On the Statistical Properties of Selection Combining Cooperative Diversity over Double Rice Fading Channels

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***Abstract*—This paper investigates the statistical properties of selection combining (SC) cooperative diversity over double Rice narrow-band fading channels under the transparent or amplify-and-forward (AF) strategy. It is assumed that the source, the destination, and relay stations are all equipped with single antenna. The Doppler power spectral density, associated with each of the double Rice fading processes, is considered to be symmetrical about the carrier frequency. Under the above conditions, analytical integral expressions are derived for the probability density function (PDF) and the cumulative distribution function (CDF) of the SC fading envelopes. Also, expressions for the level crossing rate (LCR) as well as the average duration of fades (ADF) are determined. For some cases, the integral expressions are shown to be approximated using the Laplace's method of integration. The approximations are verified to be accurate when the number of relay stations is large. The obtained results include those corresponding to the double Rayleigh and combined Rayleigh×Rice models as special cases of the double Rice fading channel. The validity of all the presented theoretical results are checked by computer simulations.**

***Index Terms*—Double Rice fading channels, selection combining (SC) cooperative diversity, probability density function (PDF), cumulative distribution function (CDF), level crossing rate (LCR), average duration of fades (ADF).**

I. Introduction

Multiplicative fading models are useful in the statistical description of multipath channels in many important scenarios such as mobile-to-mobile (M2M) transmission [1], keyhole propagation in multiple-input multiple-output (MIMO) systems [2], and multihop and cooperative communications [3], [4]. In the context of conventional multihop relaying concept, where the information signal is transmitted from the source to the destination via intermediate nodes operating as relays, the channel on each relaying segment can separately be described by the well known classical fading models, e.g. Rayleigh, Rice, Nakagami-m [5]. For convenience, however, it is desirable to consider the end-to-end fading channel as a whole with proper statistics. In this setting, the approach consisting on the modeling of multihop channels by the product of single classical models has mostly been an appropriate choice to yield the so-called compound or multiplicative fading channels [6]. The study of multiplicative fading models has therefore received a great deal of attention during the last years. For instance, the first order statistical properties of double Rayleigh channels have been reported in [7]. The second order statistics of multihop Rayleigh channels have been analyzed in [8]. The double Nakagami-m channel has been studied in [9] where the LCR and the ADF have been obtained. The analysis of the first and second order statistics of the double Rice fading model has been addressed in [10]. In addition to multihop communications, cooperative relaying, also termed cooperative diversity [1], [4], [11] has proved its ability to improve the radio link quality and the network coverage. To the best of our knowledge, the statistical characterization of cooperative diversity signals over multiplicative fading channels has not sufficiently been addressed in the literature. For instance, the statistical properties of equal gain combiner over double Rayleigh and double Rice channels have been investigated in [12] and [13], respectively. In [14], the performance of digital modulations over double Nakagami-m fading channels with maximum ratio combining diversity has been studied.

In this paper, aiming at contributing to the topic of statistical analysis of cooperative diversity over compound fading channels, we study the statistics of SC cooperative diversity over a dual-hop Rice fading channels under the AF strategy. Specifically, assuming that the fading processes are independent but not necessarily identically distributed, we derive the first order statistics in terms of the CDF and PDF of the resultant SC fading process. We then confine our attention to the investigation of the second order statistics in the form of LCR and ADF. Results corresponding to both Rayleigh×Rice and double Rayleigh fading channels with SC diversity have also been deduced as special cases. Furthermore, the validity of the derived expressions has been checked by means of computer simulations.

The rest of this paper is structured as follows. In Section II, we present some preliminaries and known results on the statistics of double Rice fading channels. In Section III, we provide the first order statistics of the underlying SC cooperative diversity whereas, in Section IV, we investigate the second order statistics. Numerical examples and results verification are presented in Section V. Finally, Section VI concludes the paper.

II. Channel model and literature review

We consider the dual hop amplify-and-forward relaying schemes studied in [10], where the multipath propagation on each of the two subchannels is described by a frequency-flat Rice fading channel. Without loss of generality, we can set the gain of the relay station to the value one since this has no impact on the fading statistics. In this case, the end-to-end fading process between the source and the destination stations, for a transmission via a relay station, can be modeled as a product of two independent but not necessarily identically distributed single Rice processes according to [10]

$$\eta(t) = \eta_1(t)\eta_2(t) \tag{1}$$

where $\eta_i(t)$ $(i = 1, 2)$ represents the single Rice fading process described as

$$\begin{aligned} \eta_i(t) &= |\mu_i(t) + m_i(t)| \\ &= |\mu_{i1}(t) + j\mu_{i2}(t) + m_i(t)| \end{aligned} \tag{2}$$

in which $\mu_i(t)$ $(i = 1, 2)$ denotes a zero-mean complex Gaussian process with a variance σ_i^2. Also, in (2), $m_i(t)$ represents the line-of-sight (LOS) component described analytically as

