Servicing Inelasticity, Leasing Resources and Pricing in 5G Networks

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***Abstract*—We consider the problem for mobile network operators (MNOs) of leasing resources, servicing and pricing mobile users, in the context of 5G systems that facilitate the use of software-defined radio access network (SD-RAN) and network function virtualization (NFV) technologies. We study the case where the service capability of a MNO cannot satisfy the total users' demand who are characterized by inelastic behavior against the servicing rate that they experience. The MNO addresses this temporal depletion of its resources and acquires dynamically, through leasing, additional resources from an infrastructure provider (InP) to adequately comply with its mobile users' demand. We model and analyze the interactions among the MNO, and the users, as a Stackelberg game. To model users' inelastic behavior, we use a sigmoid utility function. Furthermore, we show the optimal pricing decisions when MNO's supplying capacity satisfies users' demand. Given an excess on MNO's supplying capacity, we employ the generalized r-Lambert function to determine the optimal pricing. When MNO's supplying capacity is not ample, we determine, besides pricing, an approximation of the optimal amount of the additional resources to purchase, given a leasing cost imposed by the InP. An interesting finding shows that the amount of additional resources to be purchased can be larger than the MNO's minimum capacity gap. Simulation and testbed experimentation validate the feasibility of the proposed pricing and leasing scheme and demonstrate its practical application.**

***Index Terms*—Network Economics, Leasing, Pricing, Inelasticity, Stackelberg Game, Generalized r-Lambert Function.**

I. INTRODUCTION

The constant magnification of the global IP traffic [1] and the uptake of newer, advanced, real-time services, delivering virtual and augmented reality traffic and transferring performance-critical data both for consumers and large operations, put substantial strain on the network service providers[1], who strive to satisfy the users' demand and meet strict performance requirements in terms of latency, delay and/or throughput, which more often than not are characterized inelastic. Not only does this situation increase the competition among providers that need to provide reliable services with scarce available resources, but it also raises the capital and operational expenditures that an operator should invest in.

However, the emerging 5G network architecture [2] is expected to change radically the deployment and operation of modern networks. The new introduced architecture is expected to fulfill the increasing demand of users for new exigent applications. Currently, telecommunication network infrastructure is being transformed to accommodate a distributed cloud with several of network functionalities implemented as virtual network functions (VNFs) within isolated partitions of network resources called slices. Interestingly, network slicing is not limited to the core network functionalities but it involves the partitioning of RAN resources [3].

With virtualization and end-to-end network slicing, resources could be also shared and configured for the various service requirements of different tenants of the network [4], [2], [5]. This is particularly important as the assessment on capital and operational expenditures for new deployments is proving extremely costly and diminishes any potential on the return of investment for the operators [6]. Instead, in cases of high demand surges, where a network service provider faces shortcomings, the need for acquiring additional resources from infrastructure providers becomes essential, in order to provide ample services to its users. Therefore, operators in need can request and lease on demand – as a part of their service-level-agreements (SLAs) – part of the infrastructure or resources from other MNOs or InPs to provide their own services.

In this paper, we study the operation of one 5G network that consists of one virtual MNO, its associated users and one InP. The MNO faces depletion of network resources due to high users' demand and interacts with the InP requesting network resources to negotiate an SLA in the need of additional resources so as to service its users. We introduce a novel solution that allows operators to (i) dynamically negotiate SLAs and acquire by leasing the appropriate amount of resources and (ii) define pricing for the services offered to the users. The main contributions of our work can be summarized as follows:

- *A Stackelberg Model:* We propose a novel framework that is formulated as a *Stackelberg game* [7] and we use *Backward Induction* to capture the interactions among stakeholders. The MNO, being the *leader* of the game, firstly assesses the total demand that is requested by the users, evaluates his capacity availability and determines the amount of resources that needs (or not) to purchase from the infrastructure provider in order to satisfy this demand. Then, the MNO defines the optimal pricing with an aim to increase its profits and it announces the price to the users. In sequence, the users having the *followers* role in the game and taking into consideration the price that the MNO had previously announced, request, on their side, the rate that not only

[1]We use the term mobile network operator (MNO) or over-the-top network service provider interchangeably to refer to the same entity and we assume that it does not necessarily own the infrastructure and, hence, the resources that allocates to its users.

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satisfies their minimum requirements but also maximizes their benefit, as this is expressed by their *payoff* function.

- *Analytical Solutions:* In Stage III, we present an optimal solution for the users to specify their optimal demands given the MNO's announced price. We formulate their inelasticity by employing a sigmoid utility function. In Stage II, we designate MNO's dynamic pricing. Particularly, we discriminate two different cases by assessing the operator's supplying service capability and we present the solutions to determine the optimal price for the services offered by the MNO to users. When MNO's supplying capacity is ample, we employ the use of the generalized r-Lambert function [8] to determine the optimal solution and the subgame equilibrium point. In case of resources deficiency, apart from pricing, we determine in Stage I an upper bound on the amount of the resources that the MNO should purchase given a leasing cost imposed by the InP.
- *Extensive evaluation:* We conduct extensive simulations and we validate our results using real 5G equipment and testbed experimentation.

The rest of the paper is organized as follows: Section II presents prior literature, Section III describes the network model, Section IV introduces the theoretical framework, Section V presents the evaluation and numerical/experimentation results, and Section VI concludes the paper.

II. Related Work

Non-linear Utilities: A common miss-conception of research on network optimization is that traffic flows are elastic, which means that their utilities are concave and continuous, and hence the formulation of the problem under investigation can be convex-convenient. This assumption assists in the creation of tractable analytical models that can produce most of the time closed-form solutions. Notwithstanding the importance of such modelling, this approach, on the other hand, limits the applicability of the resulting algorithms or protocols to deal with real-time traffic which is mostly characterized inelastic. One of the initial works that addressed the need on this topic was [9], where authors tackle the difficulties of distributed rate allocation for inelastic flows and quantify the provision of service capacity under which non-linear discontinuous and/or sigmoidal utility functions can produce optimal solutions.

Network Capacity Expansion: The problem of flat and congestion pricing for capacity expansion in centrally planned, competitive, and monopolistic environments is considered from an economic perspective in [10]. Authors showed that expanding capacity can increase welfare if and only if the revenue from the congestion fees exceeds the value of capacity, which is valued with its marginal cost. They showed also that, in a competitive market, expanding capacity can attract new customers due to network performance improvement and delay reduction. On investigation of pricing and investments decisions in contemporary networks, authors in [11] examine jointly the optimization of Wi-Fi access points deployment and the related pricing scheme for Wi-Fi data usage in order to maximize the carrier's profit under several user demand models. Interested readers could also refer to [12]. In this work, authors present a theoretical framework to deal with optimal investment and pricing decisions of a cognitive mobile virtual network operator (C-MVNO) under spectrum supply uncertainty. This framework assesses the current user demands and evaluates spectrum sensing in unlicensed bands of secondary users against dynamic spectrum leasing to attain trade-off between cost (leasing of spectrum is costly) and uncertainty (the scanty availability of secondary networks' spectrum incurs performance deficiency). Our work is closely related to [12], in a way that it captures the interaction among stakeholders as a Stackelberg game but it extends the approach of quantifying the resource availability to a different application domain of 5G networks focusing on servicing traffic inelasticity.

