Padé approximation for coverage probability in cellular networks

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Abstract—**Coverage probability is one of the most important metrics for evaluating the performance of wireless networks. However, the spatial stochastic models for which a computable expression of the coverage probability is available are restricted (such as the Poisson based or α-Ginibre based models). Furthermore, even if it is available, the practical numerical computation may be time-consuming (in the case of α-Ginibre based model). In this paper, we propose the application of Padé approximation to the coverage probability in the wireless network models based on general spatial stationary point processes. The required Maclaurin coefficients are expressed in terms of the moment measures of the point process, so that the approximants are expected to be available for a broader class of point processes. Through some numerical experiments for the cellular network model, we demonstrate that the Padé approximation is effectively applicable for evaluating the coverage probability.**

I. INTRODUCTION

The performance of a wireless network critically depends on the configuration of wireless nodes and spatial stochastic models have been used to represent the irregularity of the node configuration, where the wireless nodes are distributed according to a spatial point processes (see, e.g., [1]–[3] and references therein). Coverage probability—the probability that the signal-to-interference-plus-noise ratio (SINR) for a wireless link achieves a target threshold—is one of the most important metrics for evaluating the performance of wireless networks and many prior works have evaluated it based on the spatial stochastic models. However, the spatial models for which a computable expression of the coverage probability is available are restricted. Most prior works assumed that the wireless nodes are deployed according to homogeneous Poisson point processes (see, e.g., [1]–[5]). While this assumption makes the models tractable and indeed the computable expressions for the coverage probability have been derived, it means that the wireless nodes are located independently of each other and their spatial correlation is ignored. Since real networks can be designed more intelligently, the models based on more general point processes are required.

Recently, the spatial models for cellular networks such that the wireless base stations (BSs) are deployed according to the Ginibre point process or its variants, α-Ginibre processes, are proposed and analyzed (see [6]–[8]). The Ginibre point process is one of the determinantal point processes and is used to account for the repulsion between particles (see, e.g., [9]–[11]). However, while the computable expressions of the coverage probability are available for the Ginibre based models, they may suffer from the time-consuming numerical computation (particularly, in the case of α-Ginibre based models).

Therefore, it would be meaningful to apply some approximation techniques to the wireless network models based on general spatial point processes. A few works have so far tackled the problems along this line. Giacomelli *et al.* [12] studied the asymptotics of the coverage probability as the density of interfering nodes goes to zero. Also, Ganti *et al.* [13] developed the series expansion for functions of interference using the factorial moment expansion. These works consider the performance metrics as functions of the intensity $\lambda > 0$ of the point process and take the asymptotics as $\lambda \downarrow 0$ in [12] or the expansion kernels around $\lambda = 0$ in [13]. Our approach is quite different from them. In this paper, we propose the application of Padé approximation to the coverage probability in the spatial stochastic models, where we expand the coverage probability around the SINR threshold θ being equal to zero. In [12], [13], when evaluating the coverage probability as a function of the SINR threshold θ, one need to compute it for each value of $\theta > 0$. In our approach, however, once the Maclaurin coefficients are computed, we can approximately evaluate the coverage probability for any value of $\theta > 0$.

We here consider two scenarios: In one scenario, we focus on a transmitter-receiver pair with a fixed distance on the plane, where additional interferers are distributed according to a spatial stationary point process. In this scenario, we show that the required Maclaurin coefficients are expressed in terms of the moment measures of the point process, so that the Padé approximants are expected to be available for a broader class of spatial stationary point processes. The other scenario represents the downlink cellular network model, where the BSs are distributed according to a spatial stationary point process and a user is associated with the nearest BS. In this scenario, the Maclaurin coefficients can be computed by using the conditional moment measures of the point process given the distance from the origin to the nearest point. We can observe that this case also reduces the computation time remarkably with sufficient agreements of approximation.

The paper is organized as follows: We describe the spatial stochastic models along the two scenarios in the next section, where the SINR is defined in each scenario and the coverage probability is given as the performance metric. In section III, we first make a brief review on the Padé approximation and then we apply it to the coverage probability. To obtain the Maclaurin expansion, we derive another easy-to-expand expression of the coverage probability. As examples, we give

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the sets of Maclaurin coefficients for the Poisson based and α-Ginibre based models of the cellular network. In section IV, through some numerical experiments for the cellular network models, we demonstrate that the Padé approximation is effectively applicable for evaluating the coverage probability.

II. Model description

We consider the following two scenarios, in each of which we define the SINR for a typical wireless link.

A. Scenario 1: A fixed transmitter-receiver pair

We focus on a typical wireless link with a fixed distance, where the receiver is located at the origin $o = (0, 0)$ on the plain $\mathbb{R}^2$ and the associated transmitter is at a point with distance $r > 0$ from the origin. There are additional interferers which are distributed according to a point process Φ on $\mathbb{R}^2$. We assume that Φ is simple and locally finite a.s. and also stationary with finite intensity $\lambda > 0$. Let X_i, $i \in \mathbb{N}$, denote the points of Φ, where the order of $X_1, X_2, \ldots$, is arbitrary. We refer to the associated transmitter as station 0 and to the interferer at X_i as station i for $i \in \mathbb{N}$. The transmission power of each station is assumed to be constant at $p > 0$. Furthermore, we assume the Rayleigh fading for the random effect of fading from the stations to receivers, so that the fading effect H_i from station i to the typical receiver is an exponential random variable with unit mean; $H_i \sim \mathrm{Exp}(1)$, where H_i, $i \in \{0\} \cup \mathbb{N}$, are mutually independent and also independent of the point process Φ. The path-loss function representing the attenuation of signals with distance is denoted by ℓ, which is nonincreasing with $\int_\epsilon^\infty x\,\ell(x)\,\mathrm{d}x < \infty$ for any $\epsilon > 0$. What we have in mind is, for example, $\ell(x) = c\,x^{-\beta}$ or $\ell(x) = c\,\min(x^{-\beta}, 1)$, $x > 0$, for some $c > 0$ and $\beta > 2$, where c and β are called respectively the path-loss coefficient and path-loss exponent. The SINR of the typical wireless link is then expressed as

$$\mathsf{SINR}_o^{(1)} = \frac{p\,H_0\,\ell(r)}{I_o(0) + w_o}, \tag{1}$$

where $I_o(0) = p\sum_{j\in\mathbb{N}} H_j\,\ell(|X_j|)$ represents the cumulative interference signal from all the interferers and w_o denotes the thermal noise at the origin, which is assumed to be a positive constant.

