A Cost Analysis of Wireless Mesh Networks

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Abstract

A Wireless Mesh Network (WMN) is composed of multiple Mesh Nodes that are connected together using the radio channel and rely on the backhaul capacity available at a limited number of Gateway Nodes. In this paper, we address the problem of assessing the generic cost of a WMN and comparing it with the cost of other supported network solutions, e.g. a cellular network. We restrict our analysis to regular topologies and make simplifying assumptions about the cost breakdown, without introducing however any bias in favour of multi-hop solutions. For any choice of the parameters, our model determines the feasible network design solutions, that satisfy the traffic demands of all Mesh Nodes thanks to optimal scheduling, and the solution having the minimum cost among all these ones. Our cost analysis takes in account the problem of interference and implies an efficient use of the available resources through spatial reuse.

I. Introduction

Wireless Mesh Networks (WMNs) rely on the multi-hop paradigm but differ from ad-hoc networks in a significant way. First of all, they are more likely to be planned, i.e. their topology and deployment characteristics can be the result of a prior design, while ad hoc networks are less supervised and more dynamic in nature [1]. Secondly, in contrast with the communication matrix characterizing ad-hoc networks, the typical traffic model for a WMN considers only traffic requirements of the mesh nodes [2]. Such requirements generates traffic flows connecting the mesh nodes to the gateway nodes. Such flows can be unidirectional or bidirectional, symmetric or not. All the gateway nodes are equivalent and there is no reason a mesh node should connect to a specific gateway node rather than any other, the choice falling for example on the nearest gateway.

A typical example is an infrastructure/backbone WMN [3], [4]: in it, gateway nodes are Access Points connected to the backbone (thanks to T1, DSL or Ethernet backhaul) and multi-hop operation provides download and/or upload access at a greater number of locations, each of those equipped with a wireless mesh node and accessed by one or multiple clients. Another example is a wireless sensor network with data retrieval gateways [5], [6]. Mesh nodes have been opportunely located and endowed with sensing capabilities. These nodes collect from the environment data which is passed over multiple hops to the gateways, able to process it. Communication from the gateways to the sensors is also possible, in the form of control messages.

WMNs requiring the deployment of gateways belong then to the class of supported networks; they are not dissimilar to cellular networks that use base stations, or other network solutions combining wired and wireless links. In this paper, we ask the question of when a WMN is less expensive than any other supported network. To answer it, we introduce a cost model assuming linear dependencies between certain network cost factors and the backhaul capacity and the wireless capacity, respectively. Section II describes such a model and its use in network planning.

Sections III and IV discuss in detail economical and technological feasibility for the line topology and for regular topologies, respectively. We restrict our analysis to regular topologies recalling that WMNs are likely to be planned and that regular topologies are those guaranteeing the best coverage. In these two sections, we derive several rules of thumb and give their validity. Finally, Section V recapitulates our main contributions.

Let us mention here that our cost analysis can be transformed into a capacity analysis once we fix the budget for the network. Then, we interpret straightforwardly the least expensive deployment as the most capacitated one. As stated before, WMNs differ from ad-hoc networks. First, the nature of flows is very clear in a WMN and communication between distant nodes typical of an unstructured network is removed. Then, the WMN can simply scale by installing more gateway. Moreover, increasing the number of nodes is likely to increase the budget for the network. This paper makes the following assumptions for WMNs: (i) increasing the backhaul capacity of gateways generates savings that motivate the deployment of a mesh network and (ii) there are two main cost factors that impact on the budget of a WMN, i.e. increasing the wireless link capacity and installing more gateways. Our cost analysis focuses on these costs.

We do not aim at formulating capacity lower bound as the result of our analysis. In their fundamental work, Gupta and Kumar [7] calculated the capacity of a random ad-hoc network with n nodes to scale as $O(1/\sqrt{n})$. The capacity lower bound was later corrected to $O(1/n)$, imposing a bounded attenuation function [8]. Moreover, it was shown that the same lower bound holds for both omni and directional antennas , even if directional antennas achieve more capacity [9]. Assuming all traffic goes through one gateway, the capacity is again $O(1/n)$, as shown by [10]. In general, a WMN will have more gateways and its capacity depends on the number of gateways m relative to the number of nodes n. [11] identified three different scaling regimes of the capacity in a WMN considering the growth of m with respect to n. If we take m to grow linearly with n,

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then the capacity no longer depends on the number of nodes.

Deployment cost of wireless network with relays and meshes was investigated for testbeds on the field [12]. Moreover, authors in [13] suggested to compute the capacity of a WMN using random topologies, Monte Carlo simulations and an optimization algorithm for the scheduling. They also computed the deployment cost of the WMN as a function of n and m. A sensibility analysis of the deployment cost of a WMN has been proposed [14], which makes use of typical WiMAX communication parameters, cost figures and relevations of effective link capacity for different distances. The authors assume a triangular mesh for carrier-grade deployment and impose fairness constraints on the user demands. They distinguish between capacity-limited and range-limited operation and compute, using simulations and tabu optimization, the cost of a WMN for different cluster formations with varying ratio of m over n and for different link capacities. We aim however at providing an analytical framework to perform cost analysis with a greater expressive power than simulation-based studies.

