Layered Coding with Non-Coherent and Coherent Layers Over Fading Channels

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***Abstract*— In this paper, we consider a novel layered coding approach with two layers. One of the two layers, denoted by the base-layer, can be received by any receiver even if it does not have reliable channel estimates. The other, refining-layer can only be received by any receiver that has channel state information. We propose signal constellations that allow the transmission of coherent and non-coherent information for the single-antenna transmitters. We derive upper bounds for the pairwise error probability for the coherent and non-coherent receivers and prove that our proposed signal constellations can achieve a diversity of order M for the $1 \times M$ system, for both the coherent and non-coherent receivers.**

I. Introduction

Single layer transmission has been the salient multimedia transmission technique in the last few decades. In single layer transmission, the receiver will either be able to decode the transmitted codeword and get all the information, or it will be unable to decode the codeword and lose all the information, an "all or nothing" situation. As opposed to the "On-Off" nature of "single-layer" transmission schemes, adding a new design element of prioritization of source information bits that can be supported by ordered error-protection levels at the physical layer (i.e., channel coding) is proven to produce significant performance gains in these cases. This approach is also known as multilayer transmission. Multilayer transmission makes it possible to partially-decode the transmitted message when the channel condition does not allow full decoding of the entire message. As a consequence, using layered multimedia transmission allows the user(s) to receive the multimedia streaming most of the time with different rates/qualities depending on their own channel states. In multilayer transmission, the multimedia source is encoded into two or more layers with each layer successively refining the description of the previous layers [1], [2]. Recently, many papers have considered the use of layered coding for multimedia data transmission in different contexts [3]–[6].

In this paper, we present a different viewpoint of layered coding that has not been addressed before to the best of authors knowledge. In any wireless communication system, the channel is estimated through the transmission of pilot signals with some frequency. Depending on the frequency of pilot transmission and the channel coherence time, some receivers might have reliable channel estimates and other receivers might not have that reliable channel estimates. This is one reason why each mobile wireless standard supports some maximum velocities for the mobile users, limited by the frequency of pilot transmission. Mobile users moving at higher speeds might not have reliable channel estimates and this means that they will not be able to receive any information, the "all or nothing" problem. In this paper, we propose a layered transmission scheme with two layers, one layer, base-layer (non-coherent-layer), that can decoded by any receiver even if it does not have reliable channel estimates, and the other, refining-layer (coherent-layer) that can be only decoded at receivers with reliable channel estimates.

In this paper, we propose signal constellations that allow the transmission of coherent and non-coherent data simultaneously. The base, non-coherent, layer bits will be encoded into the direction of the transmitted data vector and it will not be affected by the channel [7]. The other refining, coherent-layer will be transmitted using any complex constellation within the direction that requires channel knowledge at the receiver to decode it. The proposed layered coding scheme could be useful in broadcasting systems like mobile TVs, where basic bits could be transmitted on the first layer and the extra bits that improve quality could be transmitted on the second layer. This layered coding scheme could be also used in mobile technologies to improve the system mobility as at high speed channel state information is lost due to fast channel variations; however, the receiver will be able to continue to decode the bits transmitted on the base-layer.

Notations: Lower and upper boldface letters are used to denote vectors and matrices, respectively. $\mathbf{A}^T$ and $\mathbf{A}^{\mathcal{H}}$ denote the transpose and the Hermitian (conjugate) transpose of the matrix $\mathbf{A}$, respectively. The real and the imaginary parts of a complex variable c are denoted by $\Re(c)$ and $\Im(c)$, respectively.

II. System Model

We consider a $1 \times M$ communication system that operates over Rayleigh flat-fading channel as shown in Fig. 1. We assume that the channel is constant over each two consecutive time slots duration, which means that $T_c \geq 2 \times T_s$, where

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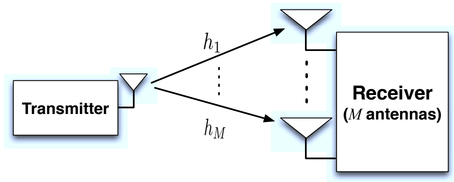

Fig. 1: The $1 \times M$ system model.

T_c is the channel coherence time and T_s is the time slot duration. Two consecutive time slots contain two types of information, the base-layer information, which is carried on the direction[1] and can be decoded by both coherent and non-coherent detectors and the second, refining-layer information which is carried on the signal constellation in each direction and this information can only be decoded by the coherent detectors. The transmitted data vector is given by $\mathbf{x} = a \cdot [d_1\ d_2]^T$, where a is carved from any complex constellation and $[d_1\ d_2]^T$ is the direction. The received signals, in the two consecutive time slots, at the i-th receive antenna are given

$$\begin{aligned} \mathbf{r}_i &= [r_{2i-1}\ r_{2i}]^T = h_i \mathbf{x} + [n_{2i-1}\ n_{2i}]^T \\ &= h_i a [d_1\ d_2]^T + [n_{2i-1}\ n_{2i}]^T,\ i = 1, 2, \cdots, M, \end{aligned} \tag{1}$$

where r_{2i-1} and r_{2i} are the received symbols in the first and the second time slots at the i-th receive antenna, respectively. The channel gains, h_i's, are modeled as independent, Rayleigh flat-fading channel, i.e., h_i is a circularly-symmetric complex Gaussian random variable with zero mean and unit variance, $h_i \sim \mathcal{CN}(0,1)$. The noise vector $\mathbf{n} = [n_{2i-1}\ \ n_{2i}]^T$ is a circularly-symmetric complex Gaussian random vector with independent elements and each element has a zero mean and a variance of N_0, $\mathbf{n} \sim \mathcal{CN}(\mathbf{0}, N_0 \mathbf{I}_2)$, where $\mathbf{I}_2$ is the 2×2 identity matrix, and $\mathbf{x}$ is the transmitted data vector. The channel gains and the noise terms at the different receive antennas are assumed to be independent.