B. Scenario 2: A downlink cellular network

We consider a downlink cellular network model (see, e.g., [5], [6]), where the stationary point process $\Phi = \{X_i\}_{i\in\mathbb{N}}$ represents the configuration of wireless BSs. The transmission power of each BS, random fading effects and path-loss function are the same as in Scenario 1 above. Each mobile user is associated with the closest BS; that is, the mobile users in the Voronoi cell of a BS are associated with that station. We focus on a typical user located at the origin o. Then, the SINR of the typical user from the associated BS is expressed as

$$\mathsf{SINR}_o^{(2)} = \frac{p\,H_{B_o}\,\ell(|X_{B_o}|)}{I_o(B_o) + w_o},$$

where B_o denotes the index of the BS associated with the typical user; that is, $\{B_o = i\} = \{|X_i| \le |X_j|, j \in \mathbb{N}\}$, and $I_o(i) = p\sum_{j\in\mathbb{N}\setminus\{i\}} H_j\,\ell(|X_j|)$ represents the cumulative interference signal from all the BSs except i. The thermal noise w_o at the origin is the same as in (1).

C. Coverage probability

As a performance metric, we consider the coverage probability, which is the probability that the SINR of the typical link achieves a predefined threshold $\theta > 0$. Exploiting that $H_i \sim \mathrm{Exp}(1)$, $i \in \{0\} \cup \mathbb{N}$ are mutually independent, we can obtain the following (see, e.g., [6]).

Proposition 1: For the wireless network model in Scenario 1, the coverage probability for the typical wireless link is given by

$$\begin{aligned}&\mathsf{P}(\mathsf{SINR}_o^{(1)} > \theta)\\ &= \exp\Bigl(-\theta\,\frac{w_0}{p\,\ell(r)}\Bigr)\,\mathsf{E}\biggl(\prod_{j\in\mathbb{N}}\Bigl(1 + \theta\,\frac{\ell(|X_j|)}{\ell(r)}\Bigr)^{-1}\biggr).\end{aligned} \tag{2}$$

On the other hand, for the cellular network model in Scenario 2, the coverage probability for the typical user is given by

$$\begin{aligned}&\mathsf{P}(\mathsf{SINR}_o^{(2)} > \theta)\\ &= \mathsf{E}\biggl(\exp\Bigl(-\theta\,\frac{w_0}{p\,\ell(|X_{B_o}|)}\Bigr)\prod_{j\in\mathbb{N}\setminus\{B_0\}}\Bigl(1 + \theta\,\frac{\ell(|X_j|)}{\ell(|X_{B_0}|)}\Bigr)^{-1}\biggr).\end{aligned} \tag{3}$$

III. Padé approximation for coverage probability

A. A brief review on Padé approximation

Suppose that a function f on $\mathbb{R}$ is $(m+n)$th differentiable for $m, n \in \mathbb{N}$ and its Maclaurin expansion $f(x) = \sum_{i=0}^{m+n} c_i\,x^i + O(x^{m+n+1})$ as $x \to 0$ is given. The Padé approximation of f is a rational fraction;

$$R_{m,n}(x) = \frac{a_0 + a_1\,x + \cdots + a_m\,x^m}{b_0 + b_1\,x + \cdots + b_n\,x^n}, \tag{4}$$

which fits $f(x)$ through the orders $1, x, x^2, \ldots, x^{m+n}$ in the sense that

$$\sum_{i=0}^{m+n} c_i\,x^i = R_{m,n}(x) + O(x^{m+n+1}). \tag{5}$$

We refer to this $R_{m,n}$ as the (m,n)-Padé approximant of f. The coefficients $a_0, a_1, \ldots, a_m$ and $b_0, b_1, \ldots, b_n$ in (4) can be obtained as follows (see, e.g., [14] for details). First, for being well-defined at $x = 0$, it must be $b_0 \ne 0$, so that we take $b_0 = 1$ without any loss of generality. The premise of Padé approximation (5) yields

$$\begin{aligned}&(1 + b_1\,x + \cdots + b_n\,x^n)\,(c_0 + c_1\,x + \cdots + c_{m+n}\,x^{m+n})\\ &= a_0 + a_1\,x + \cdots + a_m\,x^m + O(x^{m+n+1}).\end{aligned} \tag{6}$$

Thus, the coefficients of $x^{m+1}, x^{m+2}, \ldots, x^{m+n}$ on the left-hand side of (6) must be equal to zero, and we have

$$\begin{cases} b_n\,c_{m-n+1} + b_{n-1}\,c_{m-n+2} + \cdots + c_{m+1} = 0,\\ b_n\,c_{m-n+2} + b_{n-1}\,c_{m-n+3} + \cdots + c_{m+2} = 0,\\ \qquad\vdots\\ b_n\,c_m + b_{n-1}\,c_{m+1} + \cdots + c_{m+n} = 0,\end{cases} \tag{7}$$

where $c_i = 0$ for $i < 0$ (in the case of $m < n-1$). From (7), the denominator coefficients b_i, $i = 1, 2, \ldots, n$, can be obtained.

The numerator coefficients a_i, $i = 0, 1, \dots, m$, then follow from (6) by equating the coefficients of $1, x, x^2, \dots, x^m$;

$$\begin{cases} a_0 &= c_0, \\ a_1 &= c_1 + b_1\, c_0, \\ &\vdots \\ a_m &= c_m + b_1\, c_{m-1} + \cdots + b_n\, c_{m-n}, \end{cases}$$

where $c_i = 0$ for $i < 0$ (in the case of $m < n$). As a result, we have

$$\sum_{i=0}^{m} a_i\, x^i = \begin{vmatrix} c_{m-n+1} & c_{m-n+2} & \cdots & c_{m+1} \\ c_{m-n+2} & c_{m-n+3} & \cdots & c_{m+2} \\ \vdots & \vdots & \ddots & \vdots \\ c_{m-1} & c_m & \cdots & c_{m+n-1} \\ c_m & c_{m+1} & \cdots & c_{m+n} \\ \sum_{i=0}^{m-n} c_i\, x^{n+i} & \sum_{i=0}^{m-n+1} c_i\, x^{n+i-1} & \cdots & \sum_{i=0}^{m} c_i\, x^i \end{vmatrix},$$

$$\sum_{i=0}^{n} b_i\, x^i = \begin{vmatrix} c_{m-n+1} & c_{m-n+2} & \cdots & c_m & c_{m+1} \\ c_{m-n+2} & c_{m-n+3} & \cdots & c_{m+1} & c_{m+2} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ c_{m-1} & c_m & \cdots & c_{m+n-2} & c_{m+n-1} \\ c_m & c_{m+1} & \cdots & c_{m+n-1} & c_{m+n} \\ x^n & x^{n-1} & \cdots & x & 1 \end{vmatrix}.$$

As a simple example, the case of $m = n = 1$ gives

$$R_{1,1}(x) = \frac{c_0\, c_1 + ({c_1}^2 - c_0\, c_2)\, x}{c_1 - c_2\, x}.$$

B. Application to coverage probability

To apply the Padé approximation to the coverage probability, we need to have the Maclaurin expansion. The expressions (2) and (3) in Proposition 1 are, however, not so tractable. We thus transform them into other easy-to-expand expressions.