Mobile Data Offloading: Mobile data offloading has been proposed in literature as a viable solution that can significantly unburden the cellular congestion. Without the need for costly infrastructure investments, offloading exploits already existing complementary either Wi-Fi or small/femto cellular networks to deliver traffic originally targeted to macro cellular networks. It is considered ideal for delay-tolerant applications, as sophisticated pricing schemes that would provide incentives or compensation to participating users are required [13], [14]. However, its effective application is dependent on the availability of the complementary networks resources.

Moreover, it is worth mentioning the use of double-auction mechanisms for enabling mobile data offloading [15]. Those mechanisms rely on the assistance of an intermediate broker that collects the bids and payments as centralized authority for the trading of the resources between agents (involved parties). The efficiency of such schemes is given under the assumption of non-strategic bid placement to prevent influencing the market price of the resources. However, a price-anticipating behavior of agents is naturally expected and such double-auction mechanisms require targeted enhancements i.e. the use of a Stackelberg formulation with the supplying agent as a leader [16] to deal, in some extent, with such situations.

In comparison to offloading, our work departs from the same need to mitigate the capacity crunch problem and it can be treated as a 5G-enabled solution, by exploiting the latest advancements in slicing and virtualization as it seeks solutions which attach additional resources to operators facing capacity depletion for servicing inelastic demands, instead of conveying traffic to complementary networks at some expense cost.

Network Slicing, Virtualization and Software-Defined RAN: Network virtualization offers a feasible way to provide and configure a network slice tailored to the requirements of each service. Utilizing network slicing, SDN, NFV and cloud computing technologies are expected to open up the business opportunities for network operators and infrastructure providers to realizing the benefits of resources partitioning through trading in emerging 5G networks [17]. Driven by the need for dynamic configuration and flexible customization of networks slices according to the end-users requirements (e.g. mobile network operators, verticals and over-the-top service

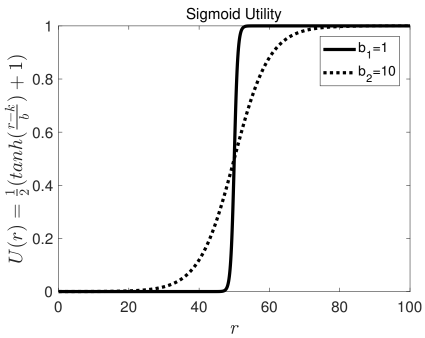


Fig. 1: Sigmoid utility function with $A = 1$. Large values of b form a smoother shape, while lower values of b make the shape of the utility steeper.

providers), authors in [3] designed FlexRAN, a flexible control plane that accommodates their needs and realizes the benefits of network slicing to enable (i) real-time and adaptive RAN control and (ii) multi-level flexibility of coordination among different RAN infrastructure entities.

III. Network Model

A. 5G Network Overview: Notations and Assumptions

We consider the downlink operation of a 5G network, which comprises one MNO, one InP and the associated users. Table I summarizes the system model parameters. The MNO provides communication services to a set of $\mathcal{N}$ mobile users up to a total capacity of C_{MNO} (bps) trying to satisfy the *minimum* aggregated demand of its associated users which equals to $\sum_{i\in\mathcal{N}} k_i = C_{\mathsf{MUs}}$. Each user $i \in \mathcal{N}$ is serviced by the MNO and her utility is described by a sigmoid hyperbolic tangent function of the transmission rate r_i that she experiences, and it is expressed by the following equation:

$$U_i(r_i) = \frac{A}{2}\left(\tanh\left(\frac{r_i - k_i}{b_i}\right) + 1\right), \qquad (1)$$

where k_i is an inflection point and denotes the user's strict demand for service, b_i is a positive design parameter that models her behavior against the service rate that she experiences, and A is a positive design parameter. The sigmoid[2] function has some interesting properties: It is a continuous, increasing, monotonic function of the transmission rate r_i and is constrained by a pair of horizontal asymptotes as $r \to \pm\infty$. Moreover, this utility function is convex for rate values r_i less than k_i, and it is concave for rate values larger than k_i. In addition, the appropriate selection of the parameter b_i can be used to adjust the steepness of the tangent as follows: the larger the value of b_i is selected, the smoother the utility function's shape. On the other hand, the smaller the value of b_i, the steeper the shape of the function. Fig. 1 illustrates the sigmoid utility as a function of rate r for two different values of b.

The MNO may temporarily or constantly face shortcomings in its services, as it is possible that the *minimum* aggregated demand C_{MUs}, requested by all users, to exceed its service capability C_{MNO}. In order to mitigate this shortcoming and fulfill the users' requirements, the MNO can request and lease at a cost c_o the additional necessary capacity C_{ADD} from the InP that owns networking resources in its vicinity.

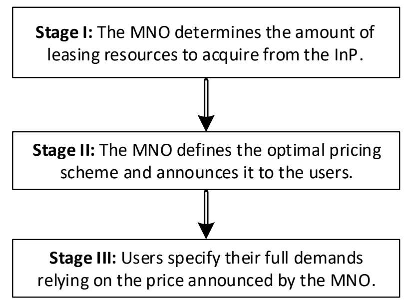


Fig. 2: A Stackelberg Game among the MNO, the InP and the users.

IV. Backward Induction Game

We model the interaction among the users and the MNO as a three-stage Stackelberg game, where the operator is the leader and the users are the followers, and we solve it by using backward induction. As shown in Fig. 2, in Stage III, each user decides how many resources (capacity) to purchase to maximize her payoff, in Stage II, the operator determines the monetary price p (per unit of rate capacity) to maximize its profit and, in Stage I, the operator decides the amount of leasing resources to acquire.

In our model we assume a monopolistic market, where the MNO operates and the leasing cost c_o for the MNO is assumed to be fixed. Therefore, a double chain monopolistic market, where the InP sets the cost based on the quantity demanded by the MNO, is not considered in this work. In the latter case, the double marginalization issue [18] may arise due to the placement of the firms in the supply chain, inducing possibly deadweight loss if the InP decides to charge in a way that it will not be beneficial for the MNO to service its users.

A. *STAGE III*: Users specify their full demands based on the announced price p.

At this Stage, the mobile users specify their *full* demands by taking into consideration the unit price p (a monetary currency) that the MNO had already announced on the previous Stage II. The payoff function of each user $i \in \mathcal{N}$ for acquiring resources (the amount that corresponds to the rate r_i) at a price p is expressed by the following equation:

$$\pi_i(r_i) = U_i(r_i) - pr_i, \qquad (2)$$

which is the difference between the user's utility function $U_i(r_i)$ of the experienced rate[3] and the total payment pr_i, which follows the linear pricing model. The solution that maximizes the above expression leads to the following result.

