

A Multi-tier Cooperative Resource Partitioning Technique for Interference Mitigation in Heterogeneous Cellular Networks

Rajkarn Singh, Sudepta Mishra, and C. Siva Ram Murthy

Indian Institute of Technology Madras, India 600036
Email: rajk@cse.iitm.ac.in, sudepta@cse.iitm.ac.in, murthy@iitm.ac.in

Abstract—In Heterogeneous cellular networks (HetNets), co-channel interference is a major concern due to the co-existence of multiple base stations with overlaid regions. Edge users are typically the victims because of high interference exposure. To counter this high interference, picocells communicate with the edge users in protected subframes (PSF). The severity of the problem intensifies in case of hotspot deployment, where picocells cannot provide coverage to the entire hotspot, thus forming a dense ring of macrocell users around picocells. We argue that these macrocell users are also victims, constituting a significant victim user population in hotspot deployment. We propose that, along with the macrocell muting during PSF, picocells should also be operated in cooperative manner with macrocell, and be barred from transmission during some of the subframes for protection of these macrocell victim users. We define a utility function to find the optimal values of PSF density for both macrocell and picocells, which would increase victim user throughput thereby enhancing system fairness. Exhaustive simulations illustrate that the proposed scheme improves victim user throughput significantly, while maintaining the overall system capacity.

Index Terms—HetNets, CRE, eICIC, Protected Subframe, Convex Optimization, Fairness

I. INTRODUCTION

Heterogeneous cellular networks (HetNets) are characterized by the coexistence of multiple base stations (BSs) such as picocells, femtocells and relays, overlaid on macrocell coverage area [1]. This brings the network closer to the end user, thereby increasing channel quality and providing better spectrum utilization. Picocells or Pico-eNodeBs (PeNB) are intelligently deployed by the operator in areas of higher user density (hotspots) for better spectrum reuse [2]. This improves coverage and capacity for such hotspot regions.

In a typical HetNet scenario, mobile user equipments (UE) have multiple choices of BSs to associate with. Association is generally done based on the value of reference signal received power (RSRP) of BSs. PeNBs have smaller coverage because of lower transmit power. Hence, fewer users get associated with PeNBs in uniform deployment. Consequently, macro-eNodeB (MeNB) experiences heavy load while PeNBs remain lightly loaded.

Cell range expansion (CRE) [3] has been proposed to offload more users to lightly loaded PeNBs by adding a

positive cell selection bias to the RSRP of PeNB. The users along the range expanded boundaries of associated PeNBs are susceptible to high interference from MeNB tier because of co-channel operation. To mitigate this problem, enhanced inter-cell interference coordination (eICIC) [4], [5] has been proposed, which exploited the concept of protected subframes (PSF) [6]. Approaches to PSF can be categorized under two classes, almost blank subframes (ABS) and reduced power subframes (RPS). In ABS mode, MeNB mutes and does not transmit any data during some of the subframes, thus providing interference free channels to PeNB so that PeNB edge users can be scheduled during these subframes. In RPS mode, MeNB transmits data symbols at a very reduced transmission power, thus providing minimal interference to the PeNB associated users.

The authors of [7] have shown that CRE can be totally ignored, if expected bitrate obtained by the UE is used as the cell selection criteria. The work shows improvement in overall cell throughput as well as cell edge throughput. However, they do not consider the current load or the bandwidth available at the BS. In [8], this factor has been added and the results have been refined. The techniques mentioned so far rely upon the static configuration of ABS density and do not take into consideration the dynamics of the system.

In case of hotspot formation in cells and deployment of PeNBs in such hotspots, CRE may not be required. Also, not all hotspot users get covered by a PeNB, and hence form a ring of MeNB UEs around the PeNB. These MeNB UEs suffer from high interference from PeNBs, resulting in degraded Quality of Service (QoS) in terms of signal to interference and noise ratio (SINR) and received bitrate. These users are considered as MeNB victim users (VUE).

The authors of [9] propose a dynamic programming solution to find optimal number of subframes to be reserved for ABS (hereafter, referred as ABS/PSF density) which would maximize overall system throughput. Additionally, the authors of [10] suggest a throughput ratio based utility function to change ABS density dynamically depending upon traffic and load distribution, but the resulting PSF value reduces MeNB throughput because of weight factor used to change PSF value. Furthermore, neither technique takes into

consideration the QoS of macro-VUEs, and the focus is upon picocell tier only.