Theorem 1: For the wireless typical link in Scenario 1, the coverage probability (2) satisfies

$$\mathsf{P}(\mathsf{SINR}_o^{(1)} > \theta) = \exp\Bigl(-\theta\, \frac{w_0}{p\, \ell(r)}\Bigr)\, \mathsf{E} \exp\Bigl(-\theta \int_0^\infty \frac{\Phi(C(r,s))}{s\,(s+\theta)}\, \mathrm{d}s\Bigr), \tag{8}$$

where $C(t,u) = \{x \in \mathbb{R}^2 \mid \ell(|x|) \ge u^{-1}\, \ell(t)\}$, $t > 0$, $u > 0$. On the other hand, for the cellular network model in Scenario 2, the coverage probability (3) satisfies

$$\mathsf{P}(\mathsf{SINR}_o^{(2)} > \theta) = \mathsf{E} \exp\Bigl\{-\theta\, \Bigl(\frac{w_0}{p\, \ell(|X_{B_o}|)} + \int_1^\infty \frac{\Phi(D(|X_{B_o}|, s))}{s\,(s+\theta)}\, \mathrm{d}s\Bigr)\Bigr\}, \tag{9}$$

where $D(t,u) = \{x \in \mathbb{R}^2 \mid |x| > t,\, \ell(|x|) \ge u^{-1}\, \ell(t)\}$, $t > 0$, $u > 1$.

For the proof of Theorem 1, we use the following lemma (see [8] for the proof).

Lemma 1: Let z_j, $j = 1, 2, \dots$ be an increasing sequence of positive reals and let $N(s) = \sum_{j=1}^\infty \mathbf{1}_{(0,s]}(z_j)$, $s > 0$. Then, for $\theta > 0$,

$$\prod_{j=1}^\infty \bigl(1 + \frac{\theta}{z_j}\bigr) = \exp\Bigl(\theta \int_0^\infty \frac{N(s)}{s\,(s+\theta)}\, \mathrm{d}s\Bigr), \tag{10}$$

and both sides above are finite if and only if $\sum_{j=1}^\infty z_j^{-1} < \infty$.

Proof of Theorem 1: In (2), let $\{Z_j\}_{j\in\mathbb{N}} = \{\ell(r)/\ell(|X_j|)\}_{j\in\mathbb{N}}$ with reordering such that $Z_1 < Z_2 < \cdots$. Then, $N(s) = \sum_{j=1}^\infty \mathbf{1}_{(0,s]}(Z_j)$, $s \ge 0$, corresponds to $\Phi(C(r,s))$, so that applying (10) in Lemma 1 to (2) yields (8).

Similarly, let $\{Z_j\}_{j\in\mathbb{N}} = \{\ell(|X_{B_o}|)/\ell(|X_j|)\}_{j\in\mathbb{N}\setminus\{B_o\}}$ in (3) and let $N(s) = \sum_{j\in\mathbb{N}} \mathbf{1}_{(0,s]}(Z_j)$, $s > 0$. Then, since X_{B_o} is the nearest point and the function ℓ is nonincreasing, we have $N(s) = 0$ for $s \le 1$ and $N(s) = \Phi(D(|X_{B_o}|, s))$ for $s > 1$. Hence, applying Lemma 1 to (3) yields (9). ■

Formulae (8) and (9) give the form of expectations of exponential functions and the Maclaurin expansions are readily obtained. For simplicity, we consider the interference-limited case (noise-free case of $w_o \equiv 0$) and expand the following $p^{(1)}(\theta)$ and $p^{(2)}(\theta)$ around $\theta = 0$;

$$p^{(1)}(\theta) = \mathsf{E} \exp\Bigl(-\theta \int_0^\infty \frac{\Phi(C(r,s))}{s\,(s+\theta)}\, \mathrm{d}s\Bigr), \tag{11}$$

$$p^{(2)}(\theta) = \mathsf{E} \exp\Bigl(-\theta \int_1^\infty \frac{\Phi(D(|X_{B_o}|, s))}{s\,(s+\theta)}\, \mathrm{d}s\Bigr). \tag{12}$$

Theorem 2: For Scenario 1, we suppose that there exists an $\epsilon > 0$ such that $\Phi((C(r,s)) = 0$, P-a.s., for $s \le \epsilon$, where $C(r,s)$ is defined in Theorem 1. The Maclaurin expansions for $p^{(1)}$ in (11) and $p^{(2)}$ in (12) are given by

$$p^{(i)}(\theta) = \sum_{h=0}^{n-1} (-\theta)^h \sum_{\ell=1}^{h} \frac{1}{\ell!} \sum_{\substack{k_1,\dots,k_\ell \ge 1 \\ k_1+\cdots+k_\ell = h}} \mathsf{E}\bigl(A_{k_1}^{(i)} \cdots A_{k_\ell}^{(i)}\bigr) + O(\theta^n), \tag{13}$$

as $\theta \downarrow 0$, for $i = 1, 2$, where

$$A_k^{(1)} = \int_\epsilon^\infty \frac{\Phi(C(r,s))}{s^{k+1}}\, \mathrm{d}s,$$

$$A_k^{(2)} = \int_1^\infty \frac{\Phi(D(|X_{B_o}|, s))}{s^{k+1}}\, \mathrm{d}s.$$

Note that, in Scenario 1, the path-loss function $\ell(x) = c\, x^{-\beta}$ does not satisfy the assumption of Theorem 2 while $\ell(x) = c\, \min(x^{-\beta}, 1)$ does (see Remark 2 below). For the proof of Theorem 2, we use the following lemma.

Lemma 2: Let $N(s)$, $s \ge 0$, denote a nondecreasing stochastic process satisfying the following.

1) There exists an $\epsilon > 0$ such that $N(s) = 0$, P-a.s., for $s \le \epsilon$.
2) For $n \in \mathbb{N}$, $\mathsf{E}\Bigl(\int_0^\infty \frac{N(s)}{s^2}\, \mathrm{d}s\Bigr)^n < \infty$.