Theorem 1. *The optimal value of demand that maximizes the user's i payoff is given by:*

$$r_i^\star(p) = b_i \operatorname{arsech}\left(\sqrt{2b_i p}\right) + k_i, \qquad (3)$$

and the following inequality $p_{\mathsf{min}} \le p \le p_{\mathsf{max}}$ determines the minimum and the maximum values that price p can take, where $p_{\mathsf{min}} = \frac{1}{2b_i}\left(1 - \tanh^2(-\frac{k_i}{b_i})\right)$ and $p_{\mathsf{max}} = \frac{1}{2b_i}$.

The detailed proof of Theorem 1 is presented in Appendix [19].

[2] This sigmoid utility contains the hyperbolic tangent function and it is a shifted and scaled version of the standard logistic function $f(x) = \frac{1}{1+e^{-x}}$, taking values within the range $x \in [0, 1]$. For $A = 1$, it is $\frac{1}{1+e^{-x}} = \frac{e^x}{e^x+1} = \frac{1}{2}\left(\tanh(x) + 1\right)$. For the rest of the paper, we assume $A = 1$.

[3] Without loss of generality, the rate r_i is a function of the allocated resources the MNO consumes to provide communication services to its users.

TABLE I: SYSTEM MODEL PARAMETERS.

Symbol	***Description***	***Symbol***	***Description***
$\mathcal{N}$	Set of mobile users	p	Monetary price
MNO	Mobile Network Operator	c	Monetary operational cost
C_{MNO}	MNOs resources-capacity	c_o	Monetary leasing cost
C_{MUs}	Aggregate mobile users *minimum* demands	$\pi_i(p)$	User i payoff $v \in \mathcal{V}$
C_{ADD}	Additional resources-capacity leased from an InP	$\sum_{i\in\mathcal{N}} r_i^\star(p)$	Total user demand
r_i	User's i service rate	$C_{\mathsf{MNO}} + C_{\mathsf{ADD}}$	Total MNOs supply capability
k_i	User's i *minimum* demand	$\Pi_{\mathsf{MNO}}, R_{\mathsf{MNO}}$	MNO's profit, MNO's revenue
b_i	User's i sensitivity factor-steepness	$U_i(r_i)$	Utility of user i

The quantity $r_i^\star(p)$ represents the total requested demand and it is the sum of the *minimum* user's i demand k_i plus a positive quantity which equals to $b_i\, \mathrm{arsech}(\sqrt{2b_i p})$ and depends both on the price p that the user pays and on her sensitivity b_i towards the received service. Now, as the user is awarded the service rate $r_i^\star(p)$ that she demanded for paying services to the MNO at price p, the benefit that she receives is expressed by plugging the optimal value $r_i^\star(p)$ to user's payoff function $\pi_i(r_i^\star)$. Therefore, the user's maximum payoff is given by:

$$\pi(r_i^\star(p)) = \frac{1}{2}\left(\tanh\left(\mathrm{arsech}(\sqrt{2b_i p})\right) + 1\right) - p\left(b_i\, \mathrm{arsech}(\sqrt{2b_i p}) + k_i\right) \quad (4)$$

The *minimum* requested rate by all users is $C_{\mathsf{MUs}} = \sum_{i=1}^{N} k_i$. We assume users are homogeneous and they adopt the same attitude against the experienced service (it is $b_i = b, \forall i \in \mathcal{N}$ i.e. they belong to the same service class). Then, the *total demand* of mobile users is expressed by the following equation:

$$\sum_{i\in\mathcal{N}} r_i^\star(p) = Nb\, \mathrm{arsech}(\sqrt{2bp}) + C_{\mathsf{MUs}}. \quad (5)$$

Next, we consider how the MNO takes the leasing and pricing decisions in Stages I-II relying on the total users' demand.

B. ***STAGE II****: The operator specifies the optimal pricing based on the total user demand and by assessing its supplying capacity to service.*

At this Stage, the MNO determines the optimal price $p^\star$ to be announced taking into account the total demand of users $\sum_{i\in\mathcal{N}} r_i^\star(p)$ as well as its total capability for capacity supply $C_{\mathsf{MNO}} + C_{\mathsf{ADD}}$, which is expressed by the sum of the capacity of its own provisioned resources and the leased capacity of the additional resources. Determining MNO's revenue is achieved either as a function of users' demand $R_{\mathsf{MNO,D}} = p\sum_{i\in\mathcal{N}} r_i^*(p)$ or as a function of MNO's capability to supply capacity $R_{\mathsf{MNO,S}} = p(C_{\mathsf{MNO}} + C_{\mathsf{ADD}})$. In this point, we discriminate two different cases, where the MNO's capability for supplying capacity is quantified against the users' demand.

a. *Excessive Supply (ES):* The MNO's capacity is larger than the users' demand, that is when $\sum_{i\in\mathcal{N}} r_i^*(p) \leq C_{\mathsf{MNO}}$ holds. Since the MNO does not need additional resources to lease, it is also $C_{\mathsf{ADD}} = 0$. As depicted in Fig. 3, the MNO's revenue, expressed as a function of users' demand (red curve) $R_{\mathsf{MNO,D}}$, has at most one intersecting point with the straight line $R^2_{\mathsf{MNO,S}}$ that represents the MNO's revenue as a function of the total supplied capacity, at a point where the revenue red curve has a positive slope.
b. *Conservative Supply (CS):* Here, the total demand of users is larger than the MNO's supplying capability to deliver services relying on its own resources. That is when $\sum_{i\in\mathcal{N}} r_i^*(p) > C_{\mathsf{MNO}}$ holds, and hence the MNO needs to purchase additional resources C_{ADD}. In this case, the point where the (red) revenue curve $R_{\mathsf{MNO,D}}$ intersects with the revenue straight line $R^1_{\mathsf{MNO,S}}$, lies in the *Conservative Supply Region* where $R_{\mathsf{MNO,D}}$ has a negative slope.

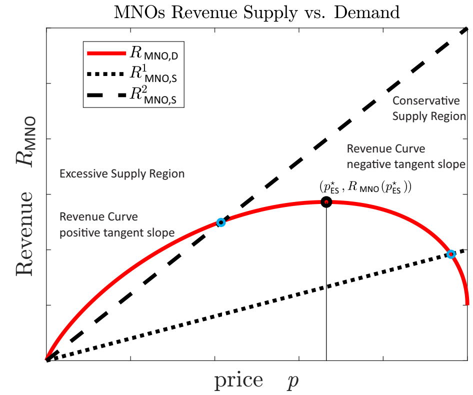


Fig. 3: *Excessive and Conservative Supply Region*: Red curve expressed by $p\sum_{i\in\mathcal{N}} r_i^\star(p)$ denotes MNO's revenue as a function of users' demand. Dotted lines show the corresponding revenue as a function of supplying resources and is expressed by $p\,(C_{\mathsf{MNO}} + C_{\mathsf{ADD}})$. The blue points where each dotted line intersects with the curve denotes the value of the MNO's revenue when the supply and demand are equalized. The black circle shows the maximum revenue that the MNO can achieve as a function of the optimal price $p^\star_{\mathsf{ES}}$ in the *Excessive Supply Region*.