$$m_i(t) = \rho_i e^{j(2\pi f_{\rho_i} t + \theta_{\rho_i})} \tag{3}$$

where the parameters ρ_i, f_{ρ_i}, and θ_{ρ_i} stand for the amplitude, Doppler frequency, and phase, respectively. The main statistical properties of the double Rice process $\eta(t)$ have been derived in [10]. The PDF of $\eta(t)$ has been obtained in [10, Eq. (12)] as

$$p_\eta(z) = \frac{z}{\sigma_1^2\sigma_2^2} \int_0^\infty \frac{1}{y} e^{-\left(\frac{y^2+\rho_2^2}{2\sigma_2^2}\right)} e^{-\left(\frac{z^2}{2\sigma_1^2 y^2} + \frac{\rho_1^2}{2\sigma_1^2}\right)} \times I_0\left(\frac{\rho_2 y}{\sigma_2^2}\right) I_0\left(\frac{\rho_1 z}{y\sigma_1^2}\right) dy, \quad z \geq 0 \tag{4}$$

where $I_0(.)$ denotes the zeroth order modified Bessel function of the first kind [15]. Regarding the corresponding CDF, it has been shown to be given by

$$F_\eta(z) = 1 - \int_0^\infty Q_1\left(\frac{\rho_1}{\sigma_1}, \frac{z}{\sigma_1 y}\right) p_{\eta_2}(y) dy, \quad z \geq 0 \tag{5}$$

where $Q_1(\cdot,\cdot)$ is the first order Marcum Q-function [15] and $p_{\eta_2}(y)$ stands for the PDF of the single Rice process $\eta_2(t)$ given by [5]

$$p_{\eta_2}(y) = \frac{y}{\sigma_2^2} e^{\left(-\frac{y^2+\rho_2^2}{2\sigma_2^2}\right)} I_0\left(\frac{\rho_2 y}{\sigma_2^2}\right). \tag{6}$$

Here, it should be mentioned that (5) describes the outage probability of radio links under double Rice fading channels. The LCR of $\eta(t)$, which describes how often the process $\eta(t)$ crosses a given level r from up to down (or from down to up), has also been determined in [10] as

$$\begin{aligned} N_\eta(r) = {} & \frac{r}{(2\pi)^{5/2}\sigma_1^2\sigma_2^2} \int_0^\infty \sqrt{\beta_2 \frac{r^2}{y^4} + \beta_1} \\ & \times e^{-\left(\frac{y^2+\rho_2^2}{2\sigma_2^2}\right)} e^{-\left(\frac{r^2}{y^2 2\sigma_1^2} + \frac{\rho_1^2}{2\sigma_1^2}\right)} \\ & \times \int_{-\pi}^{\pi} e^{\frac{r\rho_1 \cos\theta_1}{\sigma_1^2 y}} \int_{-\pi}^{\pi} e^{\frac{y\rho_2 \cos\theta_2}{\sigma_2^2}} e^{-\frac{K^2(r,y,\theta_1,\theta_2)}{2}} \\ & \times \left(1 + \sqrt{\frac{\pi}{2}} K(r, y, \theta_1, \theta_2)\, e^{\frac{1}{2}K^2(r,y,\theta_1,\theta_2)} \right. \\ & \left. \times \left\{1 + \Phi\left(\frac{K(r, y, \theta_1, \theta_2)}{2}\right)\right\}\right) d\theta_2 d\theta_1 dy \end{aligned} \tag{7}$$

where β_i $(i = 1, 2)$ stands for the negative curvature of the autocorrelation function $\Gamma_{\mu_{ij}\mu_{ij}}(\tau)$ $(i, j = 1, 2)$ of the process $\mu_{ij}(t)$ at $\tau = 0$, i.e., $\beta_i = -\ddot{\Gamma}_{\mu_{ij}\mu_{ij}}(0)$, where the double dot denotes second time derivative. Also in (7), $\Phi(\cdot)$ denotes the error function [15, Eq. 8.250(1)] and $K(\cdot,\cdot,\cdot,\cdot)$ is defined by [10]

$$K(r, y, \theta_1, \theta_2) = \frac{2\pi\rho_1 f_{\rho_1} y^2 \sin(\theta_1) + 2\pi\rho_2 f_{\rho_2} r \sin(\theta_2)}{\sqrt{\beta_1 y^4 + \beta_2 r^2}}. \tag{8}$$

Based on these statistical properties, in this work, we study the statistics of SC diversity in cooperative networks. Our focus is on the relaying systems that consist of one source, one destination, and L parallel relays, all equipped with single antenna. The relays are assumed to operate according to the time-division multiple-access (TDMA) AF protocols proposed in [16], [17]. Each end-to-end sublink of the cooperative diversity system follows the double Rice fading statistics. Assuming that all the dual-hop links are subject to the same mean noise power, the best relay can be chosen according to the highest amplitude of the L double Rice fading processes. In this case, the fading envelope available at the output of the SC can be written as

$$\eta_{SC}(t) = \max\left(\eta^{(1)}(t), ..., \eta^{(j)}(t), ..., \eta^{(L)}(t)\right) \tag{9}$$

where $\eta^{(j)}(t)$ $(j = 1, \ldots, L)$ stands for the double Rice fading process of the jth diversity branch. The first and second order statistics of $\eta_{SC}(t)$ will be investigated in sections III and IV, respectively.

