Efficient Modeling of the Routing and Spectrum Allocation Problem for Flexgrid Optical Networks

Brigitte Jaumard and Quang Anh Nguyen
Department of Computer Science and Software Engineering
Concordia University, Montreal (Qc) Canada

***Abstract*—While the problem of Routing and Spectrum Allocation (RSA) has been widely studied, very few studies attempt to solve realistic sized instances. Indeed, the state of the art is always below the standard transport capacity of a fiber link with 384 frequency slots, regardless of what the authors consider, heuristics or exact methods with a few exceptions. In this paper, we are interested in reducing the gap between realistic data sets and testbed instances that are often considered, using exact methods. Even if exact methods may fail to solve in reasonable time very large instances, they can, however, output solutions with a very good and proven accuracy. The novelty of this paper is to exploit the observations that optimal solutions contain a very large number of lightpaths associated with shortest paths or k-shortest paths with a small k. We propose an original efficient large-scale optimization model and decomposition algorithm to solve the RSA problem for flexgrid optical networks. It allows the exact or near optimal solution of much larger instances than in the literature.**

I. Introduction

To meet the demand of the networks of the future, optical transmission and networking technologies are evolving towards goals that allow greater efficiency, flexibility and scalability. Recently, elastic optical networks have been highlighted as the promising technology for future high speed optical networks [1]. An elastic optical network allocates the spectrum to lightpaths based on the bandwidth demands of its clients. The spectrum is divided into narrow slots (e.g., 12.5Ghz or 6.25Ghz) and each optical connection is allocated a given number of slots. Consequently, the network usage is greatly improved compared to DWDM (Dense Wavelength Division Multiplexing) optical networks with fixed bandwidth channels. In elastic optical networks, transmission parameters such as—optical data rate, which is fixed in former DWDM networks, is now tunable. Given that future demands indicate that high speed optical connections are needed to optimize data transport, elastic optical networks are now widely accepted as the next generation high-speed networks.

Many research efforts have been put into designing solution techniques capable of efficiently solving realistic data sets, both heuristic and exact methods. A wide range of heuristics have been proposed. Although heuristics can generally be designed to provide a solution with reasonable computational times, or if necessary in real time, they generally share the disadvantage of having little or no information on the accuracy of their solutions, or even on their quality. Most exact methods rely on compact Integer Linear Programming solutions, which while with a polynomial number of variables and constraints, are not able to scale beyond 10 nodes with a limited number of connection requests. Few proposals have been made for large-scale optimisation models, to be solved with a decomposition algorithm. Yet, very few authors have been able to solve exactly medium size instances.

Our proposal is a new large-scale optimization model, which relies on ℓ-configurations, made of lightpaths such that their first link is ℓ. The result is a model that can be efficiently solved in much shorter times than previous large-scale optimization models. Datasets with fiber optics having a standard transport capacity of 384 frequency slots can be solved exactly or nearly exactly (accuracy less than $\varepsilon < 10^{-2}$) in minutes for networks with up to 24 nodes..

The paper is organized as follows. We provide a formal statement of the Routing and Spectrum Allocation problem, together with a definition of the notations, in Section III. The original new nested decomposition model is proposed in Section IV. Solution scheme is developed in Section V. Numerical experiments are described in Section VI. Conclusions are drawn in the last section.

II. Literature Review

The literature is rich with solution techniques that range from heuristics to exact algorithms. Example of heuristics are the studies of, e.g., Goścień, Walkowiak and Klinkowski [2], Alaskar *et al.* [3], Abkenar and Rahbar [4]. For instance, Alaskar *et al.* [3] considered heuristics of the type priority allocation algorithms with the objective of minimizing the total spectrum amount needed to serve the demand. They were able to solve data instances of up 182 requests in a network of 14 nodes, 20 bidirectional links and 100 frequency slots.

One of the early ILP formulations was proposed in Christodoulopoulos *et al.* [5] and was only capable of solving very small data sets. Some other compact Integer Linear Programs (ILPs) were next proposed, but all share the lack of scalability.