III. The Optimum Non-Coherent Receiver

In this section, we derive the expressions for the optimum non-coherent detector. We also derive an upper bound on the pairwise error probability for the optimum non-coherent detector. In our pairwise analysis, we assume that there are only two directions; the first direction is

$$\mathbf{d}_A = [1\ 0]^T, \tag{2}$$

and the second direction is the nearest direction to the first one, which is given by

$$\mathbf{d}_B = \left[\cos\left(\frac{\pi}{D}\right)\ \sin\left(\frac{\pi}{D}\right)\right]^T, \tag{3}$$

in the general case of D directions.

[1]a direction is a line passing through origin, which is a subspace of $\mathbb{R}^2$.

A. The Optimum Non-Coherent SISO Receiver

In this section, we will assume that we only have the two directions as given in (2) and (3) and derive the expression for the optimum maximum likelihood (ML) receiver. Then, we will generalize the ML to the non-coherent receiver with D directions. In this section, we will focus on the case of one receive antenna.

The ML in the case of one receive antenna is given by

$$f(r_1, r_2\,|\mathbf{d}_A) \underset{\mathbf{d}_B}{\overset{\mathbf{d}_A}{\gtrless}} f(r_1, r_2|\mathbf{d}_B). \tag{4}$$

The probability distribution function for the received vector $\mathbf{r}$ is [8]

$$f(\mathbf{r}) = \frac{1}{\pi^2 \det(\mathbf{K_r})} \exp\left(-\mathbf{r}^{\mathcal{H}} \mathbf{K_r}^{-1} \mathbf{r}\right), \tag{5}$$

where $\mathbf{K_r} = E[\mathbf{r}\mathbf{r}^{\mathcal{H}}]$ is the covariance matrix of the random vector $\mathbf{r}$, since $E(\mathbf{r}) = \mathbf{0}$. Note that in this section we have assumed, without loss of generality and for clarity of presentation, that a is carved from a constant modulus constellation, such as PSK constellation.

Given that the direction $\mathbf{d}_A$ was transmitted, the received vector will be $\mathbf{r} = [r_1\ r_2]^T = [ha + n_1\ n_2]^T$ with

$$\mathbf{K_r} = \begin{pmatrix} |a|^2 + N_0 & 0 \\ 0 & N_0 \end{pmatrix}.$$

Therefore, we have

$$f(r_1, r_2\,|\mathbf{d}_A) = \frac{1}{\pi^2(|a|^2 + N_0)N_0} \exp\left(-\left(\frac{|r_1|^2}{|a|^2 + N_0} + \frac{|r_2|^2}{N_0}\right)\right). \tag{6}$$

Given that the direction $\mathbf{d}_B$ was transmitted, the received vector will be $\mathbf{r} = [r_1\ \ r_2]^T = \left[ha\cos(\frac{\pi}{D}) + n_1\ \ ha\sin(\frac{\pi}{D}) + n_2\right]^T$ with

$$\mathbf{K_r} = \begin{pmatrix} |a|^2\cos^2(\frac{\pi}{D}) + N_0 & |a|^2\cos(\frac{\pi}{D})\sin\frac{\pi}{D} \\ |a|^2\cos(\frac{\pi}{D})\sin\frac{\pi}{D} & |a|^2\sin^2(\frac{\pi}{D}) + N_0 \end{pmatrix}.$$

Therefore, we have

$$f(r_1, r_2|\mathbf{d}_B) = \frac{1}{\pi^2(|a|^2 + N_0)N_0} \exp\left(-\frac{\gamma}{N_0(|a|^2 + N_0)}\right), \tag{7}$$

where $\gamma = |r_1|^2(|a|^2\sin^2(\frac{\pi}{D}) + N_0) + |r_2|^2(|a|^2\cos^2(\frac{\pi}{D}) + N_0) - |a|^2\sin(\frac{2\pi}{D})(\Re(r_1)\Re(r_2) + \Im(r_1)\Im(r_2))$. Note that with our assumption of constant modulus constellation within the direction, then $|a|^2$ is deterministic.

Substituting from (6) and (7) in (4) yields

$$|r_1|^2 \underset{\mathbf{d}_B}{\overset{\mathbf{d}_A}{\gtrless}} |r_2|^2 + 2\cot\left(\frac{\pi}{D}\right)(\Re(r_1)\Re(r_2) + \Im(r_1)\Im(r_2)). \tag{8}$$

After some simplifications, the last inequality can be written in an inner product form as

$$|\mathbf{r}\cdot\mathbf{d}_A| \underset{\mathbf{d}_B}{\overset{\mathbf{d}_A}{\gtrless}} |\mathbf{r}\cdot\mathbf{d}_B|. \tag{9}$$

For the general number of directions case, the optimum ML detector can be written as

$$\hat{\mathbf{d}}_{ML} = \arg\max_j |\mathbf{r}\cdot\mathbf{d}_j|. \tag{10}$$

B. The Optimum Non-Coherent SIMO Receiver

In this section, we will follow the same steps as in the previous section. We will assume that we have only the two directions in (2) and (3) and derive the expression for the optimum ML receiver then we generalize it to the non-coherent receiver with D directions.