III. First order statistics of SC diversity

A. CDF of the process $\eta_{SC}(t)$

The CDF of the SC output can be obtained according to

$$F_{\eta_{SC}}(z) = \Pr\left[\eta_{SC}(t) \leq z\right] \tag{10}$$

where $\Pr[\cdot]$ denotes the probability operator. Using (9) in (10), we can write

$$F_{\eta_{SC}}(z) = \Pr\left[\eta^{(1)}(t) \leq z, ..., \eta^{(j)}(t) \leq z, ..., \eta^{(L)}(t) \leq z\right]. \tag{11}$$

Then, under the assumption that all the L sublinks of the diversity system follow independent and identically distributed (i.i.d.) fading processes, the CDF $F_{\eta_{SC}}(z)$ of the process $\eta_{SC}(t)$ is obtained as

$$F_{\eta_{SC}}(z) = (F_\eta(z))^L \quad (12)$$

where $F_\eta(z)$ is the CDF of the double Rice process given in (5). Substituting (5) in (12), we obviously get the following expression for $F_{\eta_{SC}}(z)$

$$F_{\eta_{SC}}(z) = \left(1 - \int_0^\infty Q_1\left(\frac{\rho_1}{\sigma_1}, \frac{z}{y\sigma_1}\right) p_{\eta_2}(y)dy\right)^L. \quad (13)$$

By letting $L = 1$, i.e., the non-diversity case, we obtain the CDF of double Rice fading channels given in (5). For the special case corresponding to Rayleigh×Rice fading channels, i.e., $\rho_1 = 0$, (13) can be written as

$$F_{\eta_{SC}}(z) = \left(1 - \int_0^\infty e^{\left(-\frac{z^2}{2y^2\sigma_1^2}\right)} p_{\eta_2}(y)dy\right)^L. \quad (14)$$

Setting, furthermore, $\rho_2 = 0$ in (14), and using [15, Eq. 3.471(9)], provides us with the following closed-form expression for the CDF of SC diversity over double Rayleigh fading channels

$$F_{\eta_{SC}}(z) = \left(1 - \frac{z}{\sigma_1\sigma_2} K_1\left(\frac{z}{\sigma_1\sigma_2}\right)\right)^L \quad (15)$$

where $K_1(.)$ denotes the first order modified Bessel function of the second kind [15]. Obviously, letting $L = 1$ in (15), yields the CDF of the double Rayleigh fading process given in [7, Eq. (4)]. We should add that the outage probability, with respect to a given threshold level r, is obtained from the CDF as $P_{out}(r) = F_{\eta_{SC}}(r)$.

B. PDF of the process $\eta_{SC}(t)$

Given the CDF $F_{\eta_{SC}}(z)$ of the process $\eta_{SC}(t)$, the corresponding PDF can be obtained according to

$$\begin{aligned} p_{\eta_{SC}}(z) &= \frac{d}{dz}\left(F_{\eta_{SC}}(z)\right) \\ &= L \cdot p_\eta(z) \cdot (F_\eta(z))^{L-1} \end{aligned} \quad (16)$$

where $p_\eta(z)$ and $F_\eta(z)$ are given by (4) and (5), respectively. Again, it should be noted that for the non-diversity case, i.e., $L = 1$, (16) simplifies to the PDF of double Rice processes given in (4). For the special case of SC diversity over Rayleigh×Rice channels, i.e., $\rho_1 = 0$, the PDF in (16) can be written as

$$\begin{aligned} p_{\eta_{SC}}(z) \;=\; & L\left(1 - \int_0^\infty e^{\left(-\frac{z^2}{y^2 2\sigma_1^2}\right)} p_{\eta_2}(y)dy\right)^{L-1} \\ & \times \int_0^\infty \frac{z}{(y\sigma_1)^2} e^{\left(-\frac{z^2}{y^2 2\sigma_1^2}\right)} p_{\eta_2}(y)dy. \end{aligned} \quad (17)$$

Letting $\rho_2 = 0$ in (17) and using [15, Eq. 3.471(9)], yields the following closed-form expression for the PDF $p_{\eta_{SC}}(z)$

$$p_{\eta_{SC}}(z) = \frac{Lz}{\sigma_1^2\sigma_2^2} K_0\left(\frac{z}{\sigma_1\sigma_2}\right)\left(1 - \frac{z}{\sigma_1\sigma_2} K_1\left(\frac{z}{\sigma_1\sigma_2}\right)\right)^{L-1} \quad (18)$$

where $K_0(\cdot)$ is the zeroth order modified Bessel function of the second kind [15]. Clearly, (18) stands for the PDF of the SC output in the case where the L diversity sublinks follow double Rayleigh fading statistics. Finally, for $L = 1$, (18) reduces to the PDF of double Rayleigh processes.