II. A cost model for Wireless Mesh Networks

Let us introduce our cost model for Wireless Mesh Networks. WMNs are formed by two kinds of wireless nodes (WNs): (i) mesh nodes (MNs) that are the sources of uplink traffic and destinations of downlink traffic and (ii) gateway nodes (GNs), that can process - up to a certain fixed capacity - traffic flows for any of the mesh nodes. MNs and GNs communicate together via wireless links and communication paths may require cooperation of other nodes, spanning over multiple hops. Note that a GN can also be a traffic termination, i.e. be simultaneously a MN. Then, the GN serves its own traffic and - if such traffic is lower than its capacity as a gateway - there is no need for this traffic to traverse the WMN. Traffic flows only exist between a MN, at one termination, and a GN, at the other one. Moreover, a MN which does not have an own traffic demand is called a relay node (RN), its role in the WMNs being solely to provide connectivity to other MNs.

MNs are characterized by their traffic demands - both uplink, i.e. toward GNs, and downlink, i.e. from GNs. Traffic demands are described in terms of bandwidth and usual unit of measure are bps or Bytes/s. In this analysis, we assume an homogeneous scenario, i.e. we assume that all the WNs have the same traffic demand and we call it d. We do not distinguish between uplink and downlink traffic, because, as we will show, they obey the same constraints in the regular topologies object of our study.

This homogeneity assumptions makes our cost analysis very concise. Obviously, nothing prevents high dishomogeneity in the traffic demands of the MNs. Such situations can be dealt only through network planning optimization taking as an input all the traffic demands node entries. A possible solution is to formulate the gateway placement problem and to augment it with the constraints of routing and scheduling imposed by wireless operation [15]. Our objective here is however to derive general design principles in the case traffic demands are homogeneous and exploit such regularity.

WNs communicate via wireless with each other. WNs are then also characterized by their wireless capacity. This capacity is described again in terms of bandwidth. A given wireless capacity can be reached over a link only if both the wireless sender and the wireless receiver support that capacity. As a consequence, it makes little sense for a GN to support higher wireless capacity than any other MN in the network. In the following, we suppose that all WNs have the same radio and can communicate with the same capacity, that we call $c_{wireless}$.We will introduce more assumptions on wireless communication in Section III.

Intuitively, we are interested for our design problem in values of wireless capacity such that:

$$\frac{d}{c_{wireless}} \leq 1. \tag{1}$$

Let us consider the first possible bottleneck represented by the finite capacity of the GN to serve traffic. The easiest solution deploys identical GNs, i.e. gateways characterized by the same value of backhaul capacity $c_{backhaul}$ (expressed in the bandwidth unit of measure). Because the backhaul capacity is limited, the ratio between the number of GNs and that of WNs needs to be:

$$\frac{\#_{GN}}{\#_{WN}} \geq \frac{d}{c_{backhaul}}, \tag{2}$$

in which $\#$ represents the cardinality operator.

Let us introduce cost elements in the design of the network related to CAPEX and OPEX per WN:

- $\mathcal{C}_{GN}$: the fixed cost of installing a GN with backhaul capability;
- $\mathcal{C}_{backhaul} = f(c_{backhaul})$: the backhaul cost of providing an installed GN with the given backhaul capacity;
- $\mathcal{C}_{radio} = f(c_{wireless})$: the cost of a wireless radio interface for the WN that can support the given wireless capacity;
- $\mathcal{C}_{spectrum} = f(c_{wireless})$: the cost of reserving a given wireless spectrum bandwidth in the area of coverage of the WN.

Let us suppose that the last three function dependencies are linear. However, it must be $\mathcal{C}_{GN} \neq 0$. The meaning of $\mathcal{C}_{backhaul} \propto c_{backhaul}$ is that backhaul capacity is available at least as units of traffic demand d and there are no economies of scale.[1] The meaning of $\mathcal{C}_{radio} \propto c_{wireless}$ is that there exists a variety of radio equipments realizing at least values of $c_{wireless}$ that are multiple of d. A easy wax to provide this is through multi-radio equipment. The assumption that these costs scale linearly is then conservative. Finally, the meaning of $\mathcal{C}_{spectrum} \propto c_{wireless}$ is that wireless capacity

[1]In order to justify the deployment of a mesh network, either wired or wireless, economies of scale in the backhaul are indeed necessary. Note that the minumum backhaul capacity is equal to the traffic demand d. Let us assume that a gateway with that backhaul capacity has cost $\mathcal{C}_1$. Then, we must have a gateway with backhaul capacity equal to $nd, n > 1$ for a cost $\mathcal{C}_2$ lower than $n\mathcal{C}_1$. To get our formulation back, we compute the marginal cost per minimum capacity unit as $\mathcal{C}_{backhaul}(d) = \frac{\mathcal{C}_2 - \mathcal{C}_1}{n-1}$ and the fixed cost of gateway installation as $\mathcal{C}_{GN} = \frac{n\mathcal{C}_1 - \mathcal{C}_2}{n-1}$.