Given that the direction $\mathbf{d}_A$ is transmitted, the received vector will be $\mathbf{r} = [r_1 \ r_2 \ \cdots \ r_{2M-1} \ r_{2M}]^T = [h_1 a + n_1 \ n_2 \ \cdots \ h_M a + n_{2M-1} \ n_{2M}]^T$ with

$$\mathbf{K_r} = \begin{pmatrix} |a|^2+N_0 & 0 & 0 & \cdots & 0 \\ 0 & N_0 & 0 & \cdots & 0 \\ 0 & 0 & |a|^2+N_0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & N_0 \end{pmatrix}_{2M\times 2M},$$

and

$$\det(\mathbf{K_r}) = (|a|^2+N_0)^M N_0^M. \tag{11}$$

The probability distribution function for the received vector $\mathbf{r}$ is [8]

$$f(r_1, r_2, \cdots, r_{2M} \,|\mathbf{d}_A) = \frac{1}{\pi^{2M}(|a|^2+N_0)^M N_0^M} \exp(-\alpha), \tag{12}$$

where

$$\alpha = \frac{1}{|a|^2+N_0}\sum_{i=1}^{M}|r_{2i-1}|^2 + \frac{1}{N_0}\sum_{i=1}^{M}|r_{2i}|^2. \tag{13}$$

Given that the direction $\mathbf{d}_B$ is transmitted, the received vector will be $\mathbf{r} = [r_1 \ r_2 \ \cdots \ r_{2M-1} \ r_{2M}]^T = [h_1 a\cos\frac{\pi}{D} + n_1 \ h_1 a \sin\frac{\pi}{D} + n_2 \ \cdots \ h_M a\cos\frac{\pi}{D} + n_{2M-1} \ h_M a\sin\frac{\pi}{D} + n_{2M}]^T$ with

$$\mathbf{K_r} = \begin{pmatrix} \mathbf{A} & 0 & \cdots & 0 \\ 0 & \mathbf{A} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \mathbf{A} \end{pmatrix}_{2M\times 2M},$$

where

$$\mathbf{A} = \begin{pmatrix} |a|^2\cos^2(\frac{\pi}{D})+N_0 & |a|^2\sin(\frac{\pi}{D})\cos(\frac{\pi}{D}) \\ |a|^2\sin(\frac{\pi}{D})\cos(\frac{\pi}{D}) & |a|^2\sin^2(\frac{\pi}{D})+N_0 \end{pmatrix},$$

and

$$\det(\mathbf{K_r}) = (\det(\mathbf{A}))^M = (|a|^2+N_0)^M N_0^M \tag{14}$$

and

$$\mathbf{K_r}^{-1} = \begin{pmatrix} \mathbf{A}^{-1} & 0 & \cdots & 0 \\ 0 & \mathbf{A}^{-1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \mathbf{A}^{-1} \end{pmatrix}_{2M\times 2M}$$

where

$$\mathbf{A}^{-1} = \begin{pmatrix} \frac{|a|^2\sin^2(\frac{\pi}{D})+N_0}{N_0(|a|^2+N_0)} & -\frac{|a|^2\sin(\frac{\pi}{D})\cos(\frac{\pi}{D})}{N_0(|a|^2+N_0)} \\ -\frac{|a|^2\sin(\frac{\pi}{D})\cos(\frac{\pi}{D})}{N_0(|a|^2+N_0)} & \frac{|a|^2\cos^2(\frac{\pi}{D})+N_0}{N_0(|a|^2+N_0)} \end{pmatrix}.$$

The probability distribution function for the received vector $\mathbf{r}$ is [8]

$$f(r_1, r_2, \cdots, r_{2M}|\mathbf{d}_B) = \frac{1}{\pi^{2M}(|a|^2+N_0)^M N_0^M}\exp(-\beta), \tag{15}$$

where

$$\begin{aligned}\beta =& \frac{1}{N_0(|a|^2+N_0)}\sum_{i=1}^{M}\Bigg(|r_{2i-1}|^2\left(|a|^2\sin^2\left(\frac{\pi}{D}\right)+N_0\right) \\ &- \left(r_{2i-1}^* r_{2i} + r_{2i-1}r_{2i}^*\right)|a|^2\sin\left(\frac{\pi}{D}\right)\cos\left(\frac{\pi}{D}\right) \\ &+ |r_{2i}|^2\left(|a|^2\cos^2\left(\frac{\pi}{D}\right)+N_0\right)\Bigg).\end{aligned} \tag{16}$$

After some manipulations, the ML detector can be formulated as

$$\sum_{i=1}^{M}|r_{2i-1}|^2 \underset{\mathbf{d_B}}{\overset{\mathbf{d_A}}{\gtrless}} \sum_{i=1}^{M}\left|r_{2i-1}\cos\left(\frac{\pi}{D}\right)+r_{2i}\sin\left(\frac{\pi}{D}\right)\right|^2, \tag{17}$$

which can be put in an inner product form as

$$\sum_{i=1}^{M}|\mathbf{r}_i \cdot \mathbf{d}_A|^2 \underset{\mathbf{d}_B}{\overset{\mathbf{d}_A}{\gtrless}} \sum_{i=1}^{M}|\mathbf{r}_i \cdot \mathbf{d}_B|^2, \tag{18}$$

where $\mathbf{r}_i = [r_{2i-1} \ r_{2i}]^T$, $i = 1, 2, \cdots, M$, is the received vector at the i-th receive antenna and r_{2i-1} and r_{2i} are the received symbols in the first and the second time slots, respectively.

