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A new strategy for the selection of communication technologies in VANETs with fully controllable vehicles

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***Abstract*—Vehicular communications are laying the foundations for new research areas. Embedded systems within vehicles allow the management of information in movement situations. This article focuses on the choice of the communication technology used by the vehicles with controllable trajectories. The objective is to maximize the throughput, in the centralized way, provided by these vehicles. The study area is a map divided into zones. The choice of communication technologies depends on the crossing zone of the map. In the studied scenario, the vehicle paths and their communication technologies are defined according to this environment. The number of communication technologies usable in the same zone, c, may be restricted. The problem is first formulated as an optimization problem. The complexity of this problem is then proven as $\mathcal{NP}-hard$ and there is no possibility of constant factor approximation algorithm generally. Assuming $c \geq k$, with k the number of controllable vehicles, the problem remains $\mathcal{NP}-hard$, but a new polynomial time α-approximation algorithm is analyzed. The variable α is equal to $(1-\frac{1}{e})$ with e being the base of the natural logarithm. This ratio is the best possible, unless $\mathcal{NP} \subseteq \mathcal{DTIME}(n^{loglog(n)})$.**

***Index Terms*—VANETs, Optimization, Polynomial Algorithm, Graph Theory, Approximation**

I. Introduction

Technological advances have led to the emergence of vehicle networks, a fundamental part of tomorrow's Smart Cities. Cooperative Intelligent Transport Systems (C-ITS) rely on vehicular communication for increasing road safety and efficiency [1]. Services provided by the vehicles in [2] have been grouped in three parts: safety with pre-crash alerts, traffic efficiency with speed management and infotainment with internet access.

In vehicular networks, the moving of vehicles can be predictable and even fully controllable. Vehicles can also communicate with an infrastructure (V2I) and each other (V2V). This background is completed by a recent concept of HETVNET (HETeregeneous Vehicular NETwork), based on the ability of vehicles to embed multiple Radio Access Technologies (RAT) [3].

Numerous communication technologies can be used within the framework of VANETs [4]. Their effectiveness varies according to the scenarios encountered. The communication technology DSRC (Dedicated Short Range Communication) is based on the IEEE 802.11p standard [5] and allows vehicle-to-vehicle communications. Although specifically designed to enable vehicular communications with mobility, DSRC is not optimal for all scenarios. Besides, current cellular communication technologies such as the fourth generation (4G) with the Long-Term Evolution (LTE) offer high data rates on the up and down links with a low latency. Many manufacturers are now favoring the joint use of DSRC short-range, LTE and LTE-Advanced devices for long ranges as detailed in [6].

Vehicles may periodically exchange messages containing information on its position, speed, acceleration and direction [7]. Emergency messages are broadcast by a source vehicle to alert the others. Infotainment services allow information to be transmitted to a customer. The drastic increase in the amount of information exchanged, as well as the diversity of communication technologies, is laying the groundwork for new problems. Our solution maximizes the ability of vehicles to exchange, relay and disseminate information in a region, such as along a road, a vehicle can use several communication technologies. For example, this system could disseminate essential services, such as health instructions, through fully controllable vehicles.

A vehicle travelling from a point of departure to a point of arrival can therefore select the communication technologies used and the road path taken. The strategy of our paper focuses on throughput maximization for one or more vehicles, with a limitation of the total latency associated to the change of communication technology. To the best of our knowledge, the problem has not yet been addressed in the literature. The proposed strategy models the studied areas in the form of a map. This map is meshed in such a manner that the communications technologies chosen vary according to the areas crossed. Thanks to reductions using variants of the maximum coverage problem, our problem is analyzed as $\mathcal{NP}-hard$ and there is no possibility of constant factor approximation algorithm. However, in the special case where $c \geq k$, with c the number of usable technologies for a same zone and k the number of fully controllable vehicles, our problem is still proven as $\mathcal{NP}-hard$ but an approximation algorithm in polynomial time is proposed, with a performance ratio of $1-\frac{1}{e}$. This ratio

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is the best possible, unless $\mathcal{NP} \subseteq \mathcal{DTIME}(n^{loglog(n)})$.

To answer the question *"How to optimize the use of k fully controllable vehicles in a defined area ?"*, the paper is organized as follows. Related works are analyzed in Section II. The framework of the problem is formalized in Section III, enriched by the classification of this problem as $\mathcal{NP}-hard$. Finally, the polynomial time approximation algorithm is described in Section IV.

II. Related Work

Several works have been focused on the multiple communication technologies available for VANETs. The approach of this paper uses the grid of an area to know which technology should be allowed according to the place, in a context where vehicles are fully controllable. This approach has, to our knowledge, never been realized before.

Previous strategies ([8]–[10]) have focused only on the choice of communication technology. For example, the strategy illustrated in [8] chooses the communication technology according to the best throughput. This idea was improved by taking into account the mobility of vehicles [9]. Finally, the approach detailed in [10] uses various information to motivate the selection of a communication technology according to an integrated probabilistic model. Although these models adapt to the situation in real time, they suffer from the lack of a global vision. These strategies cannot predict in advance which communication technologies to use and every new calculation to choose technologies costs time.

Another approach detailed in [11] divides a geographic area by streets. The objective of delivery planning service is to connect the nodes through a tree. The shortest path tree does not optimize the use of bandwidth. The min tree Steiner used through the constrained Steiner tree offers an approach favoring the flow provided and respecting time constraints. A Steiner tree is a minimum coverage tree that allows new nodes to be added. The min relay intersection tree used enriches this approach thanks to an increased adaptation to the area concerned. The idea presented in [12] focuses on the minimization of interference which are particularly related to the geographic areas. However, in order to adapt to the individual demands, maximizing throughput instead of minimizing interference promotes a large choice of services. Similarly, the use of a grid instead of streets allows a more adaptive approach in time and space.