In many scenarios, macro-VUEs face high interference from the picocell tier. It has also been observed that most of the UEs receiving poor SINR lie not only among the intersecting boundaries of macrocell and picocell tier, but also in between two neighbouring PeNB regions. Such a region creates a coverage hole for the MeNB users, which receive high co-channel interference from both the PeNBs. It is beneficial to associate the users located in such regions with the macrocell tier and protect them from interference caused by the picocell tier. Improvement in the QoS of the macro-VUEs by picocell muting has not been studied much in the literature. One of the reasons is that these macro-VUEs can get re-associated to the picocell tier by adding bias and hence obtain better SINR. However, the coverage expansion due to biasing is omni-directional, and would associate additional unnecessary MeNB UEs to the PeNB. The authors of [11] suggest the muting of femtocells for protection of macro-VUEs that are present in the vicinity of closed access femtocell.

In this paper, we propose that, along with the MeNB muting on specific subframes to protect pico-VUEs, the PeNBs should also refrain from transmitting on certain subframes to ensure interference free communication of the macro-VUEs. MeNB and PeNBs adopt a cooperative strategy to decide the pattern of subframe blanking. We assume synchronous mode of operation in which all PeNBs mute during the same period thus allowing all the macro-VUEs to be scheduled during those subframes. The configured PSF duty cycle for which the subframes are blanked depends upon the victim user density and should be chosen carefully. We propose a product-rate utility function based on [12] which maximizes the product of bitrates of all UEs. The utility function converges to a convex function, which can be solved by any standard convex optimization algorithm [13], hence improving victim user throughput as well as system fairness.

The rest of this paper is organized as follows. The system model is described in Section II, with focus on bitrate calculation and existing user association techniques. The proposed method for calculation of PSF density is explained in Section III. Section IV describes the results obtained. Finally, the work is concluded in Section V with remarks on future work.

II. SYSTEM MODEL

We consider a two-tier heterogeneous network model which consists of a single MeNB, M , overlaid with C low power, outdoor PeNBs. Let N_M and N_j denote the number of UEs associated with the MeNB and the j^{th} PeNB, respectively. The macrocell region consists of randomly distributed hotspot traffic locations, with each PeNB deployed at the center of a hotspot [14]. The users are either assigned to a hotspot with probability p and or uniformly placed in the macrocell region with probability $1-p$. A reference scenario is given in Figure 1, where $p = 2/3$ [10]. Here, victim and

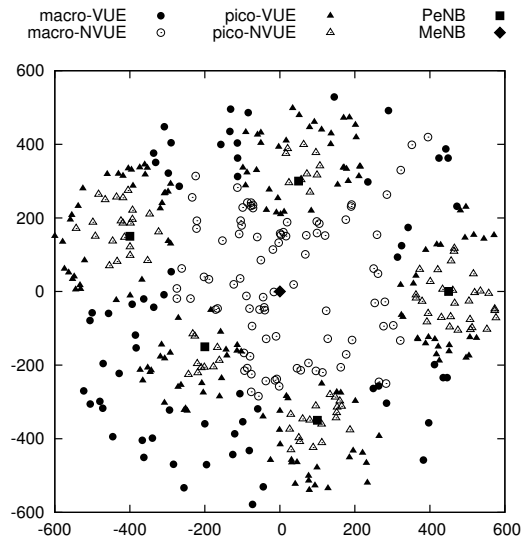


Fig. 1. System Scenario showing Victim & Non-victim Users

non-victim UEs are represented by solid and empty symbols, respectively. The MeNB is considered to be located at the origin $(0,0)$ and is surrounded by five hotspot zones, each with its own PeNB.

UEs present at the intersecting boundaries of MeNB and PeNBs that face higher interference are considered as victim UEs including both macro-VUEs and pico-VUEs, respectively. UEs present at the respective cell centres are non-victim (NVUE) which include macro-NVUE and pico-NVUE. We consider RPS mode of transmission in which BSs transmit at the reduced power during protected subframes. Further, Orthogonal Frequency Division Multiple Access (OFDMA) and round robin scheduling are considered. A simple scheduling strategy is employed, where VUEs are scheduled during PSF, thus reducing interference, while NVUEs, which are cell centre users, are scheduled during non-PSF subframes. Rayleigh flat fading path loss model is considered.