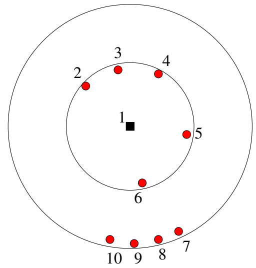

Fig. 1. One-GN wireless topology

is available at least as units of d and there are no economies of scale. Obviously, $\mathcal{C}_{spectrum} = 0$ when using the unreserved spectrum.

Then we can write:

$$\mathcal{C}_{tot} = \#_{GN} \cdot \mathcal{C}_{GN} + \#_{WN} \cdot \mathcal{C}_{backhaul}(d) + \\ + \#_{WN} \cdot (\mathcal{C}_{radio}(d) + \mathcal{C}_{spectrum}(d)) \cdot \frac{c_{wireless}}{d} \tag{3}$$

and

$$\overline{\mathcal{C}}_{WN} = \frac{\#_{GN}}{\#_{WN}} \cdot \mathcal{C}_{GN} + \mathcal{C}_{backhaul}(d) + \\ + (\mathcal{C}_{radio}(d) + \mathcal{C}_{spectrum}(d)) \cdot \frac{c_{wireless}}{d}. \tag{4}$$

Let us coin:

- $x = \frac{d}{c_{wireless}}, 0 < x \leq 1$
- $y = \frac{\#_{GN}}{\#_{WN}}, \frac{d}{c_{backhaul}} \leq y \leq 1.$

Let us further introduce:

$$\rho = \frac{\mathcal{C}_{radio}(d) + \mathcal{C}_{spectrum}(d)}{\mathcal{C}_{GN}} \tag{5}$$

in order to write:

$$\frac{\overline{\mathcal{C}}_{WN}}{\mathcal{C}_{GN}} = y + \frac{\rho}{x} + \frac{\mathcal{C}_{backhaul}(d)}{\mathcal{C}_{GN}}. \tag{6}$$

In order to justify the deployment of a wireless network, you must have $\overline{\mathcal{C}}_{WN} < \mathcal{C}_{GN} + \mathcal{C}_{backhaul}(d)$, i.e. wireless transmission should be more convenient than providing backhaul capability to all nodes. This is equivalently stated as:

$$\rho < x(1-y). \tag{7}$$

Since the wireless medium is shared among all the WNs, we have a possible bottleneck at the radio interface of the GNs. Figure 1 represents a topology with WNs placed at two different distances from the AP node 1. In the following we will evaluate the capacity of a given topology using the concept of bottleneck collision domain [10]. Irrespectively of whether multi-hop relaying for nodes in the second circle is used or not, because of the traffic demand of the nodes, it must be that:

$$x < \frac{1}{\frac{1}{y} - 1}, \tag{8}$$

$\frac{1}{y}$ being the number of WNs in the one-GN network. This relationship can be interpreted as a design constraint by isolating y:

$$y > \frac{x}{x+1}. \tag{9}$$

By employing this constraint, we obtain:

$$\rho < \frac{x}{x+1}, \tag{10}$$

where the right term attains its maximum for $x = 1$. The deployment of a wireless network can be then justified only if:

$$\rho < 0.5. \tag{11}$$

Figure 2 shows inequality (9) plotted on the xy plane. Below this line there is the region of unfeasible design. The design plane, in fact, is characterized by two directions, i.e. the horizontal direction of decreasing wireless capacity $c_{wireless}$ with respect to the traffic demand d and the vertical direction of increasing the number of GNs over the total of WNs. It is intuitive that a design with too few wireless capacity and too few GNs is infeasible. The figure shows also contour lines of equation (6) for the limit case $\rho = 0.5$. The value attained by each contour line should be interpreted as $\overline{\mathcal{C}}_{WN} - \mathcal{C}_{backhaul}(d)$ expressed in fractional unit of $\mathcal{C}_{GN}$. The intersection between the contour line with unitary value and the region described by inequality (9) corresponds to the point of coordinates $x = 1$ and $y = 0.5$, i.e. to a design having one GN every two WNs and WN capacity equal to the traffic demand. Inequality (9) is not the only constraint to which wireless networks can be subject, but we must consider also *interference*. Interference depends on the specific topology of the wireless network. Let us focus our study on regular topologies, for which we can derive general feasibility constraints.

III. Analysis of relaying in 1-D topology

Let us now focus on a particular topology: a 1-D topology is such a one that locates wireless nodes along a line. The best coverage of a 1-D topology is given by a series of equally spaced node and we concentrate our study on an infinite topology without losing generality.