The border between *Excessive* and *Conservative Supply Region* is indicated by the black dot circle which shows also the maximum revenue that the MNO can achieve as a function of the optimal price $p^\star_{\mathsf{ES}}$. The MNO's profit Π_{MNO} is the difference between its revenue $p\min\left(\sum_{i\in\mathcal{N}} r_i^*(p), C_{\mathsf{MNO}} + C_{\mathsf{ADD}}\right)$ and the incurred fixed cost $C_{\mathsf{MNO}}c + C_{\mathsf{ADD}}c_o$. The use of the *min* term denotes that the MNO's revenue is related to its service capability for capacity supply. The MNO can provide services up to the demand that its supply capability supports and leases any additional resource needed. Hence, providing services with the sole use of MNO's infrastructure comes at a cost c per capacity unit, whilst leasing the additional capacity costs c_o. The first term of the fixed cost expression refers to the operational expenditures (OPEX) of the MNO, while the second term refers to the capital expenditures (CAPEX). Consequently, the *MNO's profit* is expressed by:

$$\Pi_{\mathsf{MNO}}\left(p, C_{\mathsf{MNO}}, C_{\mathsf{ADD}}, C_{\mathsf{MUs}}, c, c_o\right) = p\min\left(\sum_{i\in\mathcal{N}} r_i^*(p), C_{\mathsf{MNO}} + C_{\mathsf{ADD}}\right) - C_{\mathsf{MNO}}c - C_{\mathsf{ADD}}c_o \quad (6)$$

The goal now for the MNO is to determine the optimal price $p^\star$ that maximizes the above profit, that is

$$p^\star = \arg\max_{p\geq 0} \Pi_{\mathsf{MNO}}\left(p, C_{\mathsf{MNO}}, C_{\mathsf{ADD}}, C_{\mathsf{MUs}}, c, c_o\right). \quad (7)$$

Depending on whether the MNO operates in the *Excessive* or in the *Conservative Supply* region, MNO's pricing differs, as it should account for the extra acquired resources. The above observations lead to the following finding.

Theorem 2. *The operator's optimal choice for price $p^\star$ depends on its service capability for capacity supply and it*

TABLE II: Excessive vs. Conservative Supply Region & Mobile Network Operator's Profits.

Region	***Obtained Resources***	***MNOs Profit*** $\Pi^{\star}_{\mathsf{MNO}}$
Excessive Supply (C_{ADD}=0)	$\frac{1}{e^{\frac{2}{bN}(C_{\mathsf{MNO}}-C_{\mathsf{MUs}})}-1} < W_{-e^{2\frac{C_{\mathsf{MUs}}}{bN}}-1}\left(2e^{2\frac{C_{\mathsf{MUs}}}{bN}}-1\right)$	$\Pi^{\star}_{\mathsf{MNO,ES}}=p^{\star}_{\mathsf{ES}}\sum_{i\in\mathcal{N}}r_i(p^{\star}_{\mathsf{ES}})-C_{\mathsf{MNO}}c$
Conservative Supply	$\frac{1}{e^{\frac{2}{bN}(C_{\mathsf{MNO}}+C_{\mathsf{ADD}}-C_{\mathsf{MUs}})}-1} \geq W_{-e^{2\frac{C_{\mathsf{MUs}}}{bN}}-1}\left(2e^{2\frac{C_{\mathsf{MUs}}}{bN}}-1\right)$	$\Pi^{\star}_{\mathsf{MNO,CS}}=p^{\star}_{\mathsf{CS}}(C^{\star}_{\mathsf{ADD}})\left(C_{\mathsf{MNO}}+C^{\star}_{\mathsf{ADD}}\right)-C_{\mathsf{MNO}}c-C^{\star}_{\mathsf{ADD}}c_o$

is different considering the following two cases:

a. *In the Excessive Supply (ES) Region, the optimal price $p^{\star}$ is given by:*
$$p^{\star}_{\mathsf{ES}}=\frac{1}{2b}\left(1-\frac{1}{\left(1+W_{-e^{2\frac{C_{\mathsf{MUs}}}{bN}}-1}\left(2e^{2\frac{C_{\mathsf{MUs}}}{bN}}-1\right)\right)^2}\right), \quad (8)$$
where $W_r(\cdot)$ denotes the generalized r-Lambert function [8]. The optimal price $p^{\star}_{\mathsf{ES}}$ is bounded within the inequality $p_{\mathsf{min}} \leq p^{\star}_{\mathsf{ES}} < p_{\mathsf{max}}$.

b. *In the Conservative Supply (CS) Region, the optimal price $p^{\star}$ is given by:*
$$p^{\star}_{\mathsf{CS}}=\frac{1}{2b}\left(1-\tanh^2(\theta)\right), \quad (9)$$
where $\theta=\frac{1}{Nb}\left(C_{\mathsf{MNO}}+C_{\mathsf{ADD}}-C_{\mathsf{MUs}}\right)$. The optimal price $p^{\star}_{\mathsf{CS}}$ is bounded within the inequality $p_{\mathsf{min}} \leq p^{\star}_{\mathsf{CS}} \leq p_{\mathsf{max}}$.

The detailed proof of Theorem 2 is given in Appendix [19]. The following two Lemmas determine accordingly the conditions for which optimal price $p^{\star}_{\mathsf{ES}}$ and $p^{\star}_{\mathsf{CS}}$ are limited between the minimum and maximum values that price p can take. Those conditions refer (*i*) to rate $r(p)$ as a function of p taking positive values, that is when $r(p) \geq 0$ and (*ii*) to the definition domain of the demand function which determines the set of prices p that rate $r(p)$, as an inverse hyperbolic secant function, can be defined.

Lemma 1. *The optimal price $p^{\star}_{\mathsf{ES}}$ is higher than the lower feasible price p_{min}, that is $p_{\mathsf{min}} \leq p^{\star}_{\mathsf{ES}}$, when the following inequality $1+W_{-e^{2\frac{C_{\mathsf{MUs}}}{bN}}-1}\left(2e^{2\frac{C_{\mathsf{MUs}}}{bN}}-1\right) \geq \coth(-\frac{k}{b})$ holds. Accordingly, the optimal price $p^{\star}_{\mathsf{ES}}$ is always lower than the maximum feasible price p_{max}, that is $p^{\star}_{\mathsf{ES}} < p_{\mathsf{max}}$.*

The proof of Lemma 1 is given in Appendix [19] and stems directly from the findings in Theorem 1.

Lemma 2. *The price $p^{\star}_{\mathsf{CS}}$ is higher than the lower feasible price p_{min}, that is $p_{\mathsf{min}} \leq p^{\star}_{\mathsf{CS}}$ when the following inequality $-\frac{k}{b} \geq \theta$ holds. In addition, the optimal price $p^{\star}_{\mathsf{CS}}$ has always a lower value than the maximum feasible price p_{max}, that is $p^{\star}_{\mathsf{CS}} \leq p_{\mathsf{max}}$, where $\theta=\frac{1}{Nb}(C_{\mathsf{MNO}}+C_{\mathsf{ADD}}-C_{\mathsf{MUs}})$.*

The proof of Lemma 2 is given also in Appendix [19]. After having specified the bounds within which the optimal prices $p^{\star}_{\mathsf{ES}}, p^{\star}_{\mathsf{CS}}$ are valued, we specify the border between the *Excessive Supply* and *Conservative Supply* regions. This finding is summarized by the following Theorem.