A. Characterization of Bitrate Based on ABS Density

SINR is a function of signal strengths of serving BS and various interfering BSs, and also depends upon channel conditions such as pathloss, channel gain, and thermal noise. For the edge users with high pathloss values and high neighborhood interference, the SINR received is often insufficient to maintain the communication link. Schemes such as eICIC have been proposed to provide improved channel quality to these UEs. eICIC is a time domain technique [4] in which a fraction of the subframes (ABS) do not transmit any data, thus providing interference free channel and opportunity to the UEs of the affected tier to be scheduled during these ABS subframes. SINR received by a UE i associated with BS j ,

scheduled during non-ABS subframes can be represented as,

$$\Gamma_{i,j}^{nonABS} = \frac{P_{tx}^j G_i^j}{\sum_{k \in \{\Omega_P \cup M\}, k \neq j} P_{tx}^k G_i^k + \sigma_i^2} \quad (1)$$

where P_{tx}^j and P_{tx}^k represent the transmitted power of serving BS j and the interfering BS k , respectively. G_i^j (and G_i^k) is the channel gain coefficient from the BS j (and k) to the UE i . Ω_P is the set of all PeNBs in the neighbouring region. σ_i^2 represents the thermal noise of UE i .

Similarly, SINR received by a UE i associated with BS j , scheduled during ABS subframes and receiving interference free signals will be,

$$\Gamma_{i,j}^{ABS} = \begin{cases} \frac{P_{tx}^j G_i^j}{\sum_{k \in \Omega_P, k \neq j} P_{tx}^k G_i^k + \sigma_i^2} & \text{if } j \in \Omega_P \\ \frac{P_{tx}^j G_i^j}{\sigma_i^2} & \text{if } j = M \end{cases} \quad (2)$$

If W is the bandwidth allocated to each UE in Hertz and α is the fraction of the subframes reserved for ABS, then bitrate R_i^j obtained by UE i from the BS j , scheduled during ABS and non-ABS subframes, respectively, is given by,

$$R_i^j = \begin{cases} \alpha W \log_2(1 + \Gamma_{i,j}^{ABS}) & \text{if ABS} \\ (1 - \alpha) W \log_2(1 + \Gamma_{i,j}^{nonABS}) & \text{if non-ABS} \end{cases} \quad (3)$$

B. UE Association Techniques

Considering users' perspective, the most important parameter is the received bitrate. However, the bitrate expected to be received by a user depends upon the received SINR and available bandwidth at the target BS. For any resource allocation strategy, the received bitrate at any user depends upon the number of users associated with its target BS. Since, the bitrate obtained by a user is inversely proportional to the total number of users associated with the BS, it is of interest to look at the various cell association techniques.

1) *RSRP*: Here, a UE gets associated with a BS providing highest Reference Signal Received Power (RSRP). This technique, however, results in high degree of user association in macrocell tier than in picocell tier. Transmit power difference between macro and pico BSs results in uneven load distribution and reduces fairness. Using *RSRP*, a UE i gets associated with BS j if,

$$servingID_i = \arg \max_j \{RSRP_{i,j}\} \quad (4)$$

2) *RSRP + Bias* [3]: In order to improve UE offloading and resource utilization in small cells, cell biasing is suggested. Adding a positive bias value, λ , to RSRP from small cells allows more users to offload to small cells. Using *RSRP + Bias*, a UE i gets associated with BS j if,

$$servingID_i = \arg \max_j \{RSRP_{i,j} + \lambda_j\} \quad (5)$$

However, these newly offloaded users now receive high interference from macrocell. In order to protect their signal quality, some enhanced interference mitigation is necessary.