Let us suppose an ideal wireless protocol, that is contention-free and permits assigning the total of the wireless capacity to user traffic demands. We assume nodes to be single-radio. As a consequence, they cannot transmit and received at the same time. To schedule transmissions at the same time on the same channel, we must ensure that the schedule will avoid the *interference*. Let us adopt the protocol interference model [7]. Wireless links that do not share any termination can be simultaneously active iff receivers are located within a distance r_1 from their reciprocal sources and no receiver is within a distance $r_2 > r_1$ from a source other than the one from which is receiving. We call r_1 and r_2 the communication range and the interference range, respectively.

Let the communication range r_1 coincide with the inter-node separation in our analysis. The most spatial reuse is reached when the interference range $r_2 < 2r_1$. With this assumption, we obtain the following constraints: if $x > 1/2$, then $y \geq 1/2$ (i.e. we need at least one GN every two nodes to satisfy the traffic demands) and, if $x \in (1/3, 1/2]$, then $y \geq 1/3$ (i.e. we need at least one GN every three nodes to satisfy them).

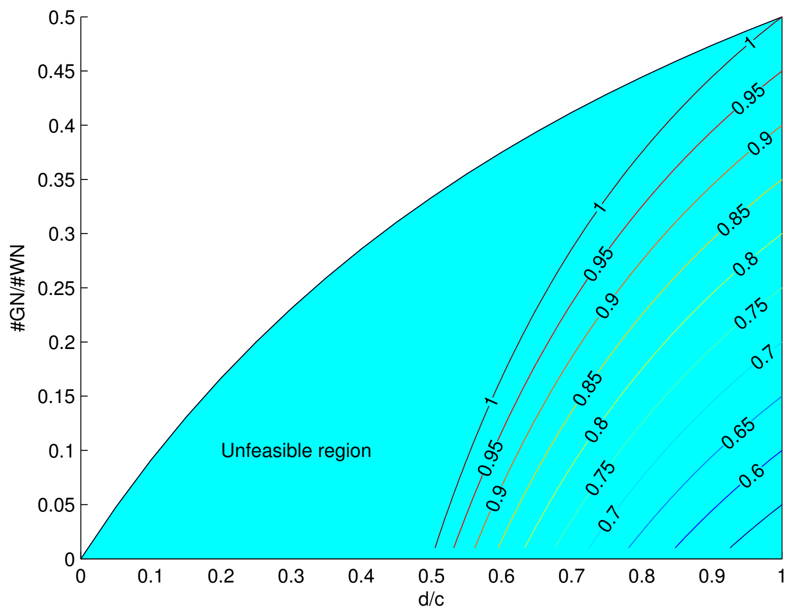

Fig. 2. xy plane for wireless design. WN average cost for the limit case when $\rho = 0.5$ is expressed by the contour lines as a fraction of the fixed cost of installing a node as a gateway. In this limit case, no wireless design that is economical (i.e. WN average cost below the unit) is also feasible.

If $x \leq 1/3$ and wireless nodes can relay traffic for other nodes, we can build a multi-hop mesh network. GNs are then equally spaced and there is only one solution to the routing, i.e. using as next hop the neighbor nearest to a GN. However, when relaying traffic over multiple hops, the bottleneck in the network can move from the radio interface of the GW to the radio interface of the MN adjacent to the GN. We refer to Fig. 3, where the collision domain of such a node is considered. The collision domain comprises all communications that interfere with the reception or are interfered by the transmission of a particular node [10].

The collision domain of the WN adjacent to the GN give rise to the following additional design constraints: if $x \in (1/4, 1/3]$, then $y \geq 1/4$; if $x \in (1/6, 1/4]$, then $y \geq 1/5$; if $x \in (1/9, 1/6]$, then $y \geq 1/7$; if $x \in (1/12, 1/9]$, then $y \geq 1/9$; if $x \in (1/15, 1/12]$, then $y \geq 1/11$; and so on. By taking $b \in \mathbb{N}, b \geq 2$, we can write that, if $x \in (\frac{1}{3(b+1)}, \frac{1}{3b}]$, then $y \geq \frac{1}{2b+3}$.

These constraints are tighter than inequality (9). The same constraints apply for both uplink and downlink traffic, and, with the assumption of Time-division Duplexing (TDD), for any combination of uplink and downlink traffic.

It is possible to interpolate with an hyperbole points marking transitions in the number of GWs in the mesh, obtaining as a lower bound the inequality:

$$y \geq \frac{3x}{2+9x}, 0 < x \leq \frac{1}{7} \tag{12}$$

A natural question will rise about the meaning of points lying on the curve of inequality (12) when they are different from the stair functions. We can provide three possible explanations for them:

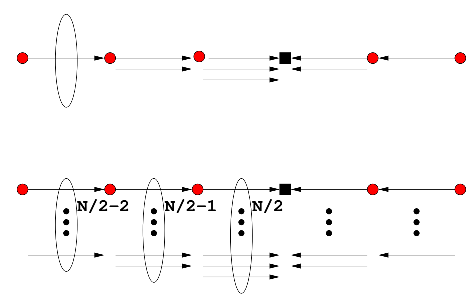

Fig. 3. Linear mesh topologies presenting a bottleneck in the collision domain of the WN adjacent to the GN.