Theorem 3. *The Excessive and the Conservative Supply Region are characterized by the inequalities given in Table II.*

The proof of Theorem 3 is given in Appendix [19]. Next, we proceed in with the assessment of the MNO's capability for ample service provision and we specify the amount of additional resources that the MNO needs to purchase.

*C. **STAGE I**: The MNO determines the optimal amount of the leasing resources.*

In this Stage, the MNO decides the amount of the leasing resources that needs to acquire from a nearby InP in order to be able to satisfy its associated users' demand, as well as to maximize its profit given a fixed leasing cost c_o. Here, in this Stage, as it has been previously stated, the *Excessive Supply* case is non applicable. On the contrary, in this Stage, the operator performs in the *Conservative Supply Region* and its profit equals to
$$\Pi_{\mathsf{MNO,CS}}\left(p^{\star}_{\mathsf{CS}}, C_{\mathsf{MNO}}, C_{\mathsf{ADD}}, C_{\mathsf{MUs}}, c, c_o\right)=p^{\star}_{\mathsf{CS}}(C_{\mathsf{MNO}}+C_{\mathsf{ADD}})-C_{\mathsf{MNO}}c-C_{\mathsf{ADD}}c_o, \quad (10)$$
which can be written as $\Pi_{\mathsf{MNO,CS}}\left(p^{\star}_{\mathsf{CS}}, C_{\mathsf{MNO}}, \theta, C_{\mathsf{MUs}}, c, c_o\right)$ $=\left(\frac{1}{2b}\left(\tanh(\theta)\right)^{'}-c_o\right)(Nb\theta+C_{\mathsf{MUs}})-C_{\mathsf{MNO}}(c-c_o)$ by setting $\theta=\frac{1}{Nb}\left(C_{\mathsf{MNO}}+C_{\mathsf{ADD}}-C_{\mathsf{MUs}}\right)$. Before determining the optimal resource amount from the above expression of MNO's profit, first, we need to examine its concavity. We consider those values of θ for which the second derivative of the profit expression is negative $\frac{\partial^2\Pi_{\mathsf{MNO,CS}}}{\partial\theta^2} \leq 0$. The following Corollary determines the case, when the aforementioned profit expression is concave down and has a local maximum point.

Corollary 1. *In Stage I, the second partial derivative of the MNO's profit expression is negative $\frac{\partial^2\Pi_{\mathsf{MNO,CS}}}{\partial\theta^2} \leq 0$, when* $\tanh(\theta)-\operatorname{csch}(2\theta) \geq \frac{Nb}{Nb\theta+C_{\mathsf{MUS}}}$.

Proof. Taking the second derivative of the MNO's profit with respect to θ leads directly to the result. □

Now, taking the first derivative of the above profit expression with respect to θ and setting it equal to zero yields
$$\left(\tanh(\theta)\right)^{''}(Nb\theta+C_{\mathsf{MUs}})-Nb\left(\left(2bc_o-\tanh(\theta)\right)^{'}\right)=0. \quad (11)$$
Setting $\lambda=\tanh(\theta)$, we have the following second order differential equation with variable coefficients:
$$\lambda^{''}=\frac{Nb}{Nb\ \theta+C_{\mathsf{MUs}}}\left(2bc_o-\lambda^{'}\right), \quad (12)$$
with $\tanh(\theta)-\operatorname{csch}(2\theta) \geq \frac{Nb}{Nb\theta+C_{\mathsf{MUS}}}$. The solution to Eq. (12) determines the amount of resources (capacity), as expressed by θ (normalized capacity), for which the MNO's profit is maximized and it is given in the following Lemma.

Lemma 3. *The solution to the differential Equation (12) is:*
$$\lambda=2bc_o\theta+\left(\tanh(1)-2bc_o\right)\frac{\ln\left(\frac{Nb}{C_{\mathsf{MUs}}}\theta+1\right)}{\ln\left(\frac{Nb}{C_{\mathsf{MUs}}}+1\right)}, \quad (13)$$
with $\lambda=\tanh(\theta)$.

The detailed proof of Lemma 3 is shown in Appendix [19].

Approximation of optimal θ: The Eq. (13) in Lemma 3 involves a hyperbolic tangent function of θ, which hinders an analytical solution with a closed form to calculate directly the optimal $\theta^{\star}$ and, hence, $C^{\star}_{\mathsf{ADD}}(\theta^{\star})$. Therefore, we take advantage of the fact that we can approximate linearly the hyperbolic tangent close to an arbitrary positive value $\theta_o \geq 0$ by using Taylor's Theorem. This value can be considered as a pre-configured telecommunication's system parameter. Given a twice continuously differentiable function (e.g. f) of one real variable (e.g. θ), we can use Taylor's theorem, so that $f(\theta)=f(\theta_o)+f^{'}(\theta_o)(\theta-\theta_o)+R_2$, where R_2 is the remainder term, which is the approximation error. Dropping

TABLE III: MNO's and Users' Decisions.

Strategies \ Regions	***Conservative Supply Region*** $\alpha,\gamma,\delta>0$		$\alpha,\delta>0,\gamma<0$		***Excessive Supply Region*** (*Subgame Perfect Equilibrium*)	
Stage I - Approximation of optimal θ	$\theta_{W_0}(c_o)$	Eq. (15)	$\theta_{W_{-1}}(c_o)$	Eq. (16)	N/A	
Stage I - Additional Resources $C_{\mathsf{ADD}}(\theta)$	$C_{\mathsf{ADD}}\left(\theta_{W_0}(c_o)\right)$		$C_{\mathsf{ADD}}(\theta_{W_{-1}}(c_o))$		$C_{\mathsf{ADD}}=0$	
Stage II - Price p	$p_{\mathsf{CS}}(\theta_{W_0}(c_o))$	Eq. (9)	$p_{\mathsf{CS}}(\theta_{W_{-1}}(c_o))$	Eq. (9)	$p^{\star}_{\mathsf{ES}}$	Eq. (8)
Stage II - Profit Π_{MNO}	$\Pi_{\mathsf{MNO,CS}}(\theta_{W_0}(c_o))$		$\Pi_{\mathsf{MNO,CS}}(\theta_{W_{-1}}(c_o))$		$\Pi^{\star}_{\mathsf{MNO,ES}}(p^{\star}_{\mathsf{ES}})$	
Stage III - User Rate r_i	$r_i(p_{\mathsf{CS}}(\theta_{W_0}(c_o)))$	Eq. (3)	$r_i(p_{\mathsf{CS}}(\theta_{W_{-1}}(c_o)))$	Eq. (3)	$r^{\star}_i(p^{\star}_{\mathsf{ES}})$	Eq. (3)
Stage III - User Payoff π	$\pi(r_i(p_{\mathsf{CS}}(\theta_{W_0}(c_o))))$	Eq. (4)	$\pi(r_i(p_{\mathsf{CS}}(\theta_{W_{-1}}(c_o))))$	Eq. (4)	$\pi^{\star}(r_i(p_{\mathsf{ES}}))$	Eq. (4)