3) *Expected Bitrate (E[R])* [7]: Mobile users only care for high bitrate and better battery life. However, the above mentioned techniques do not incorporate user traffic in cell selection criteria, leading to imbalance of user traffic in different BSs. It has been suggested that, instead of introducing bias, the available bandwidth should be considered for user association. This technique considers received bitrate from BS as the cell selection criteria. Hence, a UE i gets associated with BS j , if it provides the UE with maximum expected (per hertz) bitrate, $E[R_i^j]$, as,

$$servingID_i = \arg \max_j \{E[R_i^j]\} \quad \forall j \in \{\Omega_P \cup M\} \quad (6)$$

where

$$E[R_i^M] = (1 - \alpha) \log_2(1 + \Gamma_{i,M}^{nonABS}) \quad (7)$$

if the UE i is connected to the MeNB, and

$$E[R_i^k] = (1 - \alpha) \log_2(1 + \Gamma_{i,k}^{nonABS}) + \alpha \log_2(1 + \Gamma_{i,k}^{ABS}) \quad (8)$$

if the UE i is connected to k^{th} PeNB.

This technique improves overall system throughput considerably when compared with others. However, just considering total bandwidth rather than free bandwidth may lead to sub-optimal results. Additionally, considering only pico-VUEs while ignoring macrocell tier reduces fairness in the system.

III. PROBLEM FORMULATION

A. PSF Density and Effective Data Rates

UEs are associated with MeNB or one of the PeNBs based on the highest downlink RSRP received along with the corresponding bias value. VUEs associated with MeNB are comparatively near to PeNBs, and receive relatively lower bitrates. Our aim is to improve the bitrates of these macro-VUEs, while maintaining a decent bitrate for picocell users. The overall system capacity should also not degrade. Achieving this would require improved fairness in the network, by providing equivalent bitrates to UEs that are proportional to their data rate requests. Because of this, using PSF in PeNB tier becomes inevitable.

Therefore, we propose that PeNBs should also provision for PSF subframes. Figure 2 represents the interference scenario and the resource block allocation scheme for MeNB and PeNBs. The channels being shared among MeNB and PeNBs experience high interference and hence are used to schedule NVUEs only. Both MeNB and PeNBs transmit with reduced power or defer transmission during PSF as shown, so that the VUEs of the other tier can be scheduled.

Let α_M and α_P be the respective PSF densities of MeNB and PeNBs. The effective data rate $\mathcal{R}_i^{\mathcal{X}}$ achieved by i^{th} UE, while associated with MeNB or PeNB, \mathcal{X} , will be,

$$\mathcal{R}_i^{\mathcal{X}} = \begin{cases} (1 - \alpha_M - \alpha_P) * \nabla_i^{\mathcal{X}} & \text{for NVUE} \\ \alpha_P * \nabla_i^M & \text{for macro-VUE} \\ \alpha_M * \nabla_i^P & \text{for pico-VUE} \end{cases} \quad (9)$$

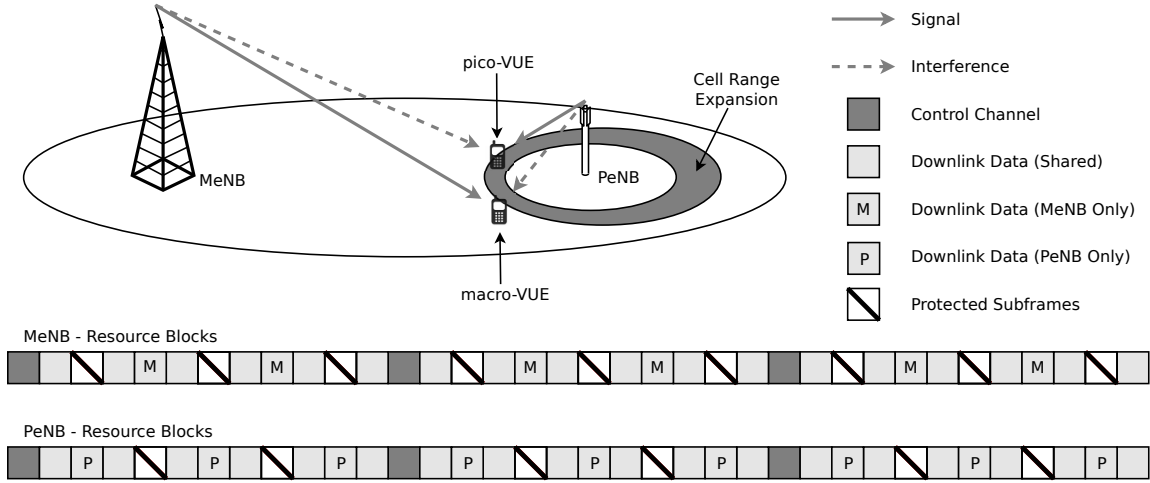


Fig. 2. PSF Pattern and Resource Block Allocation

where $\mathcal{X} \in \{\Omega_p \cup M\}$, $\nabla_i^{\mathcal{X}}$ represents the achievable average data rate of UE i over long term with full spectrum reuse. This rate can be represented in terms of average link rate of UE $r_i^{\mathcal{X}}$ as [15],