1) y maintains the same meaning, however x becomes only the average of the satisfied bandwidth demands. Some nodes (which have to share the backhaul capacity of a GN $c_{backhaul}$ in a greater number) receive less bandwidth than others;
2) maintaining same node density, nodes are no longer equally spaced and nodes that have to share a GN in a greater number are positioned closer to each other, permitting to achieve more capacity in the transmission;
3) it is possible to connect a MN to multiple GNs (in case of 1-D topology, to two GNs). This requires multi-path to be a feasible solution for the WMN.

Only in the case of accepting one of these assumptions, it makes sense to adopt the continuous line in the subsequent analysis.

We want to determine the most economical design for a

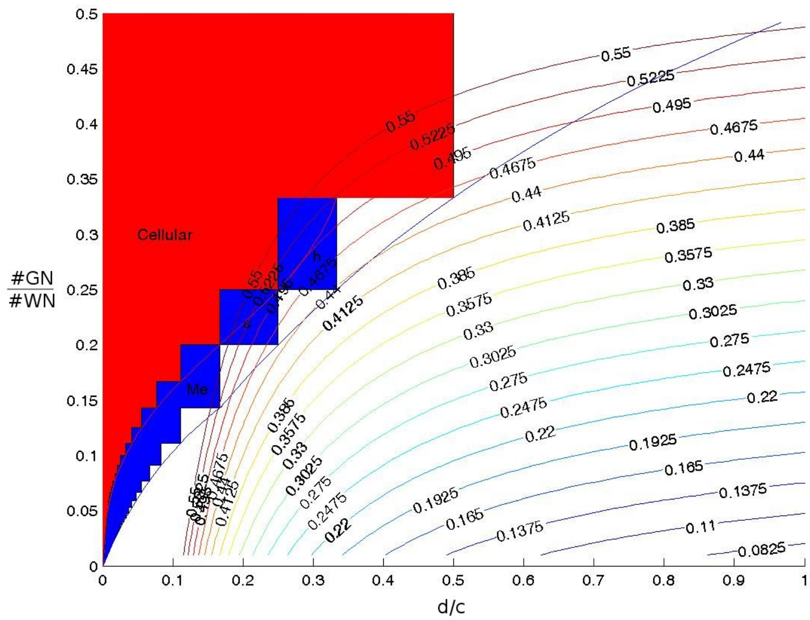

Fig. 4. Feasibility regions for 1-D multi-hop mesh and cellular deployment with $\alpha = 1$ and $r_2/r_1 < 2$ and WN average cost when $\rho = 0.0625$.

wireless network for any given value of ρ. For this purpose, let us model also cellular deployment, in which nodes communicate directly with the nearest GN, even if it is placed at a distance greater than the inter-node separation r_1. We suppose that this communication is possible, but then the capacity achievable on the wireless link will be lower than $c_{wireless}$. Let us model the wireless capacity as a continuous function of the communication distance r and adopt a generic decrease function to model attenuation:

$$c_{wireless}(r) = c_{wireless} \max \left\{ \left(\frac{r_1}{r}\right)^{\alpha}, 1 \right\}, \tag{13}$$

with r_1 being the reference distance and $\alpha \geq 1$ the attenuation exponent.

This assumption may seem unrealistic, as it requires the technology to assign a wireless capacity to every communication distance. However, there exist several wireless technologies supporting different data rates and rate adaptation [16] such that transmissions over a distance r_1 can attain the highest wireless capacity; while transmissions over a distance $2r_1$ will get a lower wireless capacity; while again transmissions over a distance $3r_1$ or greater cannot be supported. Then, writing a relationship like (13) has the advantage of isolating the attenuation exponent α, which contains information about the communication model in use. In particular, the case $\alpha = 1$ corresponds to the use of directional antennas [17]; while the case $\alpha = 2$ corresponds to free-space propagation with omnidirectional antennas; while $\alpha > 2$ is encountered in the presence of shadowing, multi-path reflections and scattering phenomenons. Multi-hop has the potential to save wireless resources, because transmissions will attain, on every hop, the highest possible wireless capacity. Fixing $\alpha = 1$ in equation (13) corresponds then to the most conservative choice for mesh compared to cellular deployment.

By taking $\alpha = 1$, we get the following design constraints for cellular deployment: if $x \in (1/4, 1/2]$, then $y \geq 1/3$; if $x \in (1/6, 1/4]$, then $y \geq 1/4$; and so on. By taking $b \in \mathbb{N}, b \geq 1$, we can write that, if $x \in (\frac{2}{6+(b+1)(b+2)}, \frac{2}{6+b(b+1)}]$, then $y \geq \frac{1}{b+3}$.