Then, it holds that

$$\begin{aligned}&\mathsf{E}\exp\Bigl(-\theta\int_0^\infty \frac{N(s)}{s\,(s+\theta)}\,\mathrm{d}s\Bigr)\\&= \sum_{h=0}^{n-1}(-\theta)^h\sum_{\ell=1}^{h}\frac{1}{\ell!}\sum_{\substack{k_1,\dots,k_\ell\ge 1\\ k_1+\dots+k_\ell=h}}\mathsf{E}(A_{k_1}\cdots A_{k_\ell})+O(\theta^n),\end{aligned}\tag{14}$$

as $\theta\downarrow 0$, where

$$A_k=\int_\epsilon^\infty \frac{N(s)}{s^{k+1}}\,\mathrm{d}s.$$

Proof: By Taylor's theorem for e^{-x}, we have

$$\begin{aligned}&\Bigl|\mathsf{E}\exp\Bigl(-\theta\int_0^\infty \frac{N(s)}{s\,(s+\theta)}\,\mathrm{d}s\Bigr)\\&\qquad -\sum_{\ell=0}^{n-1}\frac{(-\theta)^\ell}{\ell!}\,\mathsf{E}\Bigl(\int_0^\infty \frac{N(s)}{s\,(s+\theta)}\,\mathrm{d}s\Bigr)^\ell\Bigr|\\&\le \frac{\theta^n}{n!}\,\mathsf{E}\Bigl(\int_0^\infty \frac{N(s)}{s\,(s+\theta)}\,\mathrm{d}s\Bigr)^n\\&\le \frac{\theta^n}{n!}\,\mathsf{E}\Bigl(\int_0^\infty \frac{N(s)}{s^2}\,\mathrm{d}s\Bigr)^n = O(\theta^n),\end{aligned}\tag{15}$$

under Assumption 2). On the other hand, applying $(\theta+s)^{-1}=s^{-1}\sum_{k=0}^{n-1}(-\theta/s)^k+O(\theta^n)$, we have

$$\begin{aligned}&\mathsf{E}\Bigl(\int_0^\infty \frac{N(s)}{s\,(s+\theta)}\,\mathrm{d}s\Bigr)^\ell\\&=\mathsf{E}\Bigl(\sum_{k=1}^{n}(-\theta)^{k-1}A_k+O\Bigl(\theta^n\int_0^\infty\frac{N(s)}{s^{n+2}}\,\mathrm{d}s\Bigr)\Bigr)^\ell,\end{aligned}$$

where Assumption 1) is used. Thus, applying this to (15) yields

$$\begin{aligned}&\mathsf{E}\exp\Bigl(-\theta\int_0^\infty \frac{N(s)}{s\,(s+\theta)}\,\mathrm{d}s\Bigr)\\&=\sum_{\ell=0}^{n-1}\frac{1}{\ell!}\sum_{k_1=1}^{n}\cdots\sum_{k_\ell=1}^{n}\mathsf{E}(A_{k_1}\cdots A_{k_\ell})\,(-\theta)^{k_1+\dots+k_\ell}+O(\theta^n).\end{aligned}$$

Arranging the last expression with $k_1+\dots+k_\ell=h$, we obtain (14). ■

Proof of Theorem 2: For $p^{(1)}$, as in the proof of Theorem 1, let $\{Z_j\}_{j\in\mathbb{N}}=\{\ell(r)/\ell(|X_j|)\}_{j\in\mathbb{N}}$ and let $N(s)=\sum_{j=1}^\infty \mathbf{1}_{(0,s]}(Z_j)$, $s\ge 0$. Then, $N(s)$ in Lemma 2 corresponds to $\Phi(C(r,s))$.

For $p^{(2)}$, let $\{Z_j\}_{j\in\mathbb{N}}=\{\ell(|X_{B_o}|)/\ell(|X_j|)\}_{j\in\mathbb{N}\setminus\{B_o\}}$ and define $N(s)$, $s\ge 0$, as above. Then, we have $N(s)=0$ for $s\le 1$, which satisfies Assumption 1) in Lemma 2, and $N(s)=\Phi(D(|X_{B_o}|,s))$ for $s>1$. ■

Remark 1: Formula (13) shows that $p^{(i)}$, $i=1,2$, are completely monotone functions; that is, $(-1)^n\,\mathrm{d}^n p^{(i)}(\theta)/\mathrm{d}\theta^n\ge 0$ for $\theta>0$ and $n=0,1,2,\dots$ (see, e.g., [15]). This is, of course, confirmed by the form of (11) and (12), the right-hand sides of which have the form of Laplace transform.

Remark 2: For Scenario 1, let the path-loss function be $\ell(x)=c\,x^{-\beta}$, $x>0$ for $c>0$ and $\beta>2$. Then, $C(r,s)=b_o(r\,s^{1/\beta})$; the ball centered at the origin with radius $r\,s^{1/\beta}$, and there is no $\epsilon>0$ such that $\Phi(C(r,s))=0$, P-a.s., for $s\le\epsilon$. In this case, we do not have the finite Maclaurin coefficients. Indeed, the first-order Maclaurin coefficient is given by

$$\begin{aligned}\mathsf{E}(A_1^{(1)})&=\int_0^\infty\frac{\mathsf{E}(\Phi(b_o(r\,s^{1/\beta}))}{s^2}\,\mathrm{d}s\\&=\lambda\,\pi\,r^2\int_0^\infty s^{2/\beta-2}\,\mathrm{d}s=\infty.\end{aligned}$$

On the other hand, when $\ell(x)=c\,\min(x^{-\beta},1)$, we have $C(r,s)=\emptyset$ for $s<\min(r^{-\beta},1)$, which satisfies the assumption of Theorem 2.

For example, let $\ell(x)=c\,\min(x^{-\beta},1)$ and let $r\le 1$ for simplicity. In this case, we have $\epsilon=1$ in Theorem 2 and $C(r,s)=b_o(s^{1/\beta})$ for $s>1$. The Maclaurin coefficients for $p^{(1)}$ in (11) up to the second order are then as follows;

$$\begin{aligned}c_0^{(1)}&=p^{(1)}(0)=1,\\c_1^{(1)}&=p^{(1)\prime}(0)=-\int_1^\infty\frac{\nu_1(b_o(s^{1/\beta}))}{s^2}\,\mathrm{d}s=-\frac{\lambda\,\pi\,\beta}{\beta-2},\\c_2^{(1)}&=\frac{p^{(1)\prime\prime}(0)}{2}=\int_1^\infty\!\!\int_1^s\frac{\nu_2(b_o(s^{1/\beta})\times b_o(t^{1/\beta}))}{s^2\,t^2}\,\mathrm{d}t\,\mathrm{d}s\\&\qquad+\int_1^\infty\frac{\nu_1(b_o(s^{1/\beta}))}{s^3}\,\mathrm{d}s,\end{aligned}$$

where ν_n, $n\in\mathbb{N}$, denotes the nth-order moment measure of the point process Φ. We can see that the nth-order Maclaurin coefficient is expressed in terms of the moment measures up to the nth order. This implies that we can obtain the Padé approximants for a broader class of stationary point processes whenever the moment measures are available.