$$\nabla_i^{\mathcal{X}} = \begin{cases} G(N_{\mathcal{X}(n)}) r_i^{\mathcal{X}}/N_{\mathcal{X}(n)} & \text{for NVUE} \\ G(N_{\mathcal{X}(v)}) r_i^{\mathcal{X}}/N_{\mathcal{X}(v)} & \text{for VUE} \end{cases} \quad (10)$$

where $N_{\mathcal{X}(n)}$ and $N_{\mathcal{X}(v)}$ are the number of non-victim and victim UEs associated with BS \mathcal{X} , respectively, and $G(N_{\mathcal{X}(n)})$ and $G(N_{\mathcal{X}(v)})$ are multiuser diversity gains.

B. Product Rate Utility Function

The proposed optimization can be performed at each macro-cellular region separately. Therefore, the analysis done here for a single macrocell region with overlaid picocells remains valid for multiple macro-cellular regions as well. Both the MeNB and the corresponding overlaid PeNBs participate in computation. Note that for different cells, the values of α_M and α_P will be different.

To obtain fairness in the achieved data rates, product of the data rates of all UEs needs to be maximized [12]. Hence, we propose the following utility function which is the product of effective bitrates achieved by all UEs given as,

$$U'(\alpha_M, \alpha_P) = \prod_{k=1}^{N_M} \mathcal{R}_k^M \prod_{j=1}^C \prod_{i=1}^{N_j} \mathcal{R}_i^j \quad (11)$$

where N_M and N_j represent the number of UEs associated with MeNB and j^{th} PeNB, respectively, and $\mathcal{R}_i^{\mathcal{X}}$ represents the effective data rate of i^{th} user as derived in equation (9).

Rewriting equation (11) in logarithmic sum form, we get,

$$\begin{aligned} U(\alpha_M, \alpha_P) &= \sum_{k=1}^{N_M} \log(\mathcal{R}_k^M) + \sum_{j=1}^C \sum_{i=1}^{N_j} \log(\mathcal{R}_i^j) \\ &= \left(\sum_{k=1}^{N_{M(n)}} \log(\mathcal{R}_k^M) + \sum_{k=1}^{N_{M(v)}} \log(\mathcal{R}_k^M) \right) \\ &\quad + \sum_{j=1}^C \left(\sum_{i=1}^{N_{j(n)}} \log(\mathcal{R}_i^j) + \sum_{i=1}^{N_{j(v)}} \log(\mathcal{R}_i^j) \right) \\ &= \mathcal{U}_{macro} + \sum_{j=1}^C \mathcal{U}_{pico(j)} \end{aligned} \quad (12)$$

where \mathcal{U}_{macro} and $\mathcal{U}_{pico(j)}$ represent marginal MeNB and j^{th} PeNB utilities, respectively. Using equation (9), the utility for macrocell tier can be written as:

$$\begin{aligned} \mathcal{U}_{macro} &= \sum_{i=1}^{N_{M(n)}} \log \left\{ (1 - \alpha_M - \alpha_P) * \nabla_i^M \right\} \\ &\quad + \sum_{i=1}^{N_{M(v)}} \log \left\{ (\alpha_P) * \nabla_i^M \right\} \\ &= \log(1 - \alpha_M - \alpha_P) * N_{M(n)} + \sum_{i=1}^{N_{M(n)}} \log(\nabla_i^M) \\ &\quad + \log(\alpha_P) * N_{M(v)} + \sum_{i=1}^{N_{M(v)}} \log(\nabla_i^M) \end{aligned} \quad (13)$$

Corresponding PeNB utilities can also be written in the similar way. Equation (12) finally converges to the following

utility function.