C. Approximate solution to the PDF of $\eta_{SC}(t)$

As is known, the PDF of fading processes is an important statistics that is usually needed for establishing the performance analysis of wireless communications. The PDF in (4) is, however, not analytically tractable for further insights on channel statistical characterization and radio links performance investigation. The limitation lies in the fact that (4) is expressed in terms of a semi-infinite integral. Here, we propose to alleviate this limitation by providing an approximate solution to the underlying integral based on the use of the Laplace's method of integration [18]. This method can, in essence, be summarized by the following result

$$\int_0^\infty g(y)e^{-\lambda f(y)}dy \approx \sqrt{\frac{2\pi}{\lambda}} \frac{g(y_0)}{\sqrt{f''(y_0)}} e^{-\lambda f(y_0)} \quad (19)$$

where λ is a positive parameter that can be large or small [18], $f(y)$ and $g(y)$ represent two real-valued functions which are assumed to be infinitely differentiable, and the parameter y_0 denotes the critical point of the function $f(y)$. Also in (19), $f''(y)$ denotes the second derivative of the function $f(y)$ with respect to the variable y. Then, it can easily be seen that the semi-infinite integral in (4) is in the form of the Laplace's integral for which the various involved quantities in (19) are identified for our case as

$$\begin{cases} f(y) = \frac{y^2+\rho_2^2}{2\sigma_2^2} + \frac{z^2}{2\sigma_1^2 y^2} + \frac{\rho_1^2}{2\sigma_1^2} \\ g(y) = \frac{z}{\sigma_1^2\sigma_2^2 y} I_0\left(\frac{\rho_2 y}{\sigma_1^2}\right) I_0\left(\frac{\rho_1 z}{\sigma_2^2 y}\right) \\ f''(y) = \frac{1}{\sigma_2^2} + \frac{3z^2}{\sigma_1^2 y^4} \\ y_0 = \sqrt{\frac{z\sigma_2}{\sigma_1}}. \end{cases} \quad (20)$$

Substituting in (19) the appropriate quantities given in (20), the PDF $p_\eta(z)$ of the double Rice process $\eta(t)$ can be approximated by

$$\begin{aligned} p_\eta(z) \;\approx\; & \sqrt{\frac{\pi z}{2(\sigma_1\sigma_2)^3}} \cdot I_0\left(\rho_2\sqrt{\frac{z}{\sigma_1\sigma_2^3}}\right) \\ \times\; & I_0\left(\rho_1\sqrt{\frac{z}{\sigma_2\sigma_1^3}}\right) e^{-\left(\frac{z}{\sigma_1\sigma_2} + \frac{\rho_2^2}{2\sigma_2^2} + \frac{\rho_1^2}{2\sigma_1^2}\right)}. \end{aligned} \quad (21)$$

An approximate solution for the PDF $p_{\eta_{SC}}(z)$ can now be obtained by just replacing (21) in (16). This gives

$$\begin{aligned} p_{\eta_{SC}}(z) &\approx L\sqrt{\frac{\pi z}{2\left(\sigma_1\sigma_2\right)^3}} \cdot I_0\left(\rho_2\sqrt{\frac{z}{\sigma_1\sigma_2^3}}\right) \\ &\times I_0\left(\rho_1\sqrt{\frac{z}{\sigma_2\sigma_1^3}}\right) e^{-\left(\frac{z}{\sigma_1\sigma_2}+\frac{\rho_2^2}{2\sigma_2^2}+\frac{\rho_1^2}{2\sigma_1^2}\right)} \\ &\times \left(F_\eta(z)\right)^{L-1}. \end{aligned} \quad (22)$$

Letting $\rho_1 = 0$ in (22), i.e., Rayleigh×Rice fading model, provides us with the following approximate solution for $p_{\eta_{SC}}(z)$

$$\begin{aligned} p_{\eta_{SC}}(z) \approx &L\sqrt{\frac{\pi z}{2\left(\sigma_1\sigma_2\right)^3}} \cdot I_0\left(\rho_2\sqrt{\frac{z}{\sigma_1\sigma_2^3}}\right) \\ &\times e^{-\left(\frac{z}{\sigma_1\sigma_2}+\frac{\rho_2^2}{2\sigma_2^2}\right)} \\ &\times \left(1-\int_0^\infty e^{\left(-\frac{z^2}{2y^2\sigma_1^2}\right)} p_{\eta_2}(y)dy\right)^{L-1}. \end{aligned} \quad (23)$$

Similarly, by considering the special case given by $\rho_1 = \rho_2 = 0$, and using [15, Eq. 3.471(9)], it can be shown that (22) reduces to

$$\begin{aligned} p_{\eta_{SC}}(z) &\approx L\sqrt{\frac{\pi z}{2\left(\sigma_1\sigma_2\right)^3}} e^{-\frac{z}{\sigma_1\sigma_2}} \\ &\times \left(1-\frac{z}{\sigma_1\sigma_2}K_1\left(\frac{z}{\sigma_1\sigma_2}\right)\right)^{L-1}. \end{aligned} \quad (24)$$

which corresponds to the approximate PDF of SC fading processes in double Rayleigh channels.