For the cellular network model in Scenario 2, let

$$\begin{aligned}&\kappa_n(s_1,\dots,s_n)\\&=\mathsf{E}\Bigl(\Phi\bigl(D(|X_{B_o}|,s_1)\bigr)\cdots\Phi\bigl(D(|X_{B_o}|,s_n)\bigr)\Bigr),\end{aligned}\tag{16}$$

for $s_1,s_2,\dots,s_n>0$. Then, the Maclaurin coefficients of $p^{(2)}$ in (12) up to the fourth order are as follows.

$$c_0^{(2)}=1,$$

$$c_1^{(2)}=-\int_1^\infty\frac{\kappa_1(s)}{s^2}\,\mathrm{d}s,\tag{17}$$

$$c_2^{(2)}=\int_1^\infty\!\!\int_1^s\frac{\kappa_2(s,t)}{s^2\,t^2}\,\mathrm{d}t\,\mathrm{d}s+\int_1^\infty\frac{\kappa_1(s)}{s^3}\,\mathrm{d}s,\tag{18}$$

$$\begin{aligned}c_3^{(2)}&=-\int_1^\infty\!\!\int_1^s\!\!\int_1^t\frac{\kappa_3(s,t,u)}{s^2\,t^2\,u^2}\,\mathrm{d}u\,\mathrm{d}t\,\mathrm{d}s\\&\quad-\int_1^\infty\!\!\int_1^s\frac{(s+t)\,\kappa_2(s,t)}{s^3\,t^3}\,\mathrm{d}t\,\mathrm{d}s-\int_1^\infty\frac{\kappa_1(s)}{s^4}\,\mathrm{d}s,\end{aligned}\tag{19}$$

$$\begin{aligned}c_4^{(2)}&=\int_1^\infty\!\!\int_1^s\!\!\int_1^t\!\!\int_1^u\frac{\kappa_4(s,t,u,v)}{s^2\,t^2\,u^2\,v^2}\,\mathrm{d}v\,\mathrm{d}u\,\mathrm{d}t\,\mathrm{d}s\\&\quad+\int_1^\infty\!\!\int_1^s\!\!\int_1^t\frac{(s\,t+t\,u+u\,s)\,\kappa_3(s,t,u)}{s^3\,t^3\,u^3}\,\mathrm{d}u\,\mathrm{d}t\,\mathrm{d}s\\&\quad+\int_1^\infty\!\!\int_1^s\frac{(s^2+s\,t+t^2)\,\kappa_2(s,t)}{s^4\,t^4}\,\mathrm{d}t\,\mathrm{d}s\\&\quad+\int_1^\infty\frac{\kappa_1(s)}{s^5}\,\mathrm{d}s.\end{aligned}\tag{20}$$

Here, it should be noted that, when computing the Maclaurin coefficients for the cellular network model, we have to compute the conditional moment measures of Φ given the distance $|X_{B_o}|$ to the nearest point from the origin.

C. Examples

We give the Maclaurin coefficients for the coverage probability in some specific cases of point processes. We here consider the cellular network model in Scenario 2 only, so that we omit the superscript $^{(2)}$, and instead, we put the symbol representing the point process such that $c_i^{(\mathrm{Poi})}$ for the Poisson process and $c_i^{(\alpha)}$ for the α-Ginibre process. The path-loss function is fixed as $\ell(x) = x^{-\beta}$, $x > 0$, for $\beta > 2$.

a) Poisson based model: For the homogeneous Poisson process, the conditional moment measures given $|X_{B_o}| = r$ reduces to just the (unconditional) moment measures and we have the following.

Corollary 1: For the cellular network model in Scenario 2, suppose that the BSs are distributed according to a homogeneous Poisson process. Then, the Maclaurin coefficients of $p^{(2)}$ in (12) up to the fourth order are given by

$$\begin{aligned}
c_1^{(\mathrm{Poi})} &= -\frac{2}{\beta-2}, \\
c_2^{(\mathrm{Poi})} &= \frac{\beta^2}{(\beta-1)(\beta-2)^2}, \\
c_3^{(\mathrm{Poi})} &= -\frac{2\,\beta^2\,(\beta^2-\beta+4)}{(\beta-1)(\beta-2)^3\,(3\,\beta-2)}, \\
c_4^{(\mathrm{Poi})} &= \frac{16}{(\beta-2)^4} + \frac{12}{(\beta-2)\,(\beta-1)} + \frac{1}{(\beta-1)^2} \\
&\quad + \frac{8}{(\beta-1)\,(3\,\beta-1)} + \frac{1}{2\,\beta-1}.
\end{aligned}$$

Proof: Let $\nu_n^{(\mathrm{Poi})}$, $n \in \mathbb{N}$, denote the nth-order moment measure for the homogeneous Poisson process with intensity λ and let $\phi(s) = \lambda\,\pi\,r^2\,(s^{2/\beta}-1)$ for $s > 0$. Then, for $s > t > u > v > 0$, we have

$$\begin{aligned}
&\nu_1^{(\mathrm{Poi})}(D(r,s)) = \phi(s), \\
&\nu_2^{(\mathrm{Poi})}(D(r,s)\times D(r,t)) = \phi(s)\,\phi(t) + \phi(t), \\
&\nu_3^{(\mathrm{Poi})}(D(r,s)\times D(r,t)\times D(r,u)) \\
&\quad = \phi(s)\,\phi(t)\,\phi(u) + [\phi(s) + 2\,\phi(t)]\,\phi(u) + \phi(u), \\
&\nu_4^{(\mathrm{Poi})}(D(r,s)\times D(r,t)\times D(r,u)\times D(r,v)) \\
&\quad = \phi(s)\,\phi(t)\,\phi(u)\,\phi(v) \\
&\qquad + [\phi(s)\phi(t) + 2\,\phi(s)\,\phi(u) + 3\,\phi(t)\,\phi(u)]\,\phi(v) \\
&\qquad + [\phi(s) + 2\,\phi(t) + 4\,\phi(u)]\,\phi(v) + \phi(v).
\end{aligned}$$

Hence, applying $\mathsf{P}(|X_{B_o}| > r) = e^{-\lambda\pi r^2}$, $r > 0$, and integrating (17)–(20), we obtain the Maclaurin coefficients in Corollary 1. ∎

Note that these Maclaurin coefficients can be obtained directly from the integral representation of the coverage probability given in [5].

b) α-Ginibre based model: We first make a brief review on the α-Ginibre point processes (see e.g., [9]–[11] for details). Let Φ denote a simple point process on $\mathbb{C}$ and $\rho_n\colon \mathbb{C}^n \to \mathbb{R}_+$, $n \in \mathbb{N}$, denote its joint intensities with respect to some locally finite measure μ on $(\mathbb{C}, \mathcal{B}(\mathbb{C}))$; that is, for any continuous symmetric function f on $\mathbb{C}^n$ with compact support,

$$\begin{aligned}
&\mathsf{E}\Bigl(\sum_{\substack{X_1,\ldots,X_n\in\Phi\\ \text{distinct}}} f(X_1, X_2, \ldots, X_n)\Bigr) \\
&\quad = \int_{\mathbb{C}^n} f(z_1,\ldots,z_n)\,\rho_n(z_1,\ldots,z_n)\,\mu(\mathrm{d}z_1)\cdots\mu(\mathrm{d}z_n).
\end{aligned}$$

