Cost (J)	6	7	8
Optimal CE	436	502	571
Learned CE	453	533	590
Random Mapping	536	598	659

TABLE III: Cost against α

	α (J/s)				
Expected Cost (J)	0.1	0.3	0.5	0.7	0.9
Optimal CE	375	427	488	544	614
Learned CE	422	479	547	605	671
Random Mapping	478	542	619	699	780

TABLE IV: Cost against number of CAPs

	Number of CAPs		
Expected Cost (J)	1	2	3
Optimal CE	645	503	379
Learned CE	668	536	454
Random Mapping	794	608	553

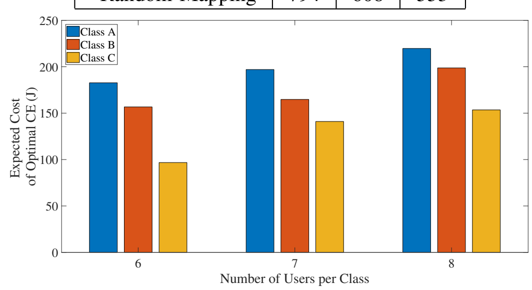

Fig. 1: Class costs against number of users

VII. Conclusion

In this work, we study the CE for a three-tier mobile offloading network. Unlike the NE, the CE is easily computable through linear programming for all systems with a finite offloading space, and does not require the existence of a potential function for its computation. This enables a more convenient solution for these systems, as opposed to the restrictive application of previous works. In the class-anonymous setting where user parameters are clustered, we propose a method to compute the optimal CE in polynomial time with respect to the number of users. Simulation results further demonstrate the feasibility of our solution method, and its performance advantage in comparison with common alternatives. There remain several open problems for future research, such as the price of anarchy and strategy-proofness. It will also be interesting to further explore computation techniques to find the CE in other forms of games relevant to MEC systems, such as graphical games.

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