$$\begin{aligned}
 U(\alpha_M, \alpha_P) = & \log(1 - \alpha_M - \alpha_P) * \left(N_{M(n)} + \sum_{j=1}^C N_{j(n)} \right) \\
 & + \log(\alpha_P) * N_{M(v)} + \log(\alpha_M) * \sum_{j=1}^C N_{j(v)} \\
 & + \sum_{i=1}^{N_{All}} \log(\nabla_i^X) \quad (14)
 \end{aligned}$$

where

$$N_{All} = N_M + \sum_{j=1}^C N_j$$

The converged equation (14) can be formulated as following standard convex optimization problem [13], which can be solved by any interior point algorithm for α_M and α_P ,

$$\begin{aligned}
 & \text{maximize } U(\alpha_M, \alpha_P) \quad (15) \\
 & \text{subject to } \alpha_M \geq 0, \\
 & \quad \alpha_P \geq 0, \\
 & \quad \alpha_M + \alpha_P < 1
 \end{aligned}$$

The computation of \mathcal{U}_{macro} and \mathcal{U}_{pico} may take place at the respective BSs thus distributing the computation load. The values of overlaid PeNB utilities are communicated to the central entity (e.g. MeNB) via X2 interface where final optimization is performed.

IV. SIMULATION RESULTS

The two-tier heterogeneous network model as shown in Figure 1 is considered with a single MeNB overlaid with the PeNBs varying from four to seven. Simulation is performed for hotspot user deployment scenario, with RSRP mode of user association. Note that the same analysis is valid for *RSRP + Bias* association as well. With N_M users in a macrocell consisting of C picocells, $N_{hotspot} = \frac{2}{3} \cdot N_M / C$ users are uniformly dropped around the each picocell, while remaining users are uniformly distributed in the macrocell. The value of MeNB ABS density (α) used for the techniques being compared is varied from 0.2 – 0.5 and a fixed bias of 6 dB is applied. Traffic is full buffer, and each MeNB and PeNB is equipped with a single omni-directional antenna. The detailed simulation parameters taken from 3GPP specification [16] are summarized in Table I.

The performance metrics used to evaluate and compare the results include overall system throughput, victim user (MeNB and PeNB) throughput and downlink SINR received by the UEs. The results are averaged over 600 different UE distribution scenarios. Note that the described framework can be easily modified to accommodate varying cell load or user distribution, or different PeNB placement strategies. The detailed analysis of the results is given below.

Figure 3 represents the total capacity achieved by the system under consideration for different values of α . The proposed scheme provides equivalent throughput for lower

TABLE I
SIMULATION PARAMETERS

Parameter	Value
Bandwidth	10 MHz
Cell Layout	1 Macrocell, {4-7} Picocells
User Density	400 per sq.km
MeNB Transmit Power	46 dBm
PeNB Transmit Power	30 dBm
Pathloss (MeNB)	$128.1 + 37.6 \log_{10}(D)$, D in km
Pathloss (PeNB)	$140.7 + 36.7 \log_{10}(D)$, D in km
Fading	Rayleigh fading
Thermal Noise Density	-174 dBm/Hz
Bias value for CRE	6 dB
ABS density	{0.2 - 0.5}
Scheduling	Proportional Fair

values of α compared to other schemes. However, there is a significant improvement for higher ABS densities, as our technique does not suffer from performance degradation with increase in α , as is seen in the case of *RSRP + Bias* and *E[R]* techniques. The capacity for *E[R]* reduces with increase in α because of two reasons. First, the number of victim users in PeNB increases, which leads to contention among pico-VUEs, thus increasing the number of blocked users. Second, since many MeNB UEs are concentrated around picocells because of hotspot distribution, more users offload to the picocell tier with an increase in α , the load for MeNB reduces considerably resulting in free subchannels in the macrocell tier resulting in underutilization of macrocell resources.