The point process Φ is said to be a determinantal point process with kernel $K\colon \mathbb{C}^2 \to \mathbb{C}$ with respect to the reference measure μ if ρ_n, $n \in \mathbb{N}$, satisfy

$$\rho_n(z_1, z_2, \ldots, z_n) = \det(K(z_i, z_j))_{1\le i,j\le n},$$

for $z_1, z_2, \ldots, z_n \in \mathbb{C}$, where $\det$ denotes the determinant. Furthermore, the determinantal point process $\Phi^{*\alpha}$ is said to be an α-Ginibre process with $\alpha \in (0,1]$ when the kernel is given by $K^{*\alpha}(z,w) = e^{z\overline{w}/\alpha}$, z, $w \in \mathbb{C}$, with respect to the (scaled) Gaussian measure $\mu^{*\alpha}(\mathrm{d}z) = \pi^{-1}\,e^{-|z|^2/\alpha}\,m(\mathrm{d}z)$, where $\overline{w}$ denotes the complex conjugate of $w \in \mathbb{C}$ and m denotes the Lebesgue measure on $(\mathbb{C}, \mathcal{B}(\mathbb{C}))$ (see [16]). The usual Ginibre point process is just the one with $\alpha = 1$ and it can be shown that $\Phi^{*\alpha}$ converges in distribution to a homogeneous Poisson point process as $\alpha \to 0$. That is, the α-Ginibre processes constitute an intermediate class between the Poisson and Ginibre point processes by tuning the value of $\alpha \in (0,1]$. It is known that the α-Ginibre processes are motion-invariant (stationary and isotropic) and their intensities are equal to π^{-1}. To make it have the intensity parameter $\lambda > 0$, we consider the scaled process $\Phi_\lambda^{*\alpha}$ which has the kernel $K_\lambda^{*\alpha}(z,w) = e^{\pi\lambda z\overline{w}/\alpha}$, with respect to $\mu_\lambda^{*\alpha}(\mathrm{d}z) = \lambda\,e^{-\pi\lambda|z|^2/\alpha}\,m(\mathrm{d}z)$. Due to the radial symmetry of α-Ginibre processes, we can apply Theorem 4.7.1 in [11] and have the following proposition, which is a generalization of Kostlan's result [17] for the usual Ginibre point process.

Proposition 2: Let X_i, $i \in \mathbb{N}$, denote the points of the α-Ginibre point process with intensity λ. Then, the set $\{|X_i|^2\}_{i\in\mathbb{N}}$ has the same distribution as $\check{\boldsymbol{Y}} = \{\check{Y}_i\}_{i\in\mathbb{N}}$, which is constructed from $\boldsymbol{Y} = \{Y_i\}_{i\in\mathbb{N}}$ such that Y_i, $i \in \mathbb{N}$, are mutually independent and each Y_i follows the ith Erlang distribution with rate parameter $\pi\,\lambda/\alpha$ ($Y_i \sim \mathrm{Gamma}(i, \pi\,\lambda/\alpha)$) and it is included in $\check{\boldsymbol{Y}}$ with probability α independently of others.

According to Proposition 2, we can construct the α-Ginibre point process $\Phi_\lambda^{*\alpha}$ with intensity λ from the usual Ginibre point process $\Phi_{\lambda/\alpha}^{*1} = \{\overline{X}_i\}_{i\in\mathbb{N}}$ with intensity λ/α by independent α-thinning; that is, by deleting each point $\overline{X}_i$, $i \in \mathbb{N}$, of $\Phi_{\lambda/\alpha}^{*1}$ with probability $1-\alpha$ independently. Note that, by Proposition 2, the set $\{|\overline{X}_i|^2\}_{i\in\mathbb{N}}$ has the same distribution as $\boldsymbol{Y} = \{Y_i\}_{i\in\mathbb{N}}$ such that $Y_i \sim \mathrm{Gamma}(i, \pi\,\lambda/\alpha)$, $i \in \mathbb{N}$, are mutually independent. Let $\{\xi_i\}_{i\in\mathbb{N}}$ denote the set of marks of $\Phi_{\lambda/\alpha}^{*1}$ such that ξ_i, $i \in \mathbb{N}$, are mutually independent and identically distributed as $\mathsf{P}(\xi_i = 1) = \alpha$ and $\mathsf{P}(\xi_i = 0) = 1-\alpha$. Then, $\Phi_\lambda^{*\alpha}$ can be constructed by

$$\Phi_\lambda^{*\alpha}(C) = \sum_{i\in\mathbb{N}} \xi_i\,\mathbf{1}_C(\overline{X}_i), \quad C \in \mathcal{B}(\mathbb{C}). \tag{21}$$

Using this construction, we derive the Maclaurin coefficients for the α-Ginibre based model of the cellular network.

Corollary 2: For the cellular network model in Scenario 2, suppose that the BSs are distributed according to the α-Ginibre point process. Then, the Maclaurin coefficients of $p^{(2)}$ in (12) up to the fourth order are given by