Again, by interpolation:

$$x \leq \frac{2y^2}{12y^2 - 5y + 1}, 0 < y \leq \frac{1}{3}. \tag{14}$$

Figure 4 represents in two different colours on the xy plane the feasibility regions of mesh and cellular deployment, together with the contour lines obtained by fixing $\rho = 0.0625$ in equation (6).

For this specific value $\rho = 0.0625$, the best design in absolute is a multi-hop network with one GN every 4 nodes. The WN average cost excluding $\mathcal{C}_{backhaul}$ will then be $0.4375 \times \mathcal{C}_{GN}$, i.e. the value of the contour lines at the point $(1/3, 1/4)$. Employing a different number of GNs incurs in an higher cost. In particular, in the cellular design with two nodes assigned to each GN, or at point $(1/2, 1/3)$, the average cost per node will be $0.4583 \times \mathcal{C}_{GN}$ and the saving introduced by multi-hop relaying is thus 4.55%.

The value $\rho = 0.0625$ is found by imposing, at the specific point $(1/3, 1/4)$, orthogonality of the gradient of function (6) and of the tangent to the region defined by inequality (9). These vectors are respectively:

$$\left(-\frac{\rho}{x^2}\ ,\ 1\right)$$

and

$$\left(1\,,\ \frac{1}{(x+1)^2}\right).$$

By imposing single-path and thus by situating ourselves on the stair functions of Fig. 4, it is possible to determine the value of ρ for which the mesh becomes more economical than the cellular deployment. It is required that:

$$\begin{aligned}\frac{1}{4}+3\rho &< \frac{1}{3}+2\rho\\ \rho &< \frac{1}{12}.\end{aligned} \tag{15}$$

The combined use of the xy network design plane and of the contour lines makes it possible to derive the least expensive deployment given any value of ρ. By setting ρ, the contour lines get fixed in the xy plane and average WN costs in the different network solutions can be computed and compared with each other. Let us take, for example, $\rho = 1/24$, i.e. half the value of ρ previously computed as the turning point between the cellular deployment and the mesh. With $\rho = 1/24$, the gap between the cost of the least expensive mesh with one GN every 5 nodes, $0.367 \times \mathcal{C}_{GN}$, and the cost of the least expensive cellular deployment under the most optimistic propagation assumption with one AP every 4 nodes, $0.417 \times \mathcal{C}_{GN}$, indicates a saving of 12% when adopting relay. By considering $\rho = 1/48$, the gap between the least expensive mesh ($y = 1/7$) and the least expensive cellular deployment ($y = 1/5$) rises to 17.6%.

IV. Analysis of Regular Grid Topologies

We want now to study planar regular topologies. There exist three regular tessellations of the plane: the triangular, the square and the hexagonal one. These three tessellations give rise to three radio regular grid coverages. The cost of these coverages is not the same. Let us suppose we want 100% coverage of the plane, i.e. all points of the plane should be placed within a distance R from a radio station. This can be generalized by σ-coverage, with $\sigma \in (0, 1]$, meaning that a fraction σ of the plane should be within the distance R (results are found in [18]). Let also r_1 represent the minimum separation between radio stations, i.e. the edge of the regular triangle, square or regular hexagon.

In the triangular grid, for 1-coverage, $R = 1/\sqrt{3}r_1$; in the square grid $R = 1/\sqrt{2}r_1$; in the hexagonal grid $R = r_1$. Density of radio stations is also different: in the triangular grid $\frac{2}{\sqrt{3}r_1^2}$; in the square grid $\frac{1}{r_1^2}$; in the hexagonal grid $\frac{4}{3\sqrt{3}r_1^2}$.

Let us express the density of radio stations per squared R. It is respectively for the triangular, square and hexagonal grids:

$$\frac{2}{3\sqrt{3}R^2} \approx \frac{0.38}{R^2};\ \ \frac{1}{2R^2} = \frac{0.5}{R^2};\quad \frac{4}{3\sqrt{3}R^2} \approx \frac{0.77}{R^2}. \tag{16}$$

The least expensive 1-coverage is then given by the triangular grid, with the hexagonal grid being the most expensive and twice as expensive than the least expensive one.

However, interpreting the grids as a radio backhaul network, WNs are more separated in the triangular grid than in the square grid or the hexagonal grid. As a consequence, they can communicate with a lower wireless capacity. Let us take $c^{hexag}_{wireless}$ as a reference value. Then:

$$c^{triang}_{wireless} = \frac{c^{hexag}_{wireless}}{\sqrt{3}^{\alpha}}; \tag{17}$$

$$c^{square}_{wireless} = \frac{c^{hexag}_{wireless}}{\sqrt{2}^{\alpha}}, \tag{18}$$

α being the attenuation exponent in equation (13).

Taking $x = \frac{d}{c^{hexag}_{wireless}}$, the feasibility region on the xy plane will be smaller for the triangular and square grids than for the hexagonal grid.