For a general number of directions, the optimum ML detector is given by

$$\hat{\mathbf{d}}_{ML} = \arg\max_j \sum_{i=1}^{M}|\mathbf{r}_i \cdot \mathbf{d_j}|^2. \tag{19}$$

C. Pairwise Error Probability (PEP) for the Non-Coherent SISO Receiver

In this section, we derive an upper bound expression for the pairwise error probability (PEP) for the SISO receiver. The PEP can be expressed as

$$\begin{aligned} &PEP(\mathbf{d}_A \to \mathbf{d}_B) \\ &\quad= \Pr\left[\hat{\mathbf{d}}_{ML} = \mathbf{d}_B \,\middle|\, \mathbf{d}_A\right] \\ &\quad= \Pr\left[|\mathbf{r}\cdot\mathbf{d}_A| < |\mathbf{r}\cdot\mathbf{d}_B| \,\middle|\, \mathbf{d}_A\right] \\ &\quad= \Pr\left[|r_1| < \left|r_1\cos\left(\frac{\pi}{D}\right)+r_2\sin\left(\frac{\pi}{D}\right)\right| \,\middle|\, \mathbf{d}_A\right]. \end{aligned} \tag{20}$$

Let $w = r_1\cos(\frac{\pi}{D})+r_2\sin(\frac{\pi}{D})$, then the PEP can be expressed as

$$\begin{aligned} PEP(\mathbf{d}_A \to \mathbf{d}_B) &= \Pr\left[|r_1| < |w| \,\middle|\, \mathbf{d}_A\right] \\ &= \Pr\left[|r_1|^2 < |w|^2 \,\middle|\, \mathbf{d}_A\right]. \end{aligned} \tag{21}$$

To get the PEP expression, we need to get the expression for the joint distribution of r_1 and w conditioned on $\mathbf{d}_A$. The conditional distribution of w conditioned on $\mathbf{d}_A$ is given by $w \,\big|\mathbf{d}_A \sim \mathcal{CN}(0, |a|^2\cos^2(\frac{\pi}{D}) + N_0)$ and $E\{r_1^* w \,\big|\mathbf{d}_A\} = (|a|^2 + N_0)\cos(\frac{\pi}{D})$. Define $r_{1R} = \Re(r_1)$, $r_{1I} = \Im(r_1)$, $w_R = \Re(w)$ and $w_I = \Im(w)$. Let ρ_{RR} denote the correlation coefficient between r_{1R} and w_R conditioned on $\mathbf{d}_A$, and is given by

$$\rho_{RR} = \frac{E\left\{r_{1R} w_R \,\big|\mathbf{d}_A\right\}}{\sigma_{r_{1R}}\sigma_{w_R}}. \tag{22}$$

Note that r_{1R} and w_R conditioned on $\mathbf{d}_A$ are both Gaussian random variables and are distributed as follows.

$$r_{1R} \,\big|\mathbf{d}_A \sim \mathcal{G}\left(0, \sigma^2_{r_{1R}} = \frac{|a|^2 + N_0}{2}\right). \tag{23}$$

$$w_R \,\big|\mathbf{d}_A \sim \mathcal{G}\left(0, \sigma^2_{w_R} = \frac{|a|^2\cos^2(\frac{\pi}{D}) + N_0}{2}\right). \tag{24}$$

After some straightforward manipulations, we can easily get

$$E\left\{r_{1R} w_R \,\big|\mathbf{d}_A\right\} = \frac{|a|^2 + N_0}{2}\cos\left(\frac{\pi}{D}\right). \tag{25}$$

Substituting from (23), (24) and (25) in (22), we get

$$\rho_{RR} = \cos\left(\frac{\pi}{D}\right)\sqrt{\frac{|a|^2 + N_0}{|a|^2\cos^2(\frac{\pi}{D}) + N_0}} \tag{26}$$

Let

$$T_1 = |r_1|^2 \,\big|\mathbf{d}_A \sim Exp\left(\lambda_1 = \frac{1}{|a|^2 + N_0}\right); \tag{27}$$

$$T_2 = |w|^2 \,\big|\mathbf{d}_A \sim Exp\left(\lambda_2 = \frac{1}{|a|^2\cos^2(\frac{\pi}{D}) + N_0}\right). \tag{28}$$

The PEP can now be expressed as

$$PEP(\mathbf{d}_A \to \mathbf{d}_B) = P\left(T_1 < T_2 \,\big|\mathbf{d}_A\right), \tag{29}$$

where T_1 and T_2 are two jointly distributed exponential random variables as defined in (27) and (28), respectively. The joint pdf of T_1 and T_2 conditioned on $\mathbf{d}_A$ can be expressed as [9]

$$f_{T_1,T_2 \,|\mathbf{d}_A}\left(t_1, t_2 \,\big|\mathbf{d}_A\right) = \frac{\exp\left(-\frac{t_1/\sigma^2_{r_{1R}} + t_2/\sigma^2_{w_R}}{2(1-\rho^2_{RR})}\right)}{4\sigma^2_{r_{1R}}\sigma^2_{w_R}(1-\rho^2_{RR})} I_0\left(\frac{|\rho_{RR}|\sqrt{t_1 t_2}}{(1-\rho^2_{RR})\sigma_{r_{1R}}\sigma_{w_R}}\right), \tag{30}$$

where $I_0(\cdot)$ is is the modified Bessel function of the first kind of zero order. Then, the PEP can be expressed as

$$PEP(\mathbf{d}_A \to \mathbf{d}_B) = \int_0^\infty \int_{t_1}^\infty \frac{\exp\left(-\frac{t_1/\sigma^2_{r_{1R}} + t_2/\sigma^2_{w_R}}{2(1-\rho^2_{RR})}\right)}{4\sigma^2_{r_{1R}}\sigma^2_{w_R}(1-\rho^2_{RR})} \times I_0\left(\frac{|\rho_{RR}|\sqrt{t_1 t_2}}{(1-\rho^2_{RR})\sigma_{r_{1R}}\sigma_{w_R}}\right) dt_2 dt_1. \tag{31}$$

It is very difficult to get a closed form expression for the PEP based on the above expression. We resort to getting closed form PEP upper bounds.