$C_{\mathsf{ADD}}(\theta)=Nb\theta+C_{\mathsf{MUs}}-C_{\mathsf{MNO}}$, $\quad \Pi_{\mathsf{MNO,CS}}(\theta)=\left(\frac{1}{2b}\tanh'(\theta)-c_o\right)(\theta Nb+C_{\mathsf{MUs}})-C_{\mathsf{MNO}}(c-c_o)$, $\quad \Pi^{\star}_{\mathsf{MNO,ES}}(p^{\star}_{\mathsf{ES}})=p^{\star}_{\mathsf{ES}}\sum_{i\in\mathcal{N}}r_i(p^{\star}_{\mathsf{ES}})-C_{\mathsf{MNO}}c$

the remainder R_2, we can obtain the linear approximation of $\tanh(\theta)$ close to θ_o. For positive values of θ, the function $\tanh(\theta)$ is concave and a piecewise linear upper bound [20] is obtained by: $\tanh(\theta)\leq\theta\,\mathrm{sech}^2(\theta_o)-\theta_o\,\mathrm{sech}^2(\theta_o)+\tanh(\theta_o)$. Hence, applying the above linear approximation of $\tanh(\theta)$ in Lemma 3 and ensuring that Corollary 1 holds, we have:

$$2bc_o\theta+(\tanh(1)-2bc_o)\frac{\ln\left(\frac{Nb}{C_{\mathsf{MUs}}}\theta+1\right)}{\ln\left(\frac{Nb}{C_{\mathsf{MUs}}}+1\right)}\leq\theta\,\mathrm{sech}^2(\theta_o)-\theta_o\,\mathrm{sech}^2(\theta_o)+\tanh(\theta_o). \quad (14)$$

The solution to the above inequality is an upper bound approximation for θ and it is attained close to θ_o. The following Theorem summarizes the approximation.

Theorem 4. *An upper bound approximation of the amount of leasing resources that maximizes the MNO's profit* Π_{MNO} *in Stage I is determined by evaluating the sign of the following variables* $\alpha=-\theta_o\,\mathrm{sech}^2(\theta_o)+\tanh(\theta_o)$, $\beta=\ln\left(\frac{Nb}{C_{\mathsf{MUs}}}+1\right)$, $\gamma=\gamma(c_o)=\frac{1}{\tanh(1)-2bc_o}$, *and* $\delta=\delta(c_o)=2bc_o-\mathrm{sech}^2(\theta_o)$, *and considering the following two cases:*

- *When* $\alpha,\gamma,\delta>0$, *the upper bound is expressed by:*
$$\theta_{W_0}(c_o)=\frac{1}{\beta\gamma\delta}W_0\left(\frac{C_{\mathsf{MUs}}}{Nb}\beta\gamma\delta e^{\beta\gamma\left(\alpha+\frac{C_{\mathsf{MUs}}}{Nb}\delta\right)}\right)-\frac{C_{\mathsf{MUs}}}{Nb}, \quad (15)$$
$\theta_{W_0}(c_o)$ *denotes the use of the main branch of the Lambert W function [21], and the leasing cost* c_o *is between*
$$c^{\mathsf{min}}_{o,W_0}\leq c_o\leq c^{\mathsf{max}}_{o,W_0},$$
where $c^{\mathsf{min}}_{o,W_0}=\frac{\mathrm{sech}^2(\theta_o)}{2b}$ *and* $c^{\mathsf{max}}_{o,W_0}=\frac{\tanh(1)}{2b}$.
- *When* $\alpha,\delta>0$ *and* $\gamma<0$, *the upper bound is expressed by:*
$$\theta_{W_{-1}}(c_o)=\frac{1}{\beta\gamma\delta}W_{-1}\left(\frac{C_{\mathsf{MUs}}}{Nb}\beta\gamma\delta e^{\beta\gamma\left(\alpha+\frac{C_{\mathsf{MUs}}}{Nb}\delta\right)}\right)-\frac{C_{\mathsf{MUs}}}{Nb}, \quad (16)$$
$\theta_{W_{-1}}(c_o)$ *denotes the use of the negative branch of the Lambert W function, and the leasing cost* c_o *is between*
$$c^{\mathsf{min}}_{o,W_{-1}}\leq c_o\leq c^{\mathsf{max}}_{o,W_{-1}},$$
where $c^{\mathsf{min}}_{o,W_{-1}}=\frac{\tanh(1)}{2b}$ *and* $c^{\mathsf{max}}_{o,W_{-1}}=c_{o_{\{\theta_{W_{-1}}(c_o)=\theta_o\}}}$.

The analytical proof is presented in the Appendix [19]. The two different branches refer to the case whether γ is positive or negative, that is whether cost is $c_o\leq\frac{\tanh(1)}{2b}$ or $c_o>\frac{\tanh(1)}{2b}$. Moreover, $c^{\mathsf{max}}_{o,W_{-1}}$ is the maximum cost for the minimum θ_0 resources that the MNO can agree to purchase from the InP, in order to get benefit.

D. Equilibrium Points and Optimality

Table III summarizes the MNO's leasing and pricing decisions, the rate allocations to the users as well as the MNO's profits and users' payoffs. The optimal solution for the amount of additional resources expressed by Eq. (13) in Stage I requires further the determination of the optimal θ in a closed form expression to fully characterize the equilibrium points in the *Conservative Supply* region and identify in detail the system parameter dependencies that affect the optimal strategies. Though the calculation of optimal θ was not possible in a closed form, however, we were able to attain an upper bound approximation to the optimal amount of resources to be purchased close to a desired point θ_o. On the contrary, in the *Excessive Supply* region sub-game perfect equilibrium exist [22]. We observe that the optimal pricing $p^{\star}_{\mathsf{ES}}$, in this region, is derived by using the negative branch of the r-Lambert function [8] which is dependent on the number of users N, their sensitivity b and their minimum requested demand C_{MUs}.

V. Performance Evaluation

In this Section, we evaluate our model using both Matlab simulation as well as implementation on devices in a testbed.