IV. Second order statistics of SC diversity

This section is devoted to the derivation of the main second order statistics of $\eta_{SC}(t)$ in the form of the LCR and ADF. In this case of fading processes, the occurrence of a down crossing of a given level r by the process $\eta_{SC}(t)$ is attributed to the situation where one of the L fading processes $\eta^{(i)}(t)$, $(i = 1, 2, ..., L)$, goes downward through the level r while the others processes take values below r. For this situation, the following result, reported in [19, Eq. (12)], can directly be applied to get the LCR of the SC output fading process

$$N_{\eta_{SC}}(z) = \sum_{l=1}^{L} N_{\eta^{(l)}}(z) \prod_{\substack{k=1\\k\neq l}}^{L} F_{\eta^{(k)}}(z) \quad (25)$$

where $N_{\eta^{(l)}}(r)$ $(l = 1, \ldots, L)$ represents the LCR of the lth SC diversity branch available in (7), while $F_{\eta^{(k)}}(r)$ $(k = 1\ldots, L)$ is the CDF of the kth branch given in (5). Since we are studying the case where the fading processes of the diversity system are i.i.d., (25) reduces to

$$N_{\eta_{SC}}(r) = L \cdot N_\eta(r) \cdot (F_\eta(r))^{L-1}. \quad (26)$$

The application of (26) results in the following expression for the LCR $N_{\eta_{SC}}(r)$ of the process $\eta_{SC}(t)$

$$\begin{aligned} N_{\eta_{SC}}(r) = &\frac{Lr}{(2\pi)^{5/2}\sigma_1^2\sigma_2^2}\int_0^\infty \sqrt{\beta_2\frac{r^2}{y^4}+\beta_1} \\ &\times e^{-\left(\frac{y^2+\rho_2^2}{2\sigma_2^2}\right)} e^{-\left(\frac{r^2}{2\sigma_1^2 y^2}+\frac{\rho_1^2}{2\sigma_1^2}\right)} \\ &\times \int_{-\pi}^{\pi} e^{\frac{r\rho_1\cos\theta_1}{\sigma_1^2 y}} \int_{-\pi}^{\pi} e^{\frac{y\rho_2\cos\theta_2}{\sigma_2^2}} e^{-\frac{K^2(r,y,\theta_1,\theta_2)}{2}} \\ &\times \left(1+\sqrt{\frac{\pi}{2}}K\left(r,y,\theta_1,\theta_2\right) e^{\frac{1}{2}K^2(r,y,\theta_1,\theta_2)}\right. \\ &\times \left.\left\{1+\Phi\left(\frac{K\left(r,y,\theta_1,\theta_2\right)}{2}\right)\right\}\right) d\theta_2 d\theta_1 \\ &\times \left(1-\int_0^\infty Q_1\left(\frac{\rho_1}{\sigma_1},\frac{r}{y\sigma_1}\right)p_{\eta_2}(y)dy\right)^{L-1} dy. \end{aligned} \quad (27)$$

Unfortunately, the above expression involves several integrals that can be evaluated only by using numerical techniques. However, for the case where the Doppler frequencies of the LOS components are equal to zero, i.e., $f_{\rho_1} = f_{\rho_2} = 0$, (27) simplifies considerably to yield

$$\begin{aligned} N_{\eta_{SC}}(r) = &\frac{Lr}{\sqrt{2\pi}\sigma_1^2\sigma_2^2}\int_0^\infty \sqrt{\beta_2\frac{r^2}{y^4}+\beta_1} \\ &\times e^{-\left(\frac{r^2}{2\sigma_1^2 y^2}+\frac{\rho_1^2}{2\sigma_1^2}\right)} e^{-\left(\frac{y^2+\rho_2^2}{2\sigma_2^2}\right)} I_0\left(\frac{r\rho_1}{\sigma_1^2 y}\right) I_0\left(\frac{y\rho_2}{\sigma_2^2}\right) \\ &\times \left(1-\int_0^\infty Q_1\left(\frac{\rho_1}{\sigma_1},\frac{r}{y\sigma_1}\right)p_{\eta_2}(y)dy\right)^{L-1} dy. \end{aligned} \quad (28)$$