1) The First PEP Upper Bound: knowing that $\pi/D \leq \frac{\pi}{2}$, then we have

$$\left|r_1\cos\left(\frac{\pi}{D}\right) + r_2\sin\left(\frac{\pi}{D}\right)\right| \leq \cos\left(\frac{\pi}{D}\right)|r_1| + \sin\left(\frac{\pi}{D}\right)|r_2|. \tag{32}$$

Then, we have

$$\Pr\left[|r_1| < \left|r_1\cos\left(\frac{\pi}{D}\right) + r_2\sin\left(\frac{\pi}{D}\right)\right| \,\Big|\mathbf{d}_A\right] \leq \Pr\left[|r_1| < \cos\left(\frac{\pi}{D}\right)|r_1| + \sin\left(\frac{\pi}{D}\right)|r_2| \,\Big|\mathbf{d}_A\right].$$

Therefore, the PEP can be upper bounded as

$$PEP(\mathbf{d}_A \to \mathbf{d}_B) \leq \Pr\left[|r_1| < \cos\left(\frac{\pi}{D}\right)|r_1| + \sin\left(\frac{\pi}{D}\right)|r_2| \,\Big|\mathbf{d}_A\right]. \tag{33}$$

Then, we have

$$PEP_{UB_1}(\mathbf{d}_A \to \mathbf{d}_B) = \Pr\left[|r_1| < \cos\left(\frac{\pi}{D}\right)|r_1| + \sin\left(\frac{\pi}{D}\right)|r_2| \,\Big|\mathbf{d}_A\right], \tag{34}$$

where $PEP_{UB_1}(\mathbf{d}_A \to \mathbf{d}_B)$ is the first PEP upper bound and can be found as

$$\begin{aligned} PEP_{UB_1}(\mathbf{d}_A \to \mathbf{d}_B) &= \Pr\left[|r_1| < \left(\frac{\sin(\frac{\pi}{D})}{1-\cos(\frac{\pi}{D})}\right)|r_2| \,\Big|\mathbf{d}_A\right] \\ &= \Pr\left[|r_1|^2 < \left(\frac{\sin(\frac{\pi}{D})}{1-\cos(\frac{\pi}{D})}\right)^2|r_2|^2 \,\Big|\mathbf{d}_A\right] \\ &= \Pr\left[u_1 < \left(\frac{\sin(\frac{\pi}{D})}{1-\cos(\frac{\pi}{D})}\right)^2 u_2 \,\Big|\mathbf{d}_A\right], \end{aligned}$$

where

$$u_1 = |r_1|^2 \,\Big|\mathbf{d}_A \sim Exp\left(\lambda_1 = \frac{1}{|a|^2 + N_0}\right) \tag{35}$$

$$u_2 = |r_2|^2 \,\Big|\mathbf{d}_A \sim Exp\left(\lambda_2 = \frac{1}{N_0}\right). \tag{36}$$

Note that u_1 and u_2 conditioned on $\mathbf{d}_A$ are independent exponential random variables. The upper bound can be obtained as

$$PEP_{UB_1}(\mathbf{d}_A \to \mathbf{d}_B) = \Pr\left[u_1 < \left(\frac{\sin(\frac{\pi}{D})}{1-\cos(\frac{\pi}{D})}\right)^2 u_2 \,\Big|\mathbf{d}_A\right].$$

Hence, the first PEP upper bound is given by

$$\begin{aligned} PEP_{UB_1}(\mathbf{d}_A \to \mathbf{d}_B) &= \int_0^\infty \int_{u_1/\left(\frac{\sin(\frac{\pi}{D})}{1-\cos(\frac{\pi}{D})}\right)^2}^\infty \lambda_1\lambda_2 e^{-\lambda_1 u_1} e^{-\lambda_2 u_2} du_2 du_1 \\ &= \left[1 + \frac{|a|^2 + N_0}{N_0}\left(\frac{1-\cos(\frac{\pi}{D})}{\sin(\frac{\pi}{D})}\right)^2\right]^{-1} \end{aligned} \tag{37}$$

2) The Second PEP Upper Bound: The second PEP upper bound can be obtained as [10]

$$PEP_{UB_2}(\mathbf{d}_A \to \mathbf{d}_B) = \frac{1}{2}\left[1 + \frac{SNR^2(1-s^2)}{4(1+SNR)}\right]^{-1}, \quad (38)$$

where $SNR = \frac{|a|^2}{2N_0}$ and $s = |a|^2\mathbf{d}_A^{\mathcal{H}}\mathbf{d}_B$, which is given by $s = |a|^2\cos(\frac{\pi}{D})$. The second PEP upper bound can be simplified to

$$PEP_{UB_2}(\mathbf{d}_A \to \mathbf{d}_B) = \frac{1}{2}\left[1 + \frac{\left(\frac{|a|^2}{2N_0}\right)^2(1-|a|^4\cos^2(\frac{\pi}{D}))}{4(1+\frac{|a|^2}{2N_0})}\right]^{-1} \quad (39)$$