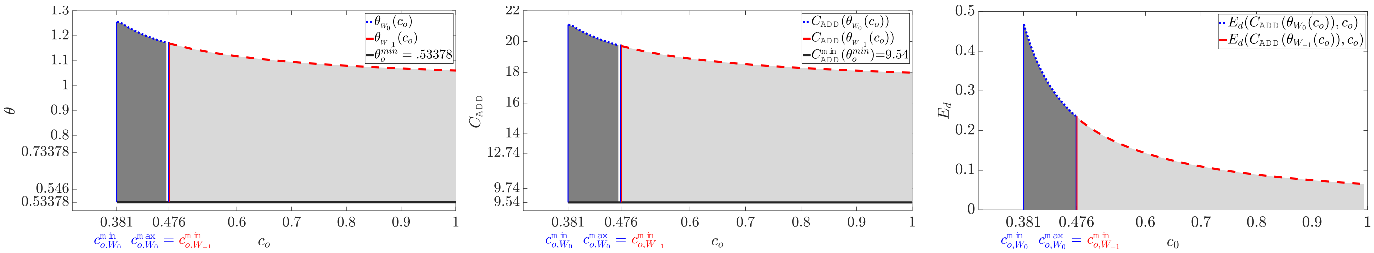
A. Software Simulations

We consider a RAN consisting of one mobile network operator, one infrastructure provider and $N=20$ mobile homogeneous users with $b=0.8$. Table IV summarizes the operational system characteristics. The MNO servicing capability cannot satisfy the requested user demand and the MNO can provide initially a maximum capacity of $C_{\mathsf{MNO}}=12$ Mbps to service its associated users. Linear pricing is applied by the MNO to its users as well as by the InP to the MNO. The minimum aggregated demand of users is $C_{\mathsf{MUs}}=13$ Mbps which over-exceeds the MNO's service capability. Therefore, we can infer that our system operates in the conservative supply region. Since $\theta^{\mathsf{min}}_o=\mathrm{arsech}(\sqrt{\tanh(1)})=0.53378$, which is the minimum acceptable value for θ_o, (see Proof of Theorem 4 in [19]), we choose an arbitrary operational point[4] $\theta_o=0.73378>\theta^{\mathsf{min}}_o$, and we recalculate the minimum leasing cost based on θ_o, thus $c^{\mathsf{min}}_{o,W_0}(\theta_o)\approx0.381\$$. The price p_{CS} is

TABLE IV: System Model Configuration Settings.

Parameter	***Value***
Number of Mobile Users	$N=20$
Mobile Users Service Sensitivity	$b=0.8$
MNO's Capacity	$C_{\mathsf{MNO}}=12$ Mbps
Minimum User's Rate Demand	$C_{\mathsf{MUs}}=13$ Mbps
Minimum Normalized Servicing Rate	$\theta^{\mathsf{min}}_o=\mathrm{arsech}(\sqrt{\tanh(1)})$
Operational Normalized Servicing Rate	$\theta_o=0.73378$
Chosen Normalized Servicing Rate	$\theta_{\mathsf{CS}}=0.546$
Price in Conservative Region	$p_{\mathsf{CS}}=0.47\$$
Total User's Rate Demand	$r=21.74$ Mbps

[4] θ_o depends on MNO's sytem configuration. This particular value is determined by the initial conditions selected to solve Eq. (12) in Lemma 3. Based on the MNOs configuration, the selection of the initial conditions may vary. e.g. instead of using $\tanh(1)$ and $\tanh(0)$, different initial conditions can be selected according to MNO's operational characteristics.



(a) Normalized rate θ vs. leasing cost c_o \$.

(b) C_{ADD} (Mbps) vs. leasing cost c_o \$.

(c) Elast. of Demand E_d vs. leasing cost c_o \$.

Fig. 4: Red (blue) dashed line indicates that the solution derived from Theorem 4 uses the negative W_{-1} (main positive W_0) branch of the Lambert identity.

also calculated using an arbitrary θ_{CS}=0.546 in Eq. (9), chosen to belong in $[\theta_o^{\min}, \bar{\theta}_o]$.

Then, the total user rate demand is calculated according to Eq. (5), which is the sum of the minimum users' demand C_{MUs} plus the extra demand that is requested from users which is $Nb\,\text{arsech}(\sqrt{2bp_{\text{CS}}})$ and it depends on the servicing price $p_{\text{CS}}(\theta_{\text{CS}}) = \frac{1}{2b}(1-\tanh^2(\theta_{\text{CS}})) = 0.47\$$. The total demand $r = Nb\,\text{arsech}(\sqrt{2bp_{\text{CS}}}) + C_{\text{MUs}} = 21.74$ Mbps is larger than the MNO's capacity C_{MNO}=12 Mbps. Thus, the MNO needs to purchase at a cost c_o additional resources from the InP to fullfil this deficiency.

The minimum required amount is definitely $C_{\text{ADD}}^{\min} = r - C_{\text{MNO}} \geq 0$ but the raising question is *"how much more should the MNO acquire to benefit from such a purchase?"*. It should be noted that the MNO not only seeks to fill at least its deficit in capacity $C_{\text{ADD}}^{\min}$ but also to maximize its profit gain. The solution in Stage I (Theorem 4) determines the answer by providing an approximation of the upper bound on the amount of resources to be purchased by the MNO at a leasing cost c_o. An interesting point is the impact of a variable leasing cost to the amount of resources that the MNO should acquire. Therefore, for different leasing costs c_o, falling inside the feasibility boundaries $[c_{o,W_0}^{\min}, c_{o,W_0}^{\max}]$ and $[c_{o,W_{-1}}^{\min}, c_{o,W_{-1}}^{\max}]$, we examine the MNO's responsiveness to the amount of additional resources C_{ADD}. That demand is determined directly using the following equation $C_{\text{ADD}}(\theta(c_o)) = Nb\theta(c_o) + C_{\text{MUs}} - C_{\text{MNO}}$ and by applying Theorem 4. Fig. 4a and Fig. 4b show the MNO's demand for the normalized rate θ and the actual rate C_{ADD} accordingly for a varying leasing cost c_o. An interesting observation reveals that the MNO's demand for additional resources changes less (decreases) than the leasing cost changes (increases). This can be numerically proved by applying the arc elasticity rule[5] between two given points of the pair (C_{ADD}, c_o) which shows that the coefficient of elasticity of demand with respect to leasing cost is $E_d < 1$ (see Fig 4c). Hence, we can infer that MNO's sensitivity to cost change is inelastic. That, has a direct positive impact on the total revenue that the InP can make which means that a rise in leasing cost c_o leads to an increase in InP's total revenue.