$$\begin{aligned}
c_1^{(\alpha)} &= -\alpha \int_0^\infty e^{-v} \bigl[M(v)\,S(v)\,\Delta F(1) \\
&\qquad - M(v)\,\nabla G(1)\bigr]\,\mathrm{d}v, \\
c_2^{(\alpha)} &= \frac{\alpha}{2} \int_0^\infty e^{-v} \bigl[M(v)\,S(v) \\
&\qquad \times \bigl((\Delta F(1))^2 - \Delta F^2(1) + 2\,\Delta F(2)\bigr) \\
&\qquad - 2\,M(v)\,\Delta F(1)\,\nabla G(1) \\
&\qquad - 2\,M(v)\,\nabla\bigl(G(2) - G^2(1)\bigr)\bigr]\,\mathrm{d}v, \\
c_3^{(\alpha)} &= -\frac{\alpha}{6} \int_0^\infty e^{-v} \bigl[M(v)\,S(v)\,\bigl((\Delta F(1))^3 + 2\,\Delta F^3(1) \\
&\qquad - 3\,\Delta F^2(1)\,\Delta F(1) + 6\,\Delta F(1)\,\Delta F(2) \\
&\qquad - 6\,\Delta F(1)\,F(2) + 6\,\Delta F(3)\bigr) \\
&\qquad - 3\,M(v)\,\bigl((\Delta F(1))^2 - \Delta F^2(1) \\
&\qquad + 2\,\Delta F(2)\,\nabla G(1)\bigr) \\
&\qquad + 2\,M(v)\,\Delta F(1)\,\nabla\bigl(G(2) - G^2(1)\bigr) \\
&\qquad - 6\,M(v)\,\nabla\bigl(G(3) - 2\,G(1)\,G(2) \\
&\qquad + G^3(1)\bigr)\bigr]\,\mathrm{d}v, \\
c_4^{(\alpha)} &= \frac{\alpha}{24} \int_0^\infty e^{-v} \bigl[M(v)\,S(v)\,\bigl((\Delta F(1))^4 - 6\Delta F^4(1) \\
&\qquad + 3\,(\Delta F^2(1))^2 - 6\,(\Delta F(1))^2\,\Delta F^2(1) \\
&\qquad + 8\,\Delta F(1)\,\Delta F^3(1) \\
&\qquad + 12\,(\Delta F(1))^2\,\Delta F(2) \\
&\qquad - 12\,\Delta F^2(1)\,\Delta F(2) + 24\,\Delta F^2(1)\,F(2) \\
&\qquad - 24\,\Delta F(1)\,\Delta F(1)\,F(2) \\
&\qquad + 24\,\Delta F(1)\,\Delta F(3) \\
&\qquad - 24\,\Delta F(1)\,F(3) + 12\,(\Delta F(2))^2 \\
&\qquad - 12\,\Delta F^2(2) + 24\,\Delta F(4)\bigr) \\
&\qquad - 4\,M(v)\,\nabla G(1)\,\bigl((\Delta F(1))^3 + 2\,\Delta F^3(1) \\
&\qquad - 3\,\Delta F^2(1)\,\Delta F(1) + 6\,\Delta F(1)\,\Delta F(2) \\
&\qquad - 6\,\Delta F(1)\,F(2) + 6\,\Delta F(3)\bigr) \\
&\qquad + 12\,M(v)\,\nabla\bigl(G^2(1) - G(2)\bigr)\,\bigl((\Delta F(1))^2 \\
&\qquad - \Delta F^2(1) + 2\,\Delta F(2)\bigr) \\
&\qquad - 24\,M(v)\,\Delta F(1) \\
&\qquad \times \nabla\,\bigl(G(3) - 2G(1)G(2) + G^3(1)\bigr) \\
&\qquad + 24\,M(v)\,\nabla\bigl(G(1)^4 - 3\,G^2(1)\,G(2) \\
&\qquad + G^2(2) + 2\,G(1)\,G(3) - G(4)\bigr)\bigr]\,\mathrm{d}v,
\end{aligned}$$

where

$$\begin{aligned}
M(v) &= M(\alpha, v) = \prod_{j=1}^\infty \Bigl(1 - \alpha + \alpha\,\frac{\Gamma(j,v)}{(j-1)!}\Bigr), \\
S(v) &= S(\alpha, v) = \sum_{k=1}^\infty \frac{v^{k-1}}{(k-1)!}\,\Bigl(1 - \alpha + \alpha\,\frac{\Gamma(k,v)}{(k-1)!}\Bigr)^{-1}, \\
\Delta F(i) &= \sum_{j=1}^\infty H(i,j), \\
\nabla G(i) &= \sum_{k=1}^\infty \frac{v^{k-1}}{(k-1)!}\,\Bigl(1 - \alpha + \alpha\,\frac{\Gamma(k,v)}{(k-1)!}\Bigr)^{-1} H(i,k), \\
H(i,n) &= \frac{\alpha}{(n-1)!}\int_v^\infty s^{n-1}\,e^{-s}\,\Bigl(\frac{v}{s}\Bigr)^{i\beta/2}\,\mathrm{d}s \\
&\qquad \times \Bigl(1 - \alpha + \alpha\,\frac{\Gamma(n,v)}{(n-1)!}\Bigr)^{-1},
\end{aligned}$$

with the incomplete gamma function;

$$\Gamma(n, v) = \int_v^\infty s^{n-1} e^{-s}\mathrm{d}s.$$

Proof: We here derive $c_1^{(\alpha)}$ only. The others are verified similarly (though the derivations are a little more complicated). We use the construction (21) of the α-Ginibre point process Φ from the usual Ginibre point process $\overline{\Phi} = \Phi^{*1}_{\lambda/\alpha} = \{\overline{X}_i\}_{i\in\mathbb{N}}$ with intensity λ/α. Note that a BS really exists at $\overline{X}_i$ only when $\xi_i = 1$, $i \in \mathbb{N}$, so that $\{B_o = i\} = \{\xi_i = 1\} \cap \mathcal{A}_i$, where $\mathcal{A}_i = \{|\overline{X}_i| < |\overline{X}_j| \text{ for } j \in \mathbb{N}_\xi \setminus \{i\}\}$ with random subset $\mathbb{N}_\xi = \{j \in \mathbb{N} \mid \xi_j = 1\}$ of $\mathbb{N}$. Thus, we have from (16) that

$$\begin{aligned}
\kappa_1(s) &= \sum_{i\in\mathbb{N}} \mathsf{E}\bigl(\Phi(D(|\overline{X}_i|, s))\,\mathbf{1}_{\{B_o = i\}}\bigr) \\
&= \alpha \sum_{i\in\mathbb{N}} \mathsf{E}\bigl(\Phi(D(|\overline{X}_i|, s))\,\mathbf{1}_{\mathcal{A}_i}\bigr), \qquad (22)
\end{aligned}$$

where we use that ξ_i is independent of others with $\mathsf{P}(\xi_i = 1) = \alpha$ in the second equality. Note here that

$$\begin{aligned}
\Phi(D(|\overline{X}_i|, s)) &= \sum_{k\in\mathbb{N}\setminus\{i\}} \mathbf{1}_{\{\xi_k = 1, |\overline{X}_i| < |\overline{X}_k| \le s^{1/\beta}|\overline{X}_i|\}}, \\
\mathbf{1}_{\mathcal{A}_i} &= \prod_{j\in\mathbb{N}\setminus\{i\}} \mathbf{1}_{\{\xi_j = 1, |\overline{X}_j| > |\overline{X}_i|\} \cup \{\xi_j = 0\}}.
\end{aligned}$$

Applying these to (22) yields

$$\begin{aligned}
&\mathsf{E}\bigl(\Phi(D(|\overline{X}_i|, s))\,\mathbf{1}_{\mathcal{A}_i}\bigr) \\
&= \alpha \sum_{k\in\mathbb{N}\setminus\{i\}} \mathsf{P}\bigl(|\overline{X}_i| < |\overline{X}_k| \le s^{1/\beta}|\overline{X}_i|, \\
&\qquad \{\xi_j = 1, |\overline{X}_j| > |\overline{X}_i|\} \cup \{\xi_j = 0\}, j \in \mathbb{N}\setminus\{i,k\}\bigr). \qquad (23)
\end{aligned}$$