D. Pairwise Error Probability (PEP) for the Non-Coherent SIMO Receiver

The PEP for the non-coherent SIMO receiver can be upper bounded as [10]

$$PEP(\mathbf{d}_A \to \mathbf{d}_B) \leq \frac{1}{2}\left[1 + \frac{SNR^2(1-s^2)}{4(1+SNR)}\right]^{-M}, \quad (40)$$

where $SNR = \frac{|a|^2}{2N_0}$ and $s = |a|^2\mathbf{d}_A^{\mathcal{H}}\mathbf{d}_B$, which is given by $s = |a|^2\cos(\frac{\pi}{D})$. From the last PEP upper bound expression in (40) it can be easily proved that the proposed signal constellation achieves a diversity of order M in the $1\times M$ system, i.e., full diversity is achieved.

IV. The Optimum Coherent Receiver

In this section, we assume that the channel state information is available at the receiver so the receiver will be able to decode all the information bits, both coherent and non-coherent. Assuming that the transmission is over D directions and using Q-ary PSK constellation, the optimum ML decoder for the SISO system is given by the minimum distance decoder as follows.

$$\hat{\mathbf{x}}_{ML} = \min_k \left[\|r_1 - x_{k1}\|^2 + \|r_2 - x_{k2}\|^2\right]. \quad (41)$$

where $k = 1, 2, \cdots, Q\times D$, x_{k1} and x_{k2} are the transmitted symbols of the k-th signal constellation point in the first and the second time slots, respectively.

For any two possible transmitted signal constellation points, $\mathbf{x_m}$ and $\mathbf{x_n}$, it is straightforward to show that the PEP for the coherent receiver is upper bounded by

$$PEP(\mathbf{x}_m \to \mathbf{x}_n) \leq \left(\frac{1}{1+\frac{1}{4N_0}\|\mathbf{x}_m - \mathbf{x}_n\|^2}\right). \quad (42)$$

For the general case of $1\times M$ system, the PEP can be upper bounded as

$$PEP(\mathbf{x}_m \to \mathbf{x}_n) \leq \left(\frac{1}{1+\frac{1}{4N_0}\|\mathbf{x}_m - \mathbf{x}_n\|^2}\right)^M. \quad (43)$$

Clearly, the diversity order of our proposed signal constellation will be M (since the distance $\|\mathbf{x}_m - \mathbf{x}_n\|^2$ for any $m \neq n$ scales linearly with the transmitted power).

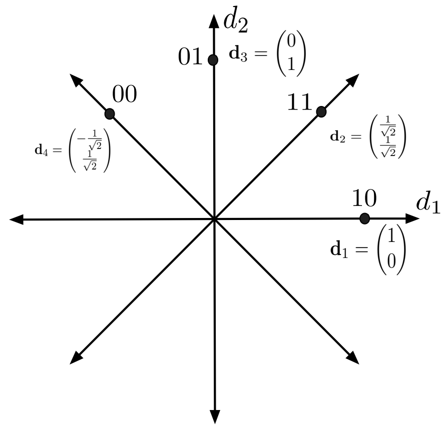

Fig. 2: Directions mapping with $D = 4$.

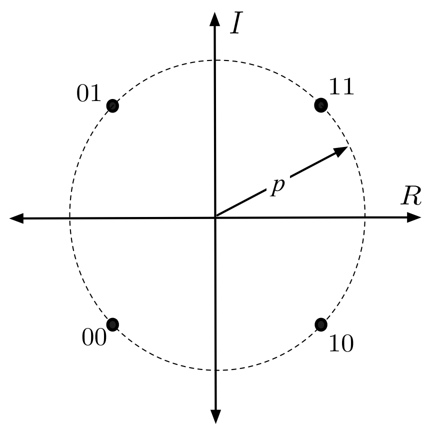

Fig. 3: QPSK signal constellation in each direction.

V. An Example: 4-Directions with QPSK Constellation

In this section, we provide an illustrating example of the proposed layered coding scheme. In this example, we have 4 directions so that each direction can be represented by 2 bits as shown in Fig. 2. These 2 "direction" bits can be decoded by both, the non-coherent and coherent receivers, so these 2 bits are supposed to have the highest priority and they represent the base-layer information. On each direction we use QPSK constellation, as shown in Fig. 3, which adds two more bits that can only be decoded by the receiver that has reliable channel state information. These 2 bits are supposed to have lower priority (refining-layer) as they can only be decoded at the coherent receivers. Therefore, we have 16 different codewords and each codeword represents 4 information bits; the first 2 bits have higher priority and they select the direction of the transmitted data and the other 2 bits, of lower priority, are transmitted by the QPSK constellation point carried on the direction. The 16 different codewords and the corresponding transmitted symbols in the two time slots are tabulated in Table I.