Previous articles have focused on maximizing network coverage. For example, the use of buses for information dissemination is discussed in [13]. In order to be adapted to areas without public transportation, article [14] proposes a system using vehicles to relay information.

Article [15] enriches these scenarios with several types of relays: static, predictable mobile and unpredictable mobile. The placement or movement of these relays is subject to a budget limit and the objective is to maximize efficiency. This efficiency is represented by a weight associated with each possible location. This representation is adapted to systems with a single communication technology.

Our work is an extension of the article [15], using several communication technologies. The scope of the study includes vehicles whose route is completely controllable. However, there are still many differences from this article. For example, the weighting system represents a concrete value with the throughput provided by one technology or a new abstraction system including several communication technologies.

III. Framework presentation

The aim of the studied problem is the maximization of throughput with a limit on both the total duration of the procedure and the number of communication technologies used simultaneously in the same area. Choices are made through the selection of communication technologies along a road path.

To offer a solution, it is necessary to transform this problem into an abstract model. This section therefore details the transformation of the problem into a graph problem. Part III-A details the choices of the modelling when transforming this real problem into a graph. Part III-B presents the creation of the graph in detail. Part III-C formally represents the problem. Finally, in the area of algorithmic complexity, this problem has been proven $\mathcal{NP}-hard$ in the part III-D. Moreover, there is no possibility of constant factor approximation algorithm.

A. Choice of modelling

Whether in model representation, or analysis in the complexity domain, many variables appear along this article. The main ones are summarized in the Table I.

TABLE I
List of variables used

Variables	Representation
$c_{z,j}$	Number of usable communication technologies per zone z and time j
c	Number of usable communications for all areas
k	Number of controllable vehicles
P	Set of (D, A)-Paths
Υ	Set of available technologies
Υ_i	A communication technology with $\Upsilon_i \in \Upsilon$
h_i	Hysteresis relating to the use of communication technology Υ_i
r_i	Range of technology Υ_i
g_i	Value of the side of the square to the grid relating to Υ_i
$w_e^{\Upsilon_i}$	Weight of the edge e with the technology Υ_i
$v_{z,j}^{\Upsilon_i}$	Point of zone z with the technology Υ_i at time j
$w_{z,j}^{\Upsilon_i}$	Weight of the outgoing edges of zone z with the technology Υ_i at time j

The modelling carried out is based on several representations. Let Υ the set of available communication technologies and each one, $\Upsilon_i \in \Upsilon$, has a communication range, denoted r_i. The variable $c_{z,j}$ represents a local restriction related to the number of communication technologies usable simultaneously in the area z at time j. The greatest lower bound of all local restrictions denoted $c = min\{c_{z,j}\}$ is a general constraint respecting all local restrictions.

Concerning the vehicles, k are fully controllable, with D as the departure point and A as the arrival point. It is possible to choose the path used by a vehicle before its departure. This idea benefits from devices embedded in a vehicle. For example, the fact that the path of a vehicle can be fully known in advance is based on the use autonomous cars as illustrated in [16] or the use of a GPS. This hypothesis appears regularly in research articles as [17].

Concerning the temporal aspect, when a vehicle switches to another communication technology a lag time appears, called hysteresis. This one is specific to the communication technology Υ_i and is a discrete value denoted $h_i \in \mathbb{N}$. The problem is studied for a time $\mathcal{T}$ and then divided into T intervals. Changing communication technologies result in a shift from one interval to another. In this paper, temporal graphs enrich the idea of evolving graphs presented in [18] by adding the hysteresis.

Finally, in the chosen model, each e edge is weighted by a $w_e^{\Upsilon_i}$ weight according to the Υ_i communication technology chosen. Previous work, notably with [8], allows to estimate this throughput in prevision, just as the range is known can be predicted. This weight $w_e^{\Upsilon_i}$ is the maximum flow that can be transferred through the e edge through the Υ_i communication technology.

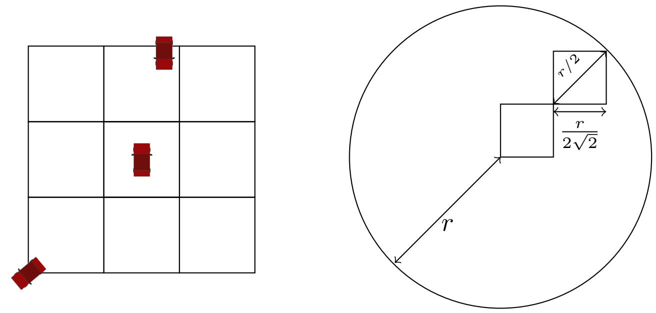
B. Graph representation

In order to take into account several communication technologies, the graph is constructed from several regular mesh grids. Indeed, a grid is associated with only one communication technology. Then, these grids are overlapped. Each area is represented by a central point. So, in a $G = (V, \mathcal{E}, E)$ graph, V is the set of central points of the grid, $\mathcal{E}$ an usable route and E represents the possible communications so that two nodes have at most $|\Upsilon|$ adjacent edges. These steps are detailed below.

The map M is divided into grid patterns. This grid is defined as follows. Let $\Upsilon = \{\Upsilon_1, \Upsilon_2, ..., \Upsilon_{|\Upsilon|}\}$ be the set of communication technologies of range r_1 for Υ_1, r_2 for Υ_2 and so on. The range of communication technologies allow to draw a regular mesh grid composed of g-sided squares, i.e. g_1 for Υ_1, g_2 for Υ_2 and so on. Variable g is defined to allow communication between a vehicle belonging to a central zone and any vehicle located in one of its eight neighboring zones as illustrated in Figure 1a.