The next generation of ILP models were large-scale optimization models, which require the use of a decomposition algorithm to solve them. In Ruiz *et al.* [6] and [7], decomposition models were proposed using column generation algorithms for their solution. The authors considered two objectives: minimizing the number of blocked demands (primary objective) and the amount of unserved bit-rate (secondary objective). They were able to solve data instances with up to 64 requests in a network of 21 nodes, 37 links and only

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96 frequency slots. In Klinkowski *et al.* [8], a clique cut generation procedure was developed and combined with a column generation procedure in order to improve the quality of the generated columns. The goal was to minimize the total amount of unserved bit rate, and data sets of 160 requests in the Spain network (21 nodes and 35 links) with again only 96 frequency slots were solved. A MILP formulation was proposed in Klinkowski *et al.* [9] which was solved using a branch-and-price algorithm that was enhanced with a simulated annealing-based heuristic. The objective was to minimize the number of used frequency slots, and they were able to solve data instances up to 60 requests in a network of 12 nodes and 20 links. Then, in Klinkowski *et al.* [10], they enhanced the algorithm with relaxations and cuts, and were able to solve data instances up to 200 requests and a network of 28 nodes and 41 links. Jaumard *et al.* [11] proposed a decomposition model based on lightpath configurations (a set of requests provisioned using a set of slots with the same lowest index frequency slot), solved using column generation. The objective was to maximize the throughput, and data instances of up to 180 requests in the Spain network (21 nodes and 35 links). Enoch *et al.* [12] recently improved the results of [11], using the same decomposition, however with reduced computational times and better accuracy.

III. RSA Provisioning: Problem Statement

An Elastic Optical Network (EON) can be represented by a directed graph $G = (V, L)$, where V is the set of nodes and L is the set of optical fiber links. The frequency spectrum of the links is divided into a set of slices (S), also called frequency slots, indexed by s.

The network traffic (demand) is represented by a set of requests, K. Each request $k \in K$ has: *(i)* a source node $v_s \in V$ and a destination node $v_d \in V$, such that $(v_s, v_d) \in \mathcal{SD}$, where $\mathcal{SD}$ is the set of source-destination node pairs with some traffic; *(ii)* a required number of slots denoted by d_k.

Provisioning a request k means: *(i)* Selecting a path from the source to the destination node of k ; *(ii)* Assigning frequency slots on every link of that path so as to satisfy the continuity and contiguity constraints, which are next described.

Continuity constraints require that a request is assigned the same frequency slots on all its path links from source to destination.

On the other hand, **Contiguity constraints** require that the assigned frequency slots are contiguous (adjacent to each other) in the spectrum.

An illustration of a request provisioning is depicted in Figure 1 with a link transport capacity of 24 frequency slots and the provisioning of 5 requests. While the objective function varies from one study to the next (see the papers cited in the literature review), we choose here to maximize the throughput as expressed by the weighted number of granted requests, with weights equal to the demand, i.e., the required number of frequency slots.

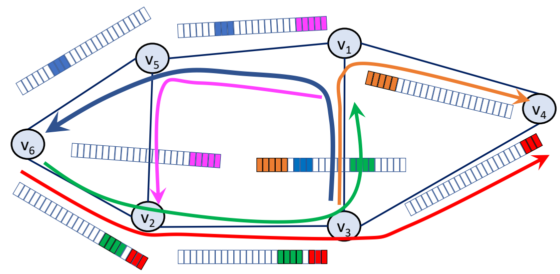


Fig. 1. An illustrative example

In the sequel, we define a lightpath by the combination of a path and a set of assigned slots satisfying the continuity and contiguity constraints.

IV. Mathematical Model

We propose here an original nested decomposition model based on the concept of link-lightpath configurations, which are next defined in Section IV-A. We expose the details of the mathematical model in Section IV-B.

A. Link-Lightpath Configuration

A Link-Lightpath configuration (or $\text{LConfig}_{\ell^\star}$ for short) is a set of lightpaths going through the same given link $\ell^\star$. Denote by Γ the overall set of configurations, indexed by γ.

A $\text{LConfig}_{\ell^\star}$ γ is characterized by a link $\ell^\star$ and a set of lightpaths all originating from the source node of link $\ell^\star$, with $\ell^\star$ being the first link of their routes. As a consequence, a $\text{LConfig}_{\ell^\star}$ is characterized by one link $\ell^\star$ and two set of parameters:

- $a^\gamma_{\ell s}$ = 1 if slot s is used on link ℓ in $\text{LConfig}_{\ell^\star}$ γ, 0 otherwise.
- a^γ_k = 1 if demand k is granted in $\text{LConfig}_{\ell^\star}$ γ, 0 otherwise.

We depict in Figure 2 two examples of $\text{LConfig}_{\ell^\star}$ going through link $\ell^\star = (v_6, v_2)$.