Here, we use Proposition 2; that is, $\{|\overline{X}_i|^2\}_{i\in\mathbb{N}} =_d \{Y_i\}_{i\in\mathbb{N}}$ with $Y_i = \alpha\,Z_i/(\pi\,\lambda)$ such that $Z_i \sim \mathrm{Gamma}(i,1)$, $i \in \mathbb{N}$, are mutually independent. Noting that $\{Z_i < Z_k \le s^{2/\beta}\,Z_i\}$ and $\{Z_j > Z_i\}$, $j \in \mathbb{N}\setminus\{i,k\}$, are conditionally independent given Z_i, (23) reduces to

$$\begin{aligned}
&\mathsf{E}\bigl(\Phi(D(|\overline{X}_i|, s))\,\mathbf{1}_{\mathcal{A}_i}\bigr) \\
&= \alpha \sum_{k\in\mathbb{N}\setminus\{i\}} \mathsf{E}\Bigl(\mathsf{P}\bigl(Z_i < Z_k \le s^{2/\beta}\,Z_i \mid Z_i\bigr) \\
&\qquad \times \prod_{j\in\mathbb{N}\setminus\{i,k\}} \bigl[1 - \alpha + \alpha\,\mathsf{P}\bigl(Z_j > Z_i \mid Z_i\bigr)\bigr]\Bigr).
\end{aligned}$$

Finally, applying the density functions of $Z_i \sim \mathrm{Gamma}(i, 1)$, $i \in \mathbb{N}$, to the above, we obtain $c_1^{(\alpha)}$ in Corollary 2 after some manipulations. ∎

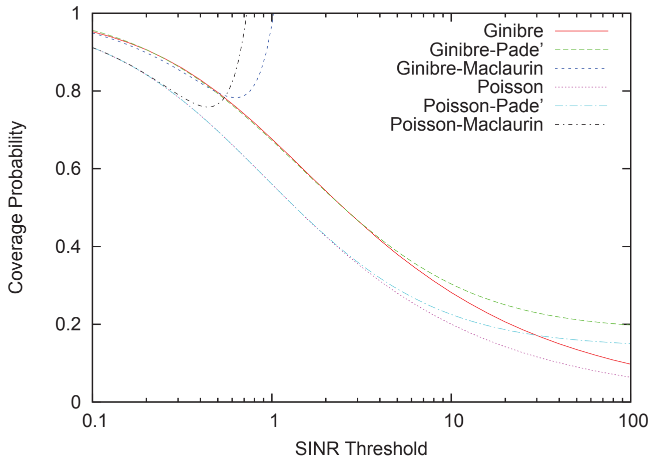

Fig. 1. Comparison of numerical integration, $(2,2)$-Padé approximation and 4th order Maclaurin approximation (Poisson and Ginibre based models with $\beta = 4$).

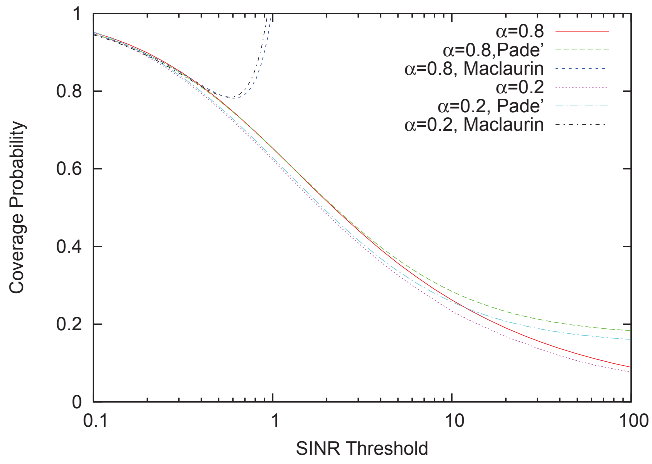

Fig. 2. Comparison of numerical integration, $(2,2)$-Padé approximation and 4th order Maclaurin approximation (α-Ginibre based model with $\alpha = 0.2$, 0.8 and $\beta = 4$).

As in the Poisson based model, the Maclaurin coefficients in Corollary 2 can be obtained directly from the integral representation of the coverage probability given in [7].

IV. Numerical experiments

We show some results of numerical experiments for the cellular network model in Scenario 2. In each experiment, we fix the path-loss function as $\ell(x) = c\,x^{-\beta}$, $x > 0$, with path-loss exponent $\beta = 4$. Note here that, in the noise-free (interference-limited) case, the coverage probability in the cellular network model does not depend on the path-loss coefficient c, the intensity λ and transmission power p (see [5]–[7]).

Figure 1 shows the numerical results for the Poisson based and Ginibre based models. The results from the direct numerical computation of the integral representations by [5] for the Poisson based model and by [6] for the Ginibre based model are compared with those of the (2,2)-Padé approximants and 4th-order Maclaurin approximants. We find that the Padé approximants agree with the direct numerical integration better than the Maclaurin approximants and furthermore that the Padé approximants are applicable for the practical use. Note here that the $(2,2)$-Padé and the fourth-order Maclaurin approximants are obtained from the same information; that is, the Maclaurin coefficients up to the fourth order.

Figure 2 shows the results for the same experiment as that of Figure 1, but for the α-Ginibre based model with $\alpha = 0.2$ and 0.8. We find the same features as Figure 1.

Figure 3 shows the numerical results of the experiment where the $(1,1)$-Padé approximants are compared with the direct numerical integrations for the Poisson based and Ginibre based models. We find that even the $(1,1)$-Padé approximants agree with the direct numerical integrations up to around $\theta = 1$, so that they seem sufficient for the practical use.

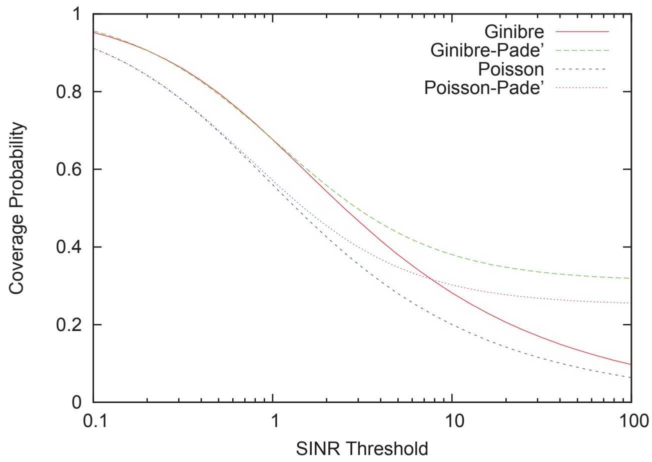

Fig. 3. Comparison of numerical integration and $(1,1)$-Padé approximation (Poisson and Ginibre based models with $\beta = 4$).

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