Regarding the special case of Rayleigh×Rice fading channels, i.e., the case where $\rho_1 = 0$, the finite range integral with respect to the variable θ_1 in (27) can be evaluated and the result becomes

$$\begin{aligned} N_{\eta_{SC}}(r) = &\frac{Lr}{(2\pi)^{3/2}\sigma_1^2\sigma_2^2}\int_0^\infty \sqrt{\beta_2\frac{r^2}{y^4}+\beta_1} \\ &\times e^{-\left(\frac{y^2+\rho_2^2}{2\sigma_2^2}\right)} e^{-\frac{r^2}{2\sigma_1^2 y^2}} \\ &\times \int_{-\pi}^{\pi} e^{\frac{y\rho_2\cos\theta_2}{\sigma_2^2}} e^{-\frac{F^2(r,y,\theta_2)}{2}} \\ &\times \left(1+\sqrt{\frac{\pi}{2}}F\left(r,y,\theta_2\right) e^{\frac{F^2(r,y,\theta_2)}{2}}\right. \\ &\times \left.\left\{1+\Phi\left(\frac{F\left(r,y,\theta_2\right)}{2}\right)\right\}\right) d\theta_2 \\ &\times \left\{1-\int_0^\infty e^{\left(-\frac{r^2}{y^2 2\sigma_1^2}\right)} p_{\eta_2}(y)dy\right\}^{L-1} dy \end{aligned} \quad (29)$$

where the function $F(\cdot,\cdot,\cdot)$ is given by

$$F(r, y, \theta_2) = \frac{2\pi\rho_2 f_{\rho_2} r \sin(\theta_2)}{\sqrt{\beta_1 y^4 + \beta_2 r^2}}. \qquad (30)$$

Also, for the special case corresponding to $f_{\rho_2} = 0$, (29) reduces to the following expression

$$N_{\eta_{SC}}(r) = \frac{Lr}{\sqrt{2\pi}\sigma_1^2\sigma_2^2} \int_0^\infty \sqrt{\beta_2 \frac{r^2}{y^4} + \beta_1} \times e^{-\frac{y^2+\rho_2^2}{2\sigma_2^2}} e^{-\left(\frac{r^2}{2\sigma_1^2 y^2}\right)} I_0\left(\frac{y\rho_2}{\sigma_2^2}\right) \times \left\{1 - \int_0^\infty e^{\left(-\frac{r^2}{y^2 2\sigma_1^2}\right)} p_{\eta_2}(y) dy\right\}^{L-1} dy. \qquad (31)$$

By letting $\rho_1 = \rho_2 = 0$ in (27), a closed-form expression for the LCR of SC diversity over double Rayleigh fading channels can be determined as

$$N_{\eta_{SC}}(r) = \frac{Lr}{\sqrt{2\pi}\sigma_1^2\sigma_2^2} \left(1 - \frac{r}{\sigma_1\sigma_2} K_1\left(\frac{r}{\sigma_1\sigma_2}\right)\right)^{L-1} \times \int_0^\infty \sqrt{\beta_2 \frac{r^2}{y^4} + \beta_1} e^{-\left(\frac{y^2}{2\sigma_2^2} + \frac{r^2}{2\sigma_1^2 y^2}\right)} dy. \qquad (32)$$

Furthermore, setting $L = 1$ in (32), i.e., the non-diversity case, we obtain the LCR of double Rayleigh fading channels given in [20, Eq. (17)]. Here, it can be noted that the semi-infinite range integral in (32) has the form of the Laplace type integral in one dimension as described in (19). Then, the application of the Laplace's method of integration [18] on (32) allows us to get the following approximate expression for the LCR $N_{\eta_{SC}}(r)$

$$N_{\eta_{SC}}(r) \approx \frac{Lr}{2(\sigma_1\sigma_2^2)} e^{-\left(\frac{r}{\sigma_2\sigma_1}\right)} \left(\sqrt{\beta_2 \frac{\sigma_1^2}{\sigma_2^2} + \beta_1}\right) \times \left(1 - \frac{r}{\sigma_1\sigma_2} K_1\left(\frac{r}{\sigma_1\sigma_2}\right)\right)^{L-1}. \qquad (33)$$

For the case where $L = 1$, (33) coincides with the already known result given in [8, Eq. (33)]. For completeness, it should be mentioned that the ADF of SC diversity over double Rice fading channels can easily be determined according to [21]

$$T_{\eta_{SC}}(r) = \frac{F_{\eta_{SC}}(r)}{N_{\eta_{SC}}(r)} = \frac{F_\eta(z)}{L N_\eta(z)}. \qquad (34)$$

That is, using (5) and (7), the underlying ADF $T_{\eta_{SC}}(r)$ can easily be evaluated from (34).