The size of the squares constituting the grid is calculated to allow communication between any adjacent squares. In the worst case, the distance between two distant vehicles is r. So there are two squares with a $r/2$ diagonal and a side size of $\frac{r}{2\sqrt{2}}$, as illustrated in Figure 1b.

From a starting map M, like the one in Figure 2a, a partition is created according to a first communication technology. Figure 2a illustrates this first partition, as a function of Υ_1, side g_1.



(a) Any vehicle in a square can communicate with any vehicle in the adjacent squares.

(b) Calculation of the side of the square according to the r range for a given communication technology.

Fig. 1. Creation of a grid for a single communication technology.

Figure 2b illustrates another partition of the M map, as a function of Υ_2, side g_2. In theses figures, D represents the departure and A, the arrival.

Then, a juxtaposition of the squares from the different communication technologies is made. Figure 2c illustrates the juxtaposition of the two grids formed by the technologies Υ_1 and Υ_2.

In order to create a graph, the nodes are defined as the centers of each overlapped regular mesh grid. The figure 2d represents the set of nodes corresponding to the M map with two communication technologies.

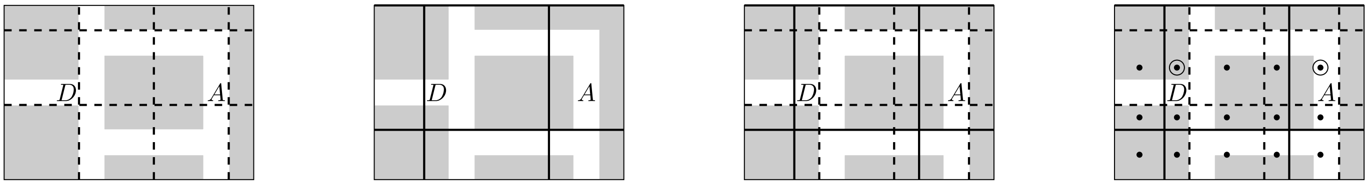
In this new map, each point is able to exchange with at least all its direct neighbors using the communication technology with the shortest range. Other communications are possible with distant neighbors using other communication technologies. An edge symbolises the possibility of communication.

From there, a graph G is created with V let be the set of central points of the grid and E, the set of edges such as $E = \cup_{\Upsilon_i \in \Upsilon} E^{\Upsilon_i}$. For two nodes $u, v \in V^2$, with the technology Υ_i the edge $(u, v) \in E^{\Upsilon_i}$ is weighted if the two points are adjacent in the M map with Υ_i technology.

Once the topology of the graph is constructed, the edges are weighted according to the available throughput using several communication technologies. The edge $e \in E$ using communication technology Υ_i has a weight $w_e^{\Upsilon_i}$. Besides, the edge $e \in \mathcal{E}$ represents a path that can be physically travelled by a vehicle. The asymmetry of this grid therefore leads to a possible slowing down of a vehicle. An edge $(u, v) \in V^2$ also belongs to $\mathcal{E}$ if there is a road in u and in v so that u and v are adjacent. Finally, the total time is divided into consecutive time intervals as analyzed in [19]. Graphs are defined as G_i so that $G_i = \{(V_i, \mathcal{E}, E_i, w_i^{\Upsilon_1} : E_i^{\Upsilon_1} \longrightarrow \mathbb{R}^+), ..., (V_i, \mathcal{E}, E_i, w_i^{\Upsilon_{|\Upsilon|}} : E_i^{\Upsilon_{|\Upsilon|}} \longrightarrow \mathbb{R}^+)\}$. A final graph represents a temporal graph $\mathcal{G} = \{G_0, ..., G_T\}$ as an entry to the problem being studied. Note that the graphs differ only by the weight of the edges in E.

C. Problem presentation

The problem studied is formalized as follows:



(a) Grid pattern for Υ_1

(b) Grid pattern for Υ_2

(c) Overlapped grid patterns

(d) Nodes based on technologies Υ_1 and Υ_2 upon map M. The departure and arrival points are circled

Fig. 2. A grid pattern of the map M according the couple of communication technologies Υ_1 and Υ_2

`Multi-Paths problem with Multi-Technologies.` (MPMT)

Input : A temporal graph $\mathcal{G}$ with k the number of paths, c the maximum number of communication technologies, pair $(D, A) \in V^2$ such that D point of departure and A the arrival.

Output : S a set of V vertices associated with a communication technology that maximizes total throughput according a time limited.

Given a map M, a Υ set of available communication technologies, the set $V = \cup_{1 \le i \le T} V_i$ and $V_i = \{v_{1,i}^{\Upsilon_1}, ..., v_{1,i}^{\Upsilon_{|\Upsilon|}}, v_{2,i}^{\Upsilon_1}, ..., v_{2,i}^{\Upsilon_{|\Upsilon|}}, ..., ..., v_{n,i}^{\Upsilon_1}, ..., v_{n,i}^{\Upsilon_{|\Upsilon|}}\}$. $v_{z,j}^{\Upsilon_i}$ corresponds to the use of Υ_i technology at point z and time j.