TABLE I: Different codewords and the corresponding transmitted vector $\mathbf{x}$

Codeword				Transmitted vector
Non-coherent bits		Coherent bits		$\mathbf{x}^T = a[d_1 \ \ d_2]$
0	0	0	0	$\left(-\frac{p}{\sqrt{2}} - i\frac{p}{\sqrt{2}}\right)\left[-\frac{1}{\sqrt{2}} \ \ \frac{1}{\sqrt{2}}\right]$
0	0	0	1	$\left(-\frac{p}{\sqrt{2}} + i\frac{p}{\sqrt{2}}\right)\left[-\frac{1}{\sqrt{2}} \ \ \frac{1}{\sqrt{2}}\right]$
0	0	1	0	$\left(\frac{p}{\sqrt{2}} - i\frac{p}{\sqrt{2}}\right)\left[-\frac{1}{\sqrt{2}} \ \ \frac{1}{\sqrt{2}}\right]$
0	0	1	1	$\left(\frac{p}{\sqrt{2}} + i\frac{p}{\sqrt{2}}\right)\left[-\frac{1}{\sqrt{2}} \ \ \frac{1}{\sqrt{2}}\right]$
0	1	0	0	$\left(-\frac{p}{\sqrt{2}} - i\frac{p}{\sqrt{2}}\right)[0 \ \ 1]$
0	1	0	1	$\left(-\frac{p}{\sqrt{2}} + i\frac{p}{\sqrt{2}}\right)[0 \ \ 1]$
0	1	1	0	$\left(\frac{p}{\sqrt{2}} - i\frac{p}{\sqrt{2}}\right)[0 \ \ 1]$
0	1	1	1	$\left(\frac{p}{\sqrt{2}} + i\frac{p}{\sqrt{2}}\right)[0 \ \ 1]$
1	0	0	0	$\left(-\frac{p}{\sqrt{2}} - i\frac{p}{\sqrt{2}}\right)[1 \ \ 0]$
1	0	0	1	$\left(-\frac{p}{\sqrt{2}} + i\frac{p}{\sqrt{2}}\right)[1 \ \ 0]$
1	0	1	0	$\left(\frac{p}{\sqrt{2}} - i\frac{p}{\sqrt{2}}\right)[1 \ \ 0]$
1	0	1	1	$\left(\frac{p}{\sqrt{2}} + i\frac{p}{\sqrt{2}}\right)[1 \ \ 0]$
1	1	0	0	$\left(-\frac{p}{\sqrt{2}} - i\frac{p}{\sqrt{2}}\right)\left[\frac{1}{\sqrt{2}} \ \ \frac{1}{\sqrt{2}}\right]$
1	1	0	1	$\left(-\frac{p}{\sqrt{2}} + i\frac{p}{\sqrt{2}}\right)\left[\frac{1}{\sqrt{2}} \ \ \frac{1}{\sqrt{2}}\right]$
1	1	1	0	$\left(\frac{p}{\sqrt{2}} - i\frac{p}{\sqrt{2}}\right)\left[\frac{1}{\sqrt{2}} \ \ \frac{1}{\sqrt{2}}\right]$
1	1	1	1	$\left(\frac{p}{\sqrt{2}} + i\frac{p}{\sqrt{2}}\right)\left[\frac{1}{\sqrt{2}} \ \ \frac{1}{\sqrt{2}}\right]$

VI. Simulation Results

In all simulations, the channels are modeled as Rayleigh flat-fading channels with unit variance each. Fig. 4 shows the performance curves for the non-coherent receiver in the cases of two, four and eight directions for the 1×1, 1×2, 1×4 and 1×8 systems.

Fig. 5 shows the performance curves for the 1×1 system using BPSK in the cases of transmission along two, four and eight directions. It also shows the corresponding pairwise error probability and the first upper bound from (37).

Performance curves for the coherent and non-coherent receivers in case of transmitting on 2 directions and using QPSK signal constellation, for coherent data transmission within the direction, for the 1×1 system are shown in Fig. 6. In this figure, we can see that the non-coherent bits have the same performance in both the coherent and non-coherent receivers which means that the loss of channel state information does not affect the performance of the first layer at any receiver.

Performance curves for the example presented in section V are shown in Fig. 7 and the same observation can be noted that the non-coherent bits BER performance is the same at the coherent and non-coherent receivers. So the base-layer can be received at any receiver with the same quality, under the same receiver SNR, and this performance is independent of whether the receiver has reliable channel estimates or not.

A comparison between the four directions and two directions cases is shown in Fig. 8, where in each direction a QPSK constellation is transmitted. We also show the performance of

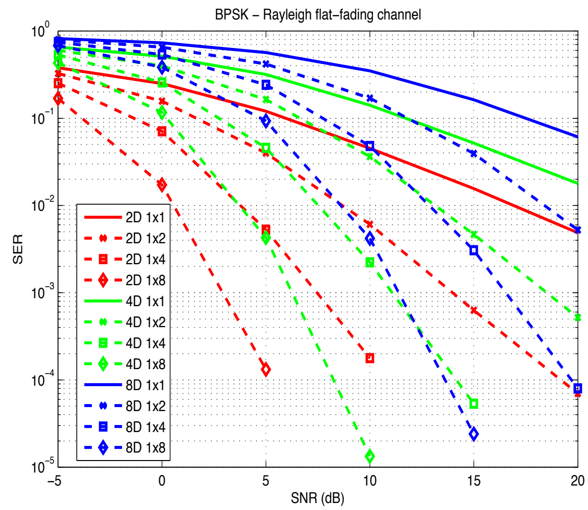

Fig. 4: Non-coherent receiver performance curves for $D = 2$, 4 and 8, and $M = 1$, 2, 4 and 8.