V. Numerical and Simulation Results

To confirm the validity and correctness of the investigated theoretical quantities, we compare them with corresponding simulation results. The generation of the envelope process

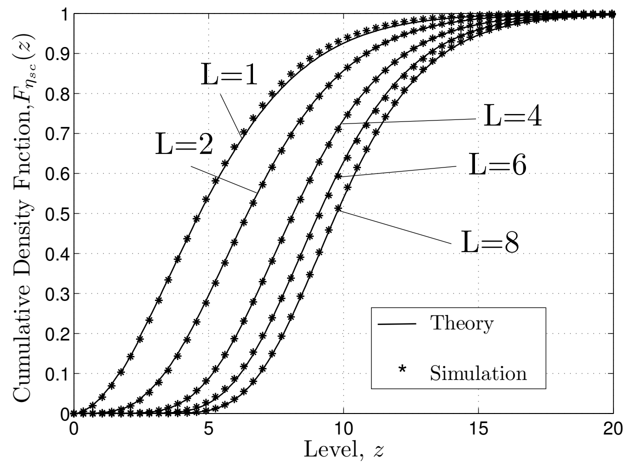

Fig. 1. The CDF of SC diversity over double Rice fading channels for different values of the number of branches L.

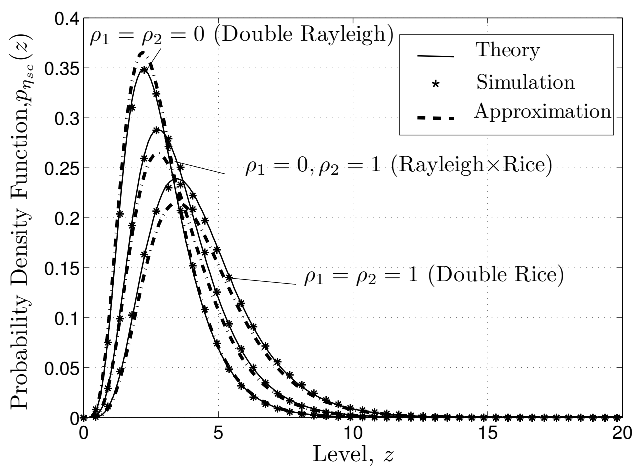

Fig. 2. The PDF of SC diversity over double Rice fading channels for $L = 4$ and various values of ρ_1 and ρ_2.

$\eta_{SC}(t)$ is obtained by using the concept of Rice's sum-of-sinusoids [21]. The method of exact Doppler spread [22] is employed in the determination of the parameters of the sinusoids. We also use Clarke's isotropic scattering model [23] for which $\beta_i = 2(\sigma_i \pi f_{\max_i})^2$ $(i = 1, 2)$, where $f_{\max_i}$ denotes the maximum Doppler frequency. Here, it should be mentioned that $f_{\max_i}$ depends on the motion of the source, relay, and destination stations. All the results to be shown are obtained for the variances $\sigma_1^2 = \sigma_2^2 = 1$, the Doppler frequencies $f_{\max_1} = f_{\max_2} = 80$ Hz, and the scenario corresponding to motionless relay stations. Also, for simplicity, the Doppler frequencies as well as the phases of the LOS components are set to zero, i.e., $f_{\rho_i} = 0$, and $\theta_{\rho_i} = 0$ $(i = 1, 2)$.

Fig. 1 illustrates the behavior of the theoretical and simulated CDF of SC diversity over double Rice fading channels for $L = 1, 2, 4, 6,$ and 8. A good fit between the theoretical and simulation results can be observed. For a fixed level z, an increase in the number of diversity branches L results in a decrease of the outage probability performance as one

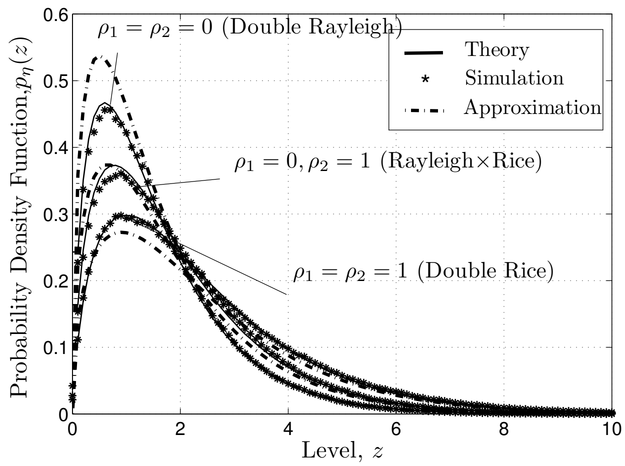

Fig. 3. A comparison between the analytical, approximate, and simulated PDFs of double Rice fading channels for different values of ρ_1 and ρ_2.

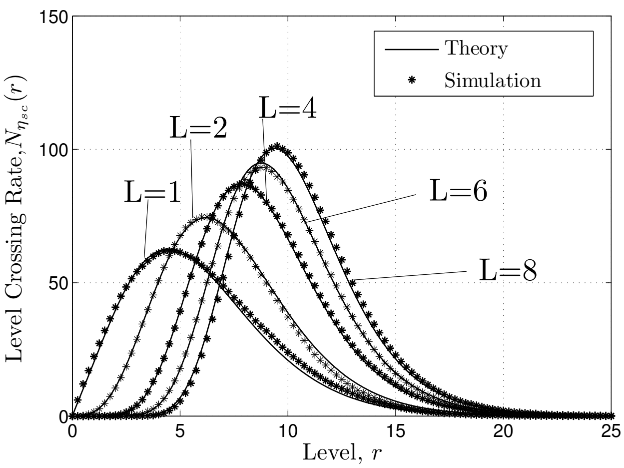

Fig. 4. The LCR of SC diversity over double Rice fading channels for different values of the number of branches L.