The weight of the edges corresponding to the zone z at time j and using communication technology Υ_i is noted $w_{z,j}^{\Upsilon_i}$. So, $w_{z,j} = \{w_{z,j}^{\Upsilon_1}, ..., w_{z,j}^{\Upsilon_\Upsilon}\}$ and $W = \{w_1, ..., w_n\}$ the set corresponding to the capacities of the arcs. If the communication technology is not available, this capacity is zero. Let be $P = \{p_1, ..., p_{|P|}\}$, the set of possible paths with p_i a subset of V. Each communication technology has a latency time called hysteresis formulated as follows:Their corresponding hysteresis is modeled by $H = \{h_1, h_2, ..., h_{|\Upsilon|}\}$.

The problem studied can be formulated with a linear program.

Definition 1: The cardinality constraint implies that in a subset $S_1 = \{..., v_{z,j}^{\Upsilon_i}, v_{z',j'}^{\Upsilon_{i'}}, ...\}$, $v_{z,j}^{\Upsilon_i}, v_{z',j'}^{\Upsilon_{i'}}$ are adjacents if :

- if $i \neq i'$, then $j' = j + h_{i'}$ with $j, j' \in [0, T]$,
- if $i = i'$, then $j' = j + 1$ with $j, j' \in [0, T]$.

Definition 2: The c technology constraint implies that in a set $P = \{p_1, ..., p_l\}$, $v_{z,j}^{\Upsilon_i} \in P_i$ if and only if $\forall z \forall j \sum_{\Upsilon_i \in \Upsilon} y_{z,j}^{\Upsilon_i} \le c$, with $y_{z,j}^{\Upsilon_i}$ indicates that $v_{z,j}^{\Upsilon_i}$ belongs to the solution.

In this linear program, variables $y_{z,j}^{\Upsilon^i}$ allows the use of the Υ^i communication technology for the area z at time j. Thus $y_{z,j}^{\Upsilon^i} = 1$, if the Υ^i communication technology is used at location z and time j and 0 otherwise. The variables $y_{z,j}^{\Upsilon_i}$ characterize the belonging of the vertex $v_{z,j}^{\Upsilon_i} \in V$ to the solution. Variables x_l allows the use of the path l. Thus $x_l = 1$, if the path l is selected, 0 otherwise.

$$
\begin{aligned}
\text{maximize} \; & \sum_{\Upsilon_i \in \Upsilon} \sum_{(z,j):\{v_{z,j}^{\Upsilon_i} \in V\}} w_{z,j}^{\Upsilon_i} \times y_{z,j}^{\Upsilon_i} \\
\text{subject to} \; & \sum_{l: v_{z,j}^{\Upsilon_i} \in p_l} x_l \ge y_{z,j}^{\Upsilon_i} \\
& \sum_{(z,j):\{v_{z,j}^{\Upsilon_i} \in V\}} y_{z,j}^{\Upsilon_i} \le c, \forall \Upsilon_i \in \Upsilon \\
& \sum_{l \in p_l} x_l \le k \\
& y_{z,j}^{\Upsilon_i} \in [0, 1] \\
& x_l \in [0, 1], l = 1, ..., |P|
\end{aligned}
$$

The objective is to maximize the total throughput provided for each communication technology. Line 2, the variable $y_{z,j}^{\Upsilon_i} = 1$ if and only if it belongs to at least one path x_l. Line 3 means that there are a maximum of c communication technologies that can be used in the same area at the same time. Line 4 limits the maximum number of paths to k.

D. Complexity

This section analyses the $MPMT$ problem from a complexity perspective. The property demonstrated is : $MPMT$ is $\mathcal{NP}-hard$ and there is no possibility of an polynomial algorithm with a constant factor approximation, unless $\mathcal{P} = \mathcal{NP}$.

The idea of proof is as follows.

Reductions preserving approximation prove these properties. A special case of the maximum coverage problem with budget called $NMCG - int$ is first defined.

The $NMCG$ problem has been proven as $\mathcal{NP} - hard$ and there is no constant approximation factor. A reduction preserving the approximation makes it possible to keep these properties in $NMCG - int$.

Finally, a second reduction preserving the approximation and going from $NMCG{-}int$ to $MPMT$, allows to prove that $MPMT$ is $\mathcal{NP} - hard$ and has no constant approximation factor.

The evidence is detailed below.

Maximum coverage problem with group budget constraints (MCG) with items weighted and without group disjoint and ($NMCG-int$)

Input : The input of $NMCG-int$ is a set $X = \{x_1, ..., x_n\}$ of n items corresponding to a set of weights $W = \{w_1, ..., w_n\}$, a collection S of k subsets of X with a cost $c_1, ...c_k$, a collection of X $G = \{G_1, ...G_l\}$ of l groups with budgets $B_1, B_2, ..., B_l$ and B the total budget.

Output : A subset H of S so that $\sum_{x_j \in H} w(x_j)$ is maximized such that $\sum_{S_j \in H} c(S_j) \leq B$, $\forall i \in [1, l] \sum_{S_j \in H \cap G_i}, c(S_j) \leq B_i$.

Theorem 1: Unless $\mathcal{P} = \mathcal{NP}$, $NMCG-int$ is $\mathcal{NP}-hard$ and there is no constant factor approximation.

Proof 1: A L-reduction from the $NMCG$ to the $NMCG-int$ is presented. This one is an approximation-preserving reduction.

Maximum coverage problem with group budget constraints (MCG) without group disjoint. ($NMCG$)

Input : The input of $NMCG$ is a set $X' = \{x'_1, ..., x'_n\}$ of n' items, a collection S' of k' subsets of X' with a cost $c'_1, ...c'_k$, a collection of S $G' = \{G'_1, ...G'_l\}$ of l' groups with budgets $B'_1, B'_2, ..., B'_l$ and B' the total budget.