Moreover, we can consider another advantage of a denser deployment like the hexagonal grid. We suppose that traffic is not a property of the WN, i.e. we do not have a fixed d, but it is a density function on the plane. By using more radio stations to cover the plane, the traffic requirement of each WNs in the hexagonal grid will be lower than for WNs in the triangular or square grid. Let us take d^{hexag} as a reference value. We derive the following relations among traffic demands:

$$d^{triang} = 2 \cdot d^{hexag}; \tag{19}$$

$$d^{square} = \frac{3\sqrt{3}}{4} d^{hexag}. \tag{20}$$

The triangular grid employs half the number of WNs than the hexagonal grid. However, the hexagonal grid attains a wireless capacity on the link a factor $\sqrt{3}^{\alpha}$ higher than the triangular grid and is characterized by a traffic demand a factor 2 lower than the triangular grid. Let us ask the question whether the hexagonal grid is more economical than the triangular one or not. Given the different densities of WNs, the hexagonal grid would be more convenient than the triangular grid if:

$$\overline{\mathcal{C}}^{triang}_{WN} > 2\overline{\mathcal{C}}^{hexag}_{WN} \tag{21}$$

The three grid deployments can be represented on a single plot, by fixing α to a particular value. Then, we assume that $x = \frac{d^{hexag}}{c^{hexag}_{wireless}}$ and we rescale in the xy plane the feasibility curves of triangular and square deployments according to the relations (17)-(20) such that a unique cost function can be used to compute the WN average cost $\overline{\mathcal{C}}_{WN}$.

Figure 5 represents the three grid deployments on a single plot when $\alpha = 1$.

The plotted feasibility constraint for grid deployment is given by inequality (9) expressing the bottleneck at the wireless interface of the GN. All WNs are within one hop from the GN (i) when $\#_{GN} \geq \#_{WN}/7$, for the triangular grid; (ii) when $\#_{GN} \geq \#_{WN}/5$, for the square grid, and (iii) when $\#_{GN} \geq \#_{WN}/4$, for the hexagonal grid. The same feasibility constraint applies however also for mesh with an higher number of the GNs and using multi-hop, if a condition on the ratio between the interference range r_2 and the node separation r_1 is satisfied. When only two hops are used, the only bottleneck is located at the radio interface of the GN, because the collision domain of the adjacent MN is smaller than

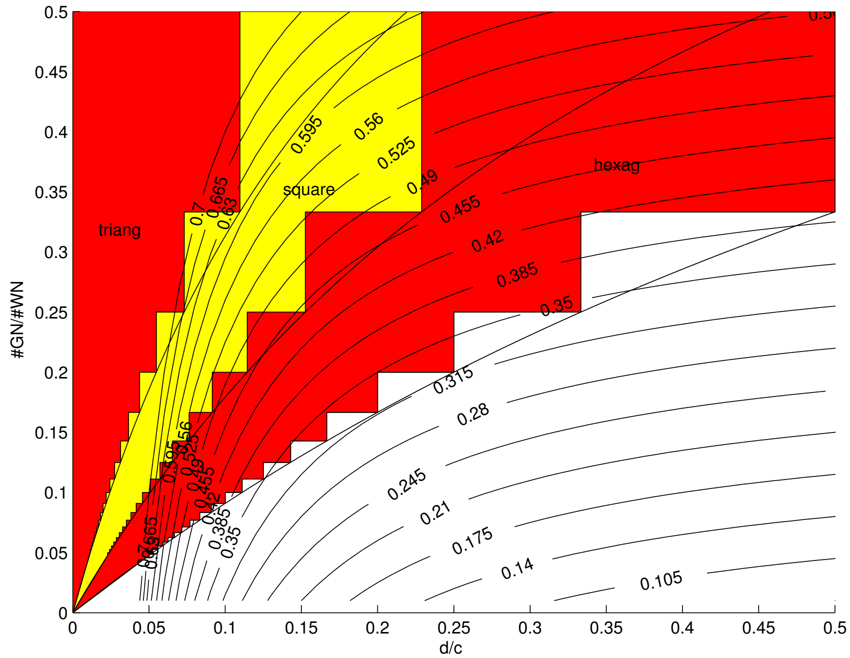

Fig. 5. Feasibility regions of the different grid deployments (triangular, square and hexagonal) with $\alpha = 1$ and $r_1/r_2 < 2$. WN average cost when $\rho = 0.03$ is expressed by the contour lines. Note that the density of WNs in the triangular grid is half of that in the hexagonal grid and that in the square grid is a factor 1.5 lower than that of the hexagonal grid.

that. Two-hops mesh topologies are shown in Fig. 6, for the three grid deployments. Inequality (9) correctly characterizes these topologies, if enough spatial reuse is possible within the mesh, i.e. if $r_2/r_1 < \sqrt{2}$ for the square grid and if $r_2/r_1 < \sqrt{3}$ for the triangular and hexagonal grid.