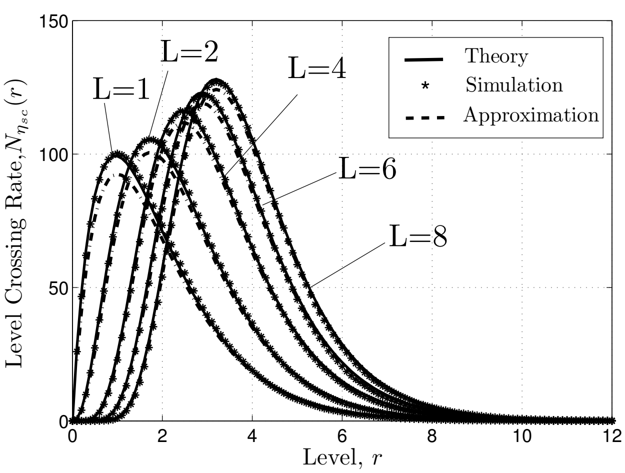

Fig. 5. The LCR of SC diversity over double Rayleigh fading channels for different values of the number of branches L.

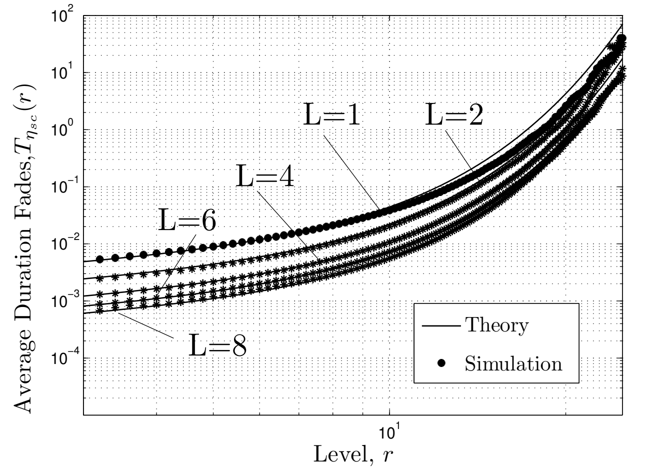

Fig. 6. The ADF of SC diversity over double Rice fading channels for different values of the number of branches L.

would expect. In Fig. 2, the exact and approximate theoretical PDFs of SC diversity over double Rice ($\rho_1 = \rho_2 = 1$), Rayleigh×Rice ($\rho_1 = 0$, $\rho_2 = 1$), and double Rayleigh ($\rho_1 = \rho_2 = 0$) channels are shown together with corresponding simulation data for $L = 4$. A perfect agreement can be observed between the exact analytical curves and the simulation results. The close correspondence between the approximation and the exact theoretical results reveals the validity of (22) as a simplified approximation for (16). Fig. 3 depicts the results of the PDF of fading processes in the non-diversity case for double Rice ($\rho_1 = \rho_2 = 1$), Rayleigh×Rice ($\rho_1 = 0$, $\rho_2 = 1$), and double Rayleigh ($\rho_1 = \rho_2 = 0$) models. Apart from the double Rayleigh channel, the approximate solution is seen to be in a reasonable agreement with the exact analytical and simulation results. Fig. 4 illustrates the impact of the number of diversity branches L on the behavior of the LCR of SC diversity over double Rice fading model. Again, a good correspondence between the analytical and simulated curves can be observed in this figure. The accuracy of the approximate solution for the LCR of double Rayleigh channels is illustrated in Fig. 5. From this figure, it can be observed that as the number of branches L increases, the approximate result tends to agree perfectly with the exact analytical solution. Indeed, from $L = 4$, we obtain a reasonable correspondence between the exact and approximate analytical results. Finally, the behavior of the ADF of SC diversity over double Rice fading channels, for various values of the number of branches L, is illustrated in Fig. 6.

VI. Conclusion

In this paper, the first and second order statistics of SC cooperative diversity over double Rice fading channels have been investigated. Analytical integral expressions for the CDF and PDF of the fading process, available at the SC output, have first been derived. Corresponding approximate solution has then been obtained for the PDF by using the Laplace's method of integration. Theoretical expressions for the LCR and ADF have subsequently been determined in the form of semi-infinite integral equations. For the case of double Rayleigh channels, the Laplace's approach of integration has as well been applied and a closed-form approximate solution for the LCR has been provided. Corresponding statistical quantities have been deduced for Rayleigh$\times$Rice and double Rayleigh models as special cases of the double Rice fading channel. The correctness of the exact analytical quantities and the accuracy of the approximations have been checked using computer simulations.

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