Output : A subset H' of S' so that $|\cup_{S'_j \in H'} S'_j|$ is maximized such that $\sum_{S'_j \in H'} c(S_j) \leq B'$, $\forall i \in [1, l], \sum_{S'_j \in H' \cap G'_i}, c(S'_j) \leq B'_i$.

Let I', an instance of $NMCG$. An instance I of $NMCG-int$ is constructed as follows. The set X corresponds to the the X'. All corresponding weights are 1. Each subset $S'_j \in S'$ has a corresponding subset $S_j \in S$, so that the costs are the sames. Each $G'_i \in G'$ corresponds to a collection of S'. Groups are created in a similar way, so that $\forall S_j \in S, G_i = \cup_{x \in S_j \cap G_i} x$. The corresponding Budgets are the same. Every feasible solution to $NMCG-int$ is feasible for $NMCG$ and vice versa. The ratio of approximation is preserved by this reduction since the solutions are identical.

Article [20] proved that Problem $NMCG$ was $\mathcal{NP}-hard$ and that there is no constant factor approximation algorithm existing. With the $L-reduction$, the $NMCG-int$ has these properties.

Theorem 2: Unless $\mathcal{P} = \mathcal{NP}$, $MPMT$ is $\mathcal{NP}-hard$ and there is no constant factor approximation.

Proof 2: A L-reduction from the $NMCG-int$ to the $MPMT$ is presented.

Let I, an instance of $NMCG-int$. An instance I_0 of $MPMT$ is constructed as follows. The set X corresponds to the the vertices of $\mathcal{G}$. $|S|$ vertices D and one A are added. The graph is directed and complete. The weights of items corresponds to the weight if outgoing edges of the vertices associated. The vertexes D and A have their outgoing edges weighted to 0. Each subset $S_j \in S$ corresponds to a feasible path. The vertices are the same that the corresponding items and the points A and D_j are added. Each $G_i \in G$ corresponds to a set of items of X. Each group corresponds to a zone at a given time and therefore B_i corresponds to the limit of communication technologies that can be used in the same place. A last group is added in order to group together all vertices A_j. This group has a total budget of k and limits the number of paths used.

Every feasible solution to $NMCG-int$ is feasible for $MPMT$ and vice versa. The ratio of approximation is preserved by this reduction since the solutions are identical.

Theorem 2 proved that Problem $NMCG-int$ was $\mathcal{NP}-hard$ and that there is no constant factor approximation algorithm existing. With the $L-reduction$, the $MPMT$ has these properties.

Since there is no constant approximation factor for the general problem, further work will focus on particular cases. If the number of communication technologies is not limiting ($c \geq k$), then there is indeed a possible α-approximation.

However, the approximation works in a very restrictive scenario. The practical use of this approximation leads to a preliminary filtering limiting the number of possible paths. This filtering weakens the result obtained. Afterwards, it will be interesting to look for other possible approximations increasing the levels of liberty .

IV. Approximation

The application of the $GBMC$ problem within the framework of VANETs requires the constraints related to the times and related to the choice of communication technology to be taken into account. However, the number of technologies that can be used in the same area is not constraining ($c \geq k$). The new definition is taken into account in a new approximation algorithm which ratio is calculated.

A. Generalized Budgeted Maximum Coverage Problem with cardinality and technologies constraints ($BMCP-CTC$) presentation

Defined as $\mathcal{NP}-hard$ by [15], the $BMCP-CC$ problem allows to add a temporal aspect with the addition of cardinality constraints.

Constraints related to communication technologies are modeled through a set filtering. The maximum coverage problem budgeted with cardinality and communication technology constraints is noted $BMCP-CTC$.

Definition 3: A $CTC-path$ is a possible path in a directed graph, as defined below.
These rules are, $\forall v_{z,j}^{\Upsilon_i} \in p$:

- stay put: $\forall p \in P, p =< ..., v_{z,j}^{\Upsilon_i}, v_{z,j+1}^{\Upsilon_i}, ... >$
- displacement: $\forall p \in P, p =< ..., v_{z,j}^{\Upsilon_i}, v_{z',j+1}^{\Upsilon_{i'}}, ... >$ so that the displacement from z to z' is possible i.e. $(z, z') \in \mathcal{E}$ and $i = i'$ or $(z, z') \in E^{\Upsilon_i}$,
- the change in communication technology: $\forall p \in P, p =< ..., v_{z,j}^{\Upsilon_i}, v_{z,j+h_{i'}}^{\Upsilon_{i'}}, ... >$.
- a D start such that D is the zone of the first element $p =< v_{D,0}^{\Upsilon_i}, ... >$;
- an arrival A such as A is the zone of the last element $p =< ...v_{A,T}^{\Upsilon_i} >$;

In order to take account of these constraints, the budgeted problem is reformulated as follows.

`Generalized Budgeted Maximum Coverage Problem with cardinality and communication technologies contraints.` (GBMC-CTC)

Input : The input of $GBMC$, with c the maximum number of communication technologies in a given place and at a given time and k the maximal number of path

Output : Feasible selection with maximum profit which respects cardinality constraints and technology constraints.

The input is a set $< \mathcal{G}, S, T, k, c >$ and the solution is a subset of S that maximizes the weight of the k paths. The graph is created to represent the number of possible paths.

First, starting from $\mathcal{G}$, an oriented graph $\mathcal{G} = (V, E)$ is built as follows $V \leftarrow V(G_0) \cup V(G_1) \cup ... \cup V(G_T)$. For each node $v^{\Upsilon_i}_{z,j}$ and $v^{\Upsilon'_i}_{z',j'}$, there is a direct arc if the edges respect the constraints of a $CTC - path$.