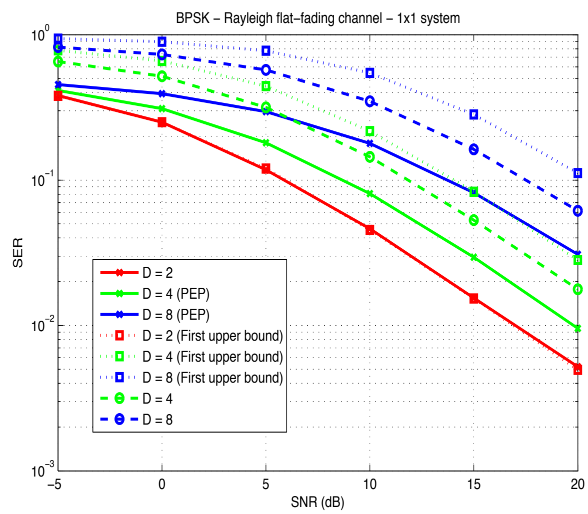

Fig. 5: Pairwise error probability and the first upper bound to the pairwise error probability curves for $D = 2$, 4 and 8.

the conventional QPSK modulation. Note that the coherent receiver will decode all the data while the non-coherent receiver will decode only the non-coherent (direction) bits.

VII. Discussion and Conclusion

In this paper, we have considered layered coding from a new viewpoint that has not been addressed before. Layered coding with a non-coherent, base-layer and a coherent, refining-layer has been considered. Receivers with unreliable channel estimates can decode the non-coherent layer and the receivers with reliable channel estimates can decode all the information and experience better service quality.

We have proposed signal constellations that will allow the transmission of the two layers for the $1 \times M$ communication systems. The non-coherent bits are sent in the direction of

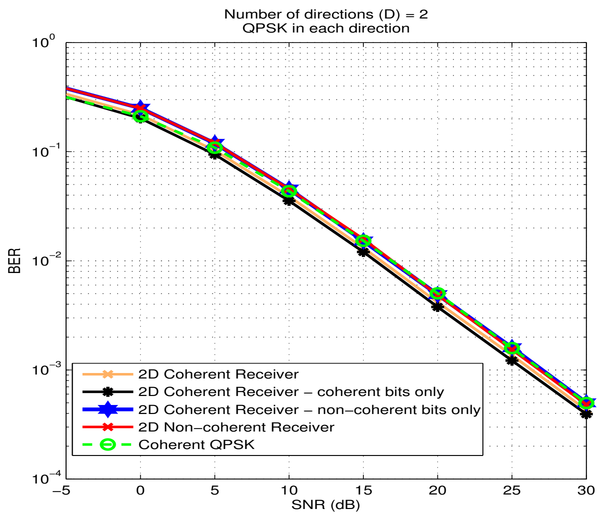

Fig. 6: Performance Curves for the coherent and non-coherent receivers in case of two directions $D = 2$.

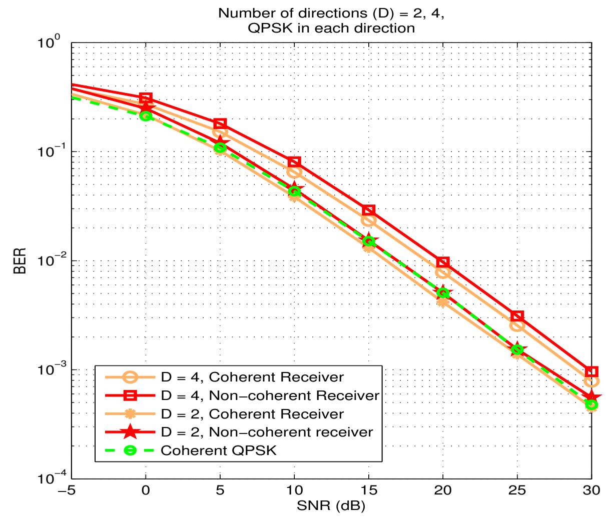

Fig. 8: Comparison between the cases of two and four directions of the non-coherent and coherent receivers.

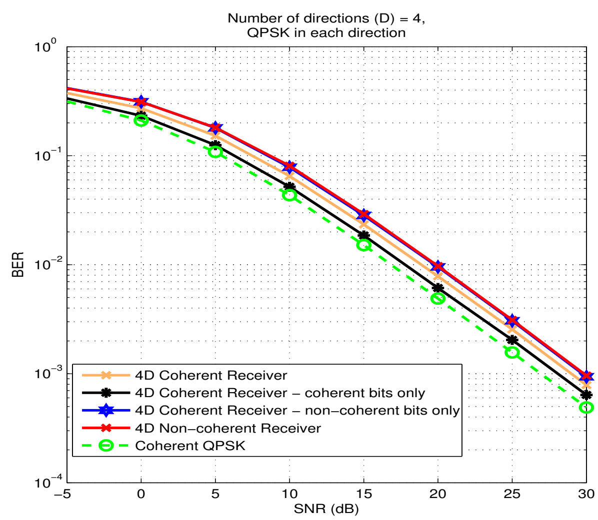

Fig. 7: Performance Curves for the coherent and non-coherent receivers in case of four directions $D = 4$.

the transmitted data vector, which is only affected by the channel noise not the channel gains. The coherent bits are transmitted by sending a constellation point, carved from any QAM constellation, in the direction selected by the non-coherent bits.

We consider this paper an initial step towards investigating this new viewpoint of layered coding. A future work could be to find signal constellations for the general $T \times M$ systems, with T transmit antennas and M receive antennas. The design of signal constellations based on Grassmann manifold can be useful in this case (this approach has been extensively used for the design of non-coherent signal constellations). Another future direction will be to examine whether there will be a tradeoff between the amounts of information sent in the non-coherent and coherent layers.

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