Figure 5 shows that the least expensive triangular deployment is more economical than the least expensive hexagonal one, as $\min \overline{\mathcal{C}}_{WN}^{triang} < 2 \min \overline{\mathcal{C}}_{WN}^{hexag}$. Fig. 5 assumes that $\rho = 0.03$, but this relationship holds for any ρ, when $\alpha = 1$. However, when $\alpha > 1.7$ (value found numerically), the inverted relationship holds, making the hexagonal grid more convenient than the triangular one.

We stated that bottom part of Fig. 5 is accurate only if the the interference range is relatively small. In the case that $r_2/r_1 \geq 2$ (or $r_2/r_1 \geq \sqrt{3}$ for the hexagonal grid), we need to introduce additional mesh design constraints. Moreover, cellular deployment can become more economical than mesh. Over a path of two-hops, traffic gets transmitted twice, i.e. first by the WN originating it and then by the relay node. If the transmission of the WN interferes with the interface of the GN, then we need to reserve double of the resources of the GN for the traffic routed over a two-hops path. Alternatively, we can compute the resources required when transmitting directly from every WN to the GN. According to our model of equation (13), the further away the WN from the GN, the lower the wireless capacity it gets. Let us consider trigonometry. The angle between any three closest nodes in the hexagonal grid is 120 degrees. With such an angle or a smaller one, one-hop

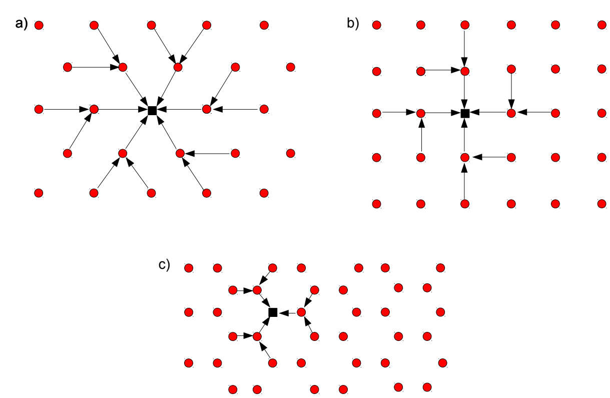

Fig. 6. The triangular (a), square (b) and hexagonal (c) grids. The hexagonal grid is twice denser than the triangular grid. Mesh configurations connecting WNs two-hops away from the GN are shown as well.

transmission is preferred to two-hops relaying when:

$$\frac{1}{\sqrt{3}^{\alpha}} > \frac{1}{2}$$
$$\alpha < 1.262. \tag{22}$$

We know how to write feasibility constraints both for mesh and cellular deployment for the grid deployments and to represent them on the xy plane. In the case of mesh, these constraints depend on the interference range, while in the case of cellular deployment, they depend on the attenuation exponent α.

V. CONCLUSIONS

In this paper, we explored topology and deployment factors of Wireless Mesh Networks (WMNs) and studied their economical impact on the network design using regular topologies, i.e. the topologies providing the best 1-D and 2-D coverage solutions. We made the following simplifying assumptions: (i) wireless nodes all have the same traffic demand; (ii) the wireless protocol is asymmetric, ideal and contention-free and (iii) the wireless protocol can achieve a capacity that decreases with communication range. We introduced a cost model with the advantage of employing only three parameters that together account for the fundamental technological constraints of the WMN. They are:

- ρ, the ratio between the cost of a radio achieving the minimum capacity (i.e. a capacity equal to the node traffic demand) and the fixed cost of installing a gateway node. This parameter characterizes the trade-off between increasing the wireless link capacity and installing more gateways;
- α, the attenuation exponent of the wireless technology. In particular, this parameter characterizes the trade-off between mesh and cellular deployment;
- r_2/r_1, the ratio between the interference range and the communication range according to the protocol interference model. This parameter determines spatial reuse in the mesh network.

As the main contribution of this work, we introduced the xy design plane, which has wireless capacity provisioning and gateway numerical incidence as its horizontal and vertical coordinates, respectively. It is possible to represent on the xy plane any network design solution to an homogeneous problem and to compute their cost using a cost function that only depends on the parameter ρ. Moreover, we were able to compute (and draw on the xy plane) feasibility constraints for mesh and cellular, linear and planar deployments. Our representation is powerful enough to represent even multi-path. The representation could be extended to consider also multi-radio, which can optimize the spatial reuse [19]. However, with our assumptions, it will not feature multi-power as we require nodes to transmit always at their maximum power to get the maximum capacity.

Thanks to the assumptions discussed, we were able to provide some useful rules of thumb. First, we found out that - in a line topology - mesh is more economical than cellular for small enough values of ρ whatever the attenuation exponent α. In particular, in the limit case with $\alpha = 1$, mesh is more economical than cellular if $\rho < 1/12$. Secondly, we derived a value of α below which the triangular grid is more economical than the hexagonal grid and above which the reverse relationship holds.

An extension of this work would include perturbed and random topologies.

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