B. Testbed Experimentation Setup

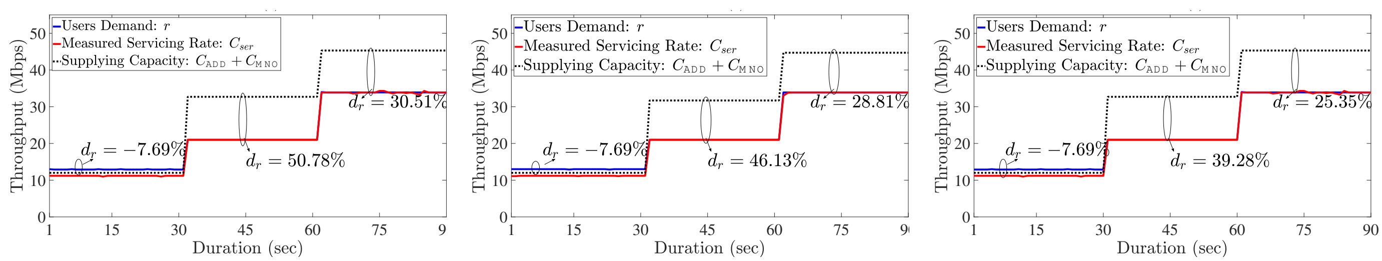
To assess the performance of realistic scenarios, we conducted experimentation by building a 5G network in the NITOS Testbed [23]. We used the OpenAirInterface (OAI) [24] framework on the USRP B210 Software Defined Radio [25] to create the core network and initiate one gNB. For the mobile users we used usb dongles (Qualcomm MDM9200 [26] chipset) to instantiate the UEs. Leveraging FlexRAN [3] platform along with OAI allowed us for the flexible and programmable control of the underlying RAN infrastructure as well as for the real-time configuration and allocation of the gNB's resources to the MNO's slice. In this scenario we assumed that the MNO has one slice that uses the resources offered by the InP that owns the gNB. For the MNO we adopted the same configuration settings as in the simulation part (see Table IV). Moreover, the gNB is configured with 20MHz bandwidth, 64 QAM, SISO in FDD mode and its total measured capacity is 54.55Mbps against the theoretical maximum of 75Mbps[6]. The MNO's slice is initially configured with a pre-defined bandwidth budget that corresponds to the 22% of the gNB's 54.55Mbps max capacity (thus $C_{\text{MNO}} \approx 12$Mbps). This setting can be re-configured on-the-fly and on demand according to the MNO's needs to service its users by using the FlexRAN API. Due to hardware limitations, we used 4 UEs but we emulated the actual number ($N = 20$ as in the simulation) by creating multiple UDP traffic flows (5 per UE) requesting download services with the $iperf$ command.

Fig. 5 and Table V summarize the collected rate results for three scenarios with different leasing costs c_o. Demand r is the same to all scenarios, and hence the average servicing rate C_{ser} at the UEs side is also measured the same in all scenarios, since the gNB services the same rate of demand. However, the MNO's total supplying capacity (and hence the amount of additional resources) changes (decreases) as the cost increases. Although this is expected, according also to the simulation results, here, our purpose is to demonstrate the responsiveness of the FlexRAN implementation for an MNO to acquire the beneficial amount of resources using our leasing scheme. Initially, in the first 30secs, the MNO's supplying capacity is configured to be less than the minimum aggregated users' demand $C_{\text{MNO}} = 12 < r = C_{\text{MUs}} = 13$ Mbps, since $C_{\text{ADD}} = 0$. We observe a negative percentage change[7] at $d_r = -7.69\%$ between the supplying capacity $C_{\text{MNO}} + C_{\text{ADD}}$ and the requested demand r, as well as a negative percentage difference between servicing rate C_{ser} and demand r at -13.07%. Then, in the time period of $31-60$secs, the MNO announces a price p=0.47\$ and users respond by requesting an extra demand based on that price according to $Nb\,\text{arsech}(\sqrt{2bp})$. As a result the total user demand rises to 21.74 Mbps. To mitigate this incident, the MNO calculates its deficit and requests to purchase additional resources. The MNO's slice is reconfigured accordingly using FlexRAN and its supplying capacity C_{ADD}+C_{MNO} is increased to accommodate the new total users' demand and the percentage change between C_{ser} and r reduces now to -1.28%. Moreover, when the cost is $c_o = 0.4\$$, the additional leased capacity (applying

[5]The arc elasticity E_d of demand C_{ADD} with respect to cost c_o is calculated as $E_d = \frac{\textit{\% change in } C_{\text{ADD}}}{\textit{\% change in } c_o} = \frac{C_{\text{ADD}}^{(0)} - C_{\text{ADD}}^{(1)}}{(C_{\text{ADD}}^{(0)} + C_{\text{ADD}}^{(1)})/2} \Big/ \frac{p^{(0)} - p^{(1)}}{(p^{(0)} + p^{(1)})/2}$.

[6]This difference is due to the type of antennas used in the testbed and the lack of an amplifier.

[7]The percentage change between x and y is $d_r(x,y) = \frac{x-y}{|y|}100\%$.



(a) Leasing cost $c_o = 0.4$ \$.   (b) Leasing cost $c_o = 0.47$ \$.   (c) Leasing cost $c_o = 0.8$ \$.

Fig. 5: Collected measurements for three different leasing costs c_o. Servicing C_{ser} and demand r rate per time period is the same to all scenarios. MNO responds differently to an increasing leasing cost c_o and its total supplying capacity $C_{\text{ADD}}+C_{\text{MNO}}$ differs according to the amount of leased resources C_{ADD}.

TABLE V: EXPERIMENTATION MEASUREMENTS - RESULTS.

sec	C_{MNO}	C_{MUs}	r	C_{ser}	$\frac{C_{\text{ser}}-r}{r}100\%$	c_o = *0.4* \$			c_o = *0.47* \$			c_o = *0.8* \$		
						C_{ADD}	$C_{\text{ADD}}+C_{\text{MNO}}$	d_r	C_{ADD}	$C_{\text{ADD}}+C_{\text{MNO}}$	d_r	C_{ADD}	$C_{\text{ADD}}+C_{\text{MNO}}$	d_r
0-30	12	13	13	11.3	-13.07%	0	12	-7.69%	0	12	-7.69%	0	12	-7.69%
31-60	12	13	21.74	21.46	-1.28%	20.78	32.78	50.78%	19.77	31.77	46.13%	18.28	30.28	39.28%
61-90	12	26	34.74	34.34	-1.15%	33.34	45.34	30.51%	32.75	44.75	28.81%	31.55	43.55	25.35%

Theorem 4, with θ_o=0.73378) is $C_{\text{ADD}}(\theta_{W_0}(c_o))$=20.78Mbps (and $C_{\text{ADD}}(\theta_{W_0}(c_o))=19.77$, $C_{\text{ADD}}(\theta_{W_{-1}}(c_o))=18.28$Mbps for c_o=0.47\$ and 0.8\$, respectively). The percentage change d_r now has a positive increase at (50.78, 46.13 and 39.28% respectively) in comparison with the previous time period but it decreases as the cost increases. Between $61-90$secs, the C_{MUs} demand is intentionally doubled (to emulate the arrival of new users) but the price p is kept the same. The MNO requests additional capacity at the same cost c_o to fulfill this extra demand and its slice is re-configured accordingly.

VI. CONCLUSION

In this paper, we studied the problem for a MNO of leasing resources, servicing and pricing mobile users in the context of 5G networks and beyond. The evolution of the underlying technologies to support flexible re-configuration of radio access resources on demand has matured [3], [4]. As operators seek ways to maximize their return on investment and offer high-reliability service for time-critical applications, leasing resources on-demand, to expand their available capacity whilst servicing real-time inelastic traffic, seems beneficial against the increasing infrastructure costs. Our analysis models the interactions among users and one MNO in a monopolistic market and proposes pricing and resources leasing that can be applied on real communication systems to help operators offer reliable services and grow their business. We have discovered the game equilibrium for the *Excessive Supply* Region where operator's capacity suffices to support users' demand, whilst obtaining an approximation to the optimal amount of the leased resources in the *Conservative Supply* Region. An interesting extension to this work is the study of the double chain monopolistic market and the relevant mechanisms (e.g non-linear pricing) to prevent the double marginalization issue among operators and providers.

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