The graph $\mathcal{G}$ has its vertexes V, so that :
$V = \{v^1_{1,0}, v^1_{2,0}, ..., v^1_{n,0}, v^2_{1,0}, v^2_{2,0}, ..., v^2_{n,0};$
$v^1_{1,1}, v^1_{2,1}, ..., v^1_{n,1}, v^2_{1,1}, v^2_{2,1}, ..., v^2_{n,1}; ...;$
$v^1_{1,T}, v^1_{2,T}, ..., v^1_{n,1}, v^2_{1,T}, v^2_{2,T}, ..., v^2_{n,T}\}$
where each $v^{\Upsilon_i}_{z,j}$ represents the node relative to the z point in the graph, depending on the j time and Υ_i communication technology. The solution A is a subset of V' such that $A = \{S_1, S_2, ..., S_m\}$ where S_i represents a possible trajectory in $\mathcal{G}$.

The figure 3 illustrates the process of creating the $\mathcal{G}$ graph. However, the $\mathcal{G}$ subgraph gathers all the paths from D to A. The other paths -even if they belong to $\mathcal{G}$- are not shown.

The algorithm works on the $\mathcal{G}$ graph.

B. Greedy algorithm presentation

The $BMCP - CTC$ is $\mathcal{NP} - hard$ because it is similar to the $BMCP - CC$ problem already defined as $\mathcal{NP} - hard$ [15]. An approximation algorithm is proposed where the number of usable communication technologies c is such that $c \geq k$, with k the number of paths. This new approximation algorithm offers an α-approximation, with $\alpha = 1 - \frac{1}{e}$. The idea is based on several previous concepts. Throughput maximization takes up this idea in an inspired way by the dynamic algorithm developed in [21].

The algorithm inputs are as follows. $\mathcal{G}$ represents the entire temporal graph. The variable k represents the maximum number of paths and c represents the maximum number of communication technologies allowed per zone. $\mathcal{S}$ represents the graph with updated weights. The solutions are grouped in A.

The $GreedyAlgo$ is used to select the path with the highest weight. In the graph $\mathcal{S}$, the sum of the edge weights of a path represents the total throughput. A path S_i is a set of vertices obeying the rules defined in a $CTC - path$, so that c is not restricting. The distance between two nodes $v^{\Upsilon_i}_{z,j}$ and $v^{\Upsilon'_i}_{z',j'}$ is noted $dist(v^{\Upsilon_i}_{z,j}, v^{\Upsilon'_i}_{z',j'})$.

Algorithm 1 Algorithm with Multi Technologies (AMT)

Require: $< \mathcal{G}, k >$
Ensure: A
1: $\mathcal{S} \leftarrow \mathcal{G}$,
2: $A \leftarrow \emptyset$,
3: $i \leftarrow 1$
4: **repeat**
5: $\quad A, \mathcal{S} \leftarrow GreedyAlgo(\mathcal{S})$
6: $\quad i \leftarrow i + 1$
7: **until** $i \leq k$
8: **return** A

This algorithm uses a dynamic program computing the longest path (in terms of weight). The method is related to maximization of submodular set functions detailed in [22].

Firstly, the $GreedyAlgo$ is presented. $\forall \Upsilon_i, \Upsilon'_i \in \Upsilon$ and $z \in [1, n]$, the impossibility of a path appears as follows : $dist(v^{\Upsilon_i}_{z,j}, v^{\Upsilon_i}_{z',j}) = 0$ and $dist(v^{\Upsilon_i}_{z,j}, v^{\Upsilon'_i}_{z,j}) = 0$. The maximization according to the choice of communication technology and the route obeys to the following equation, with $e \in E^{\Upsilon_i}$. The max variable chooses the maximum weighting among neighbors according to the $\Upsilon_i \in \Upsilon$ communication technologies and the choice of the route to be taken limited to the available paths $\in S$. The available neighborsaccording to Υ_i are noted $d^- v^{\Upsilon_i} = \{(u|(u,v) \in E^{\Upsilon_i}) \cap (u|(u,v) \in \mathcal{E})\}$. So, $max = max_{\Upsilon'_i \in \Upsilon, r \in d^- v^{\Upsilon'_i}_{z',t}}$ and

$$dist(v^{\Upsilon_i}_{z,0}, v^{\Upsilon'_i}_{z',t}) = max\{dist(v^{\Upsilon_i}_{z,0}, d) + w_e^{\Upsilon'_i}\} \qquad (1)$$

Based on above recursive relation, the longest paths from a node in $t = 0$ to a node in $t = T$. Once a path has been elected, the corresponding weights are reset to 0.

According to the loop Line 7, the operation of electing a path is repeated k times, taking into account the updating of the vertex weights.