Figures 4(a) and 4(b) represent the cumulative distribution function (CDF) of SINR of macro and pico-VUEs for various schemes under consideration. We observe that the proposed scheme provides better SINR to the victim users as it allocates interference free subchannels not only to the PeNB victim users, but to MeNB victim users as well, which were ignored in earlier works. There is a significant improvement of SINR over the basic RSRP based cell selection scheme. For higher values of α , *E[R]* scheme provides better SINR

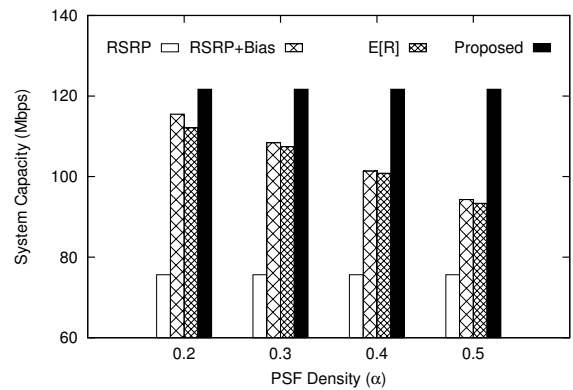


Fig. 3. Total System Throughput

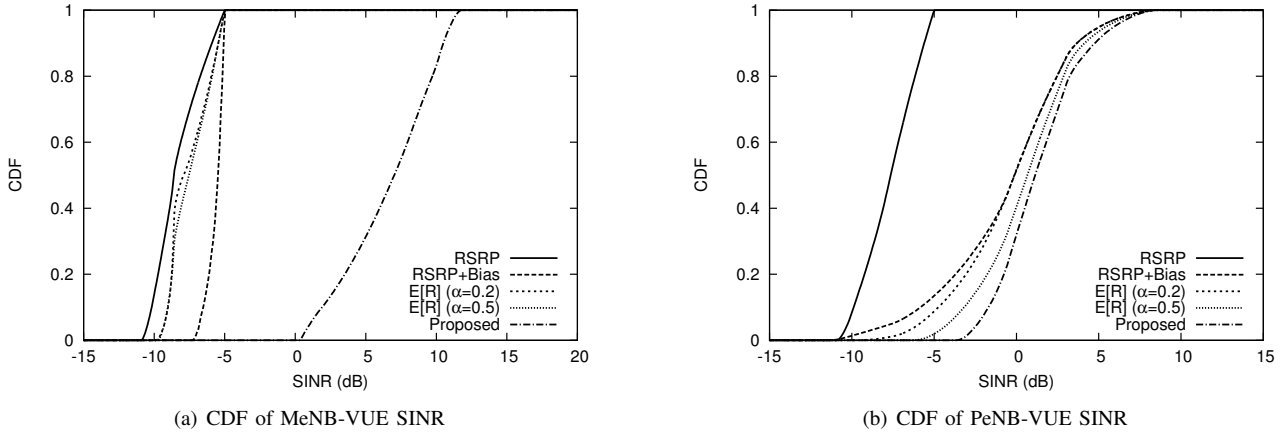


Fig. 4. CDF of Victim User SINR with REB = 6 dB and $\alpha = \{0.2, 0.5\}$

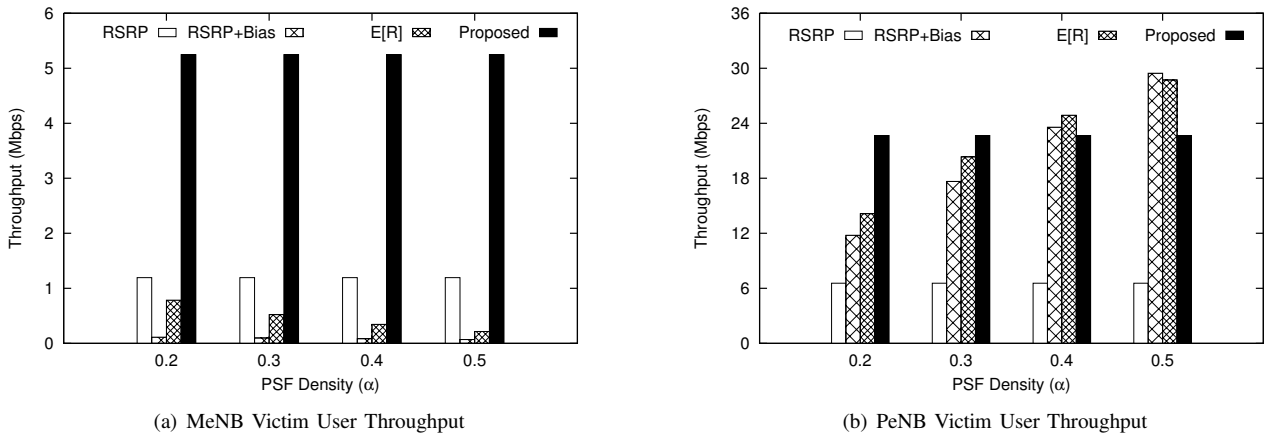


Fig. 5. Victim User Throughput with REB = 6 dB and $\alpha = [0.2 - 0.5]$

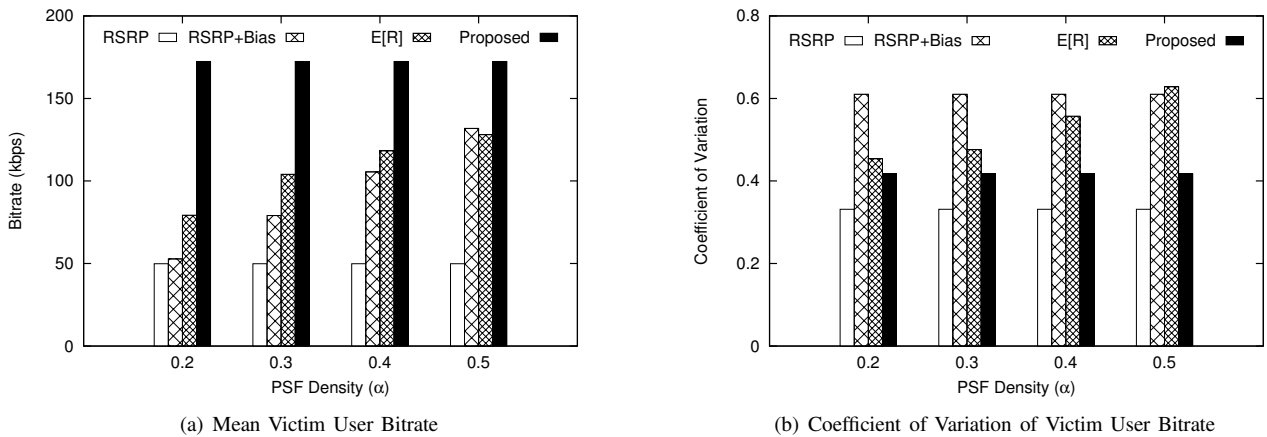


Fig. 6. Per Victim User (PeNB) Bitrate and Coefficient of Variation with REB = 6 dB and $\alpha = [0.2 - 0.5]$

to PeNB VUEs because of the availability of a large number of interference free subchannels.

Figures 5(a) and 5(b) represent throughput of victim users that belong to MeNB and all PeNBs, respectively. We find that there is a significant improvement in the capacity achieved by MeNB victim users, made possible due to the introduction of PSF in PeNB with density α_P . In case of PeNB victim users, $RSRP + Bias$ and $E[R]$ techniques achieve higher victim throughput, because of a large number of interference free subchannels being allocated to the victim users. This is also because of the large number of users being associated with PeNBs with $RSRP + Bias$ scheme and $E[R]$ scheme at higher values of α , thus resulting in higher sum throughput.

However, average bitrate per user achieved, which is represented in Figure 6(a) shows that with our proposed scheme, each PeNB VUE receives the bitrate which is equivalent to the one received by other schemes. Figure 6(b) also depicts that for higher values of α , variation among the achieved bitrates is comparatively higher for other schemes. This is indicative of the fact that some of the victim UEs end up getting very low bitrates, making these schemes unfair among victim UEs. Also, higher values of α lead to a significant decrease in the overall system throughput, as seen earlier in Figure 3.

V. CONCLUSION

In this paper, we looked at the performance of victim users in an HetNet scenario. We analyzed various interference management schemes that are based on inter cell interference coordination and use macrocell muting as the basis to provide interference free subchannels to the picocell victim users. The approach is motivated by the fact that in various scenarios, users with the poor signal quality are macrocell associated users, who suffer from co-channel interference from picocells. Based on this observation, we proposed muting of small cells. The proposed scheme provides better QoS to the macrocell victim users, eliminating macrocell coverage holes which are otherwise present in the regions between neighbouring picocells. The proposed utility function adds fairness to the system by providing equivalent bitrates to the users. Future work includes computing PSF densities for individual PeNBs depending upon the load variations among all picocell BSs.

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