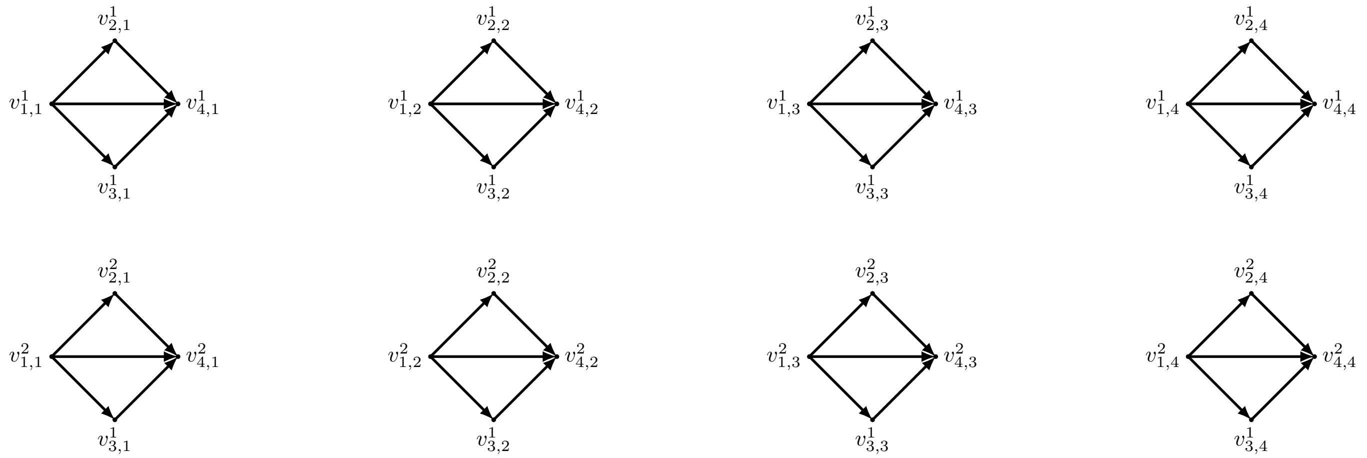
C. Example

The algorithm is applied to a simple example shown in Figure 3. Initially, there are only four zones, two communication technologies with different weights and a time of 4. The points in the zone are denoted $v^{\Upsilon_i}_{z,j}$ with z the zone, j the time and Υ_i the technology used.

Figure 4 models the graph corresponding to the previous graphs. Starting points are $v^{\Upsilon_i}_{1,1}, \forall \Upsilon_i \in \Upsilon$. End points are $v^{\Upsilon_i}_{4,4}, \forall \Upsilon_i \in \Upsilon$. To lighten the graph, only the edges allowing a path from the start to the end point are represented. Also, some edges with hysteresis greater than the maximum hysteresis, such as the edge $(v^1_{3,2}, v^2_{3,3})$ are not included.

The created sets are listed in the table II. The model is thus represented with S the sets in this table. The set of items U are the vertexes of the graph. This example assumes too that $c = 2$, only two communication technologies are available per point, and $k = 2$ two paths are required in the end.

The table is modeled in II. The S sets selected respect the constraints of cardinality and hysteresis. In this example, the



(a) Graph G_1 (b) Graph G_2 (c) Graph G_3 (d) Graph G_4

Fig. 3. Illustration of the algorithm. G_t represents the graph at time t.

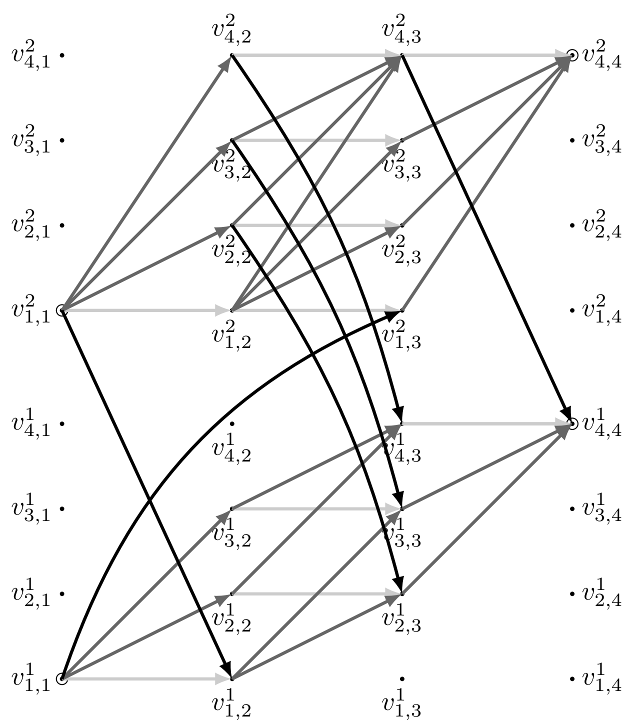


Fig. 4. This graph shows only the set of possible paths from D to A.

Technologies used	subsets	Total weight	Total weight 2
Only technology 1	$S_1 = \{v^1_{1,1}, v^1_{1,2}, v^1_{2,3}, v^1_{4,4}\}$	6	0
	$S_2 = \{v^1_{1,1}, v^1_{1,2}, v^1_{3,3}, v^1_{4,4}\}$	6	2
	$S_3 = \{v^1_{1,1}, v^1_{2,2}, v^1_{2,3}, v^1_{4,4}\}$	6	2
	$S_4 = \{v^1_{1,1}, v^1_{2,2}, v^1_{4,3}, v^1_{4,4}\}$	6	4
	$S_5 = \{v^1_{1,1}, v^1_{3,2}, v^1_{3,3}, v^1_{4,4}\}$	6	4
	$S_6 = \{v^1_{1,1}, v^1_{3,2}, v^1_{4,3}, v^1_{4,4}\}$	6	4
Only technology 2	$S_7 = \{v^2_{1,1}, v^2_{1,2}, v^2_{2,3}, v^2_{4,4}\}$	4	4
	$S_8 = \{v^2_{1,1}, v^2_{1,2}, v^2_{3,3}, v^2_{4,4}\}$	4	4
	$S_9 = \{v^2_{1,1}, v^2_{2,2}, v^2_{2,3}, v^2_{4,4}\}$	4	4
	$S_{10} = \{v^2_{1,1}, v^2_{2,2}, v^2_{4,3}, v^2_{4,4}\}$	4	4
	$S_{11} = \{v^2_{1,1}, v^2_{3,2}, v^2_{3,3}, v^2_{4,4}\}$	4	4
	$S_{12} = \{v^2_{1,1}, v^2_{3,2}, v^2_{4,3}, v^2_{4,4}\}$	4	4
	$S_{13} = \{v^2_{1,1}, v^2_{4,2}, v^2_{4,3}, v^2_{4,4}\}$	4	4
	$S_{14} = \{v^2_{1,1}, v^2_{1,2}, v^2_{4,3}, v^2_{4,4}\}$	4	4
	$S_{15} = \{v^2_{1,1}, v^2_{1,2}, v^2_{1,3}, v^2_{4,4}\}$	4	4
With changes of technologies	$S_{16} = \{v^1_{1,1}, v^2_{1,3}, v^2_{4,4}\}$	3	1
	$S_{17} = \{v^2_{1,1}, v^1_{1,2}, v^1_{2,3}, v^1_{4,4}\}$	4	0
	$S_{18} = \{v^2_{1,1}, v^1_{1,2}, v^1_{3,3}, v^1_{4,4}\}$	5	3
	$S_{19} = \{v^2_{1,1}, v^2_{2,2}, v^1_{2,3}, v^2_{2,4}\}$	4	2
	$S_{20} = \{v^2_{1,1}, v^2_{2,2}, v^2_{4,3}, v^1_{4,4}\}$	3	3
	$S_{21} = \{v^2_{1,1}, v^2_{3,2}, v^1_{3,3}, v^2_{2,4}\}$	4	4
	$S_{22} = \{v^2_{1,1}, v^2_{3,2}, v^2_{4,3}, v^1_{4,4}\}$	3	3
	$S_{23} = \{v^2_{1,1}, v^2_{4,2}, v^1_{4,3}, v^1_{4,4}\}$	4	4
	$S_{24} = \{v^2_{1,1}, v^2_{4,2}, v^2_{4,3}, v^1_{4,4}\}$	3	3

TABLE II
SUBSETS OF POSSIBLES S

weighted corresponding to communication technology 1 are all higher than those of communication technology 2. Therefore, only the sets S_1 to S_4 belongs to the final solution.

The solution proposed by the algorithm consists of the two paths S_1 and S_4. The calculation of the total weight varies according to these choices. Thus, the weights corresponding to the S_1 and S_4 sets are 6. Once the S_1 path is considered as the solution, the weight corresponding to the S_4 set becomes 4.

D. Properties

The solution provided by AMT is a α-approximation of the optimal result in polynomial time, if $c \geq k$.

Theorem 3: AMT runs in polynomial time.

Proof 3: The temporal complexity of the calculation of the k longest paths is made up of a series of elementary steps.

The table of the longest paths is of maximum size $|\Upsilon| \times T \times n$ with n, the number of zones. The calculation of the maximum weight between two neighboring vertices has a maximum complexity of $\bigcirc(|\Upsilon| \times n)$. Consequently a column of the table is calculated in time $\bigcirc(|\Upsilon|^2 \times n^2)$. As the table has T columns, the calculation time of the table is $\bigcirc(|\Upsilon|^2 \times n^2 \times T)$. Now all the distances going from point D to point A are calculated. In the worst case, the complexity is therefore $\bigcirc(|\Upsilon|^3 \times n^3 \times T)$. The weight change at each step of the loop is $(|V|^2)$, with $|V| = |\Upsilon| \times n \times T$.

Finally, these methods are embedded in a loop of k size, with k being the maximum number of paths. So, Therefore, the total complexity is $\bigcirc(k \times |\Upsilon|^3 \times n^3 \times T)$.

Theorem 4: With $c \geq k$, algorithm AMT is a $(1 - \frac{1}{e})$-approximation for $BMCP - CTC$, which is guaranteed to produce a solution at least $1 - \frac{1}{e}$ times the optimal solution.

Proof 4: The strategy of line 5 in Algorithm 1 is based on the greedy algorithm developed in [22] with the submodular set functions. The solution is a $(1 - \frac{1}{e})$-approximation of the best possible with e being the base of the natural logarithm.

The general idea is as follows. At each loop, the proposed solution can only increase, and the starting value equals 0. According to [22], the solution of the $GreedyAlgo$ algorithm is at least equal to $1 - (\frac{K-1}{K})^K$ times the optimal value.

$$\frac{value(greedyApproximation)}{value(optimal)} \geq 1 - (\frac{K-1}{K})^K \geq \frac{e-1}{e} \tag{2}$$

This solution is proven to be the best possible, unless $NP \subseteq DTIME(n^{loglog(n)})$, with n the size of the input.

V. Conclusion

This paper develops a new strategy for the choice of communication technology related to the different possible paths. The maximization of the received throughput, taking into account the latency time relative to changes in communication technology, is particularly adapted to the demands of users accustomed to a broadband offer. The choice of path by the algorithm is integrated in the expansion of autonomous vehicles. This work provides a new solution to maximize the use of VANETs with several communication technologies. The strategy is based on the choice of communication technologies used and the choice of paths. The proposed modelling unites these two aspects, and provides a new optimization problem. Finally, a solution to this problem is proposed through a polynomial time approximation algorithm.

The result of this paper lays the foundation for much future research. From a modelling perspective, the use of time windows would allow a better adaptation to road traffic hazards (accident, traffic jam, delay...). Moreover, the development of embedded systems favours the use of distributed systems at the expense of centralized systems. Finally, it would be interesting to compare the algorithm proposed in this paper through realistic simulations and define the behavior of this approach in several scenarios.

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