B. Mathematical Model

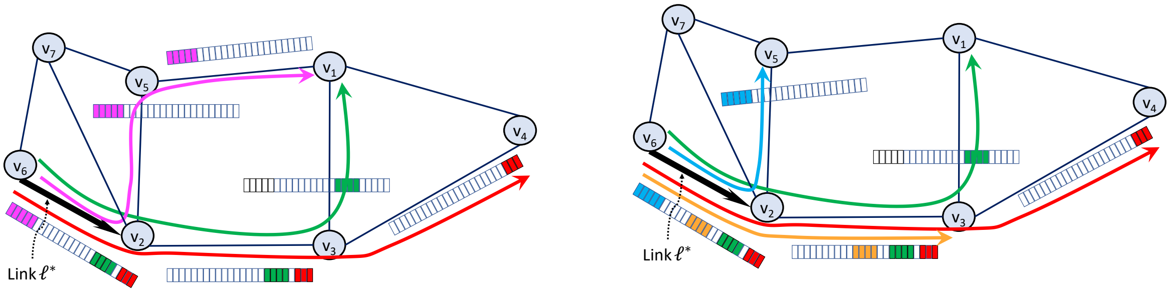
The next proposed mathematical model has two sets of variables. The first set of variables corresponds to decision variables z_γ, whose values depend on whether or not $\text{LConfig}_{\ell^\star}$ is selected. The second set of variables also corresponds to decision variables x_k, whose values depend on whether or not request k is granted.

The mathematical model is written as follows:

$$\max \sum_{k \in K} d_k x_k \qquad \text{(Throughput)} \qquad (1)$$

$$\text{subject to:} \sum_{\gamma \in \Gamma_{\ell^\star}} z_\gamma \leq 1 \qquad \ell^\star \in L \qquad (2)$$

$$\sum_{\gamma \in \Gamma} a^\gamma_{\ell s} z_\gamma \leq 1 \qquad \ell \in L, s \in S \qquad (3)$$



(a) Configuration 1.

(b) Configuration 2.

Fig. 2. Two Configuration Examples

$$x_k \leq \sum_{\gamma \in \Gamma} a_k^\gamma z_\gamma \qquad k \in K \qquad (4)$$

$$z_\gamma \in \{0,1\} \qquad \gamma \in \Gamma \qquad (5)$$

$$x_k \in \{0,1\} \qquad k \in K. \qquad (6)$$

Constraints (2) enforce the selection of at most one $\text{LConfig}_{\ell^\star}$ for each link $\ell^\star \in L$. Constraints (3) make sure that if a configuration is chosen, then all the frequency slots used in that configuration are not re-used in another selected configuration. Constraints (4) allow the identification of the connection requests that are granted and provisioned. Constraints (5) and (6) define the domains of the variables.

Note that, without loss of generality, we could assume $0 \leq x_k \leq 1, k \in K$. Considering the maximization objective and constraints (4), even if we assume $x_k \in [0,1]$, x_k can only take values 0 or 1 in the optimal solution. It has therefore the advantage of reducing the integer explicit requirements, without impacting the integer requirements of the optimal solution

V. Solution Scheme

The mathematical model proposed in the previous section has an exponential number of variables, and therefore is not scalable if solved using classical ILP (Integer Linear Programming) tools. Indeed, we need to use column generation techniques in order to manage a solution process that only requires an implicit enumeration of the $\text{LConfig}_{\ell^\star}$ (interested readers may refer to Chvatal [13]).

A. Column Generation and Integer Solution

Column generation method allows the exact solution of the linear relaxation of model (2)-(6), i.e., where constraints $z_\gamma \in \{0,1\}$ are replaced by $z_\gamma \geq 0$, for $\gamma \in \Gamma$. It consists in solving alternatively a restricted master problem (the model of Section IV-B with a very limited number of columns/variables) and the pricing problem (generation of a new $\text{LConfig}_{\ell^\star}$) until the optimality condition is satisfied (i.e., no $\text{LConfig}_{\ell^\star}$ with a negative reduced cost). In other words, when a new $\text{LConfig}_{\ell^\star}$ is generated, it is added to the current restricted master problem only if its addition implies an improvement of the optimal value of the current restricted master problem. This condition, indeed an optimality condition, can be easily checked with the sign of the reduced cost, denoted by RCOST, see (16) for its expression (the reader is referred to [13] if not familiar with linear programming), of variables z_γ.

Once the optimal solution of the LP (Linear Programming) relaxation ($\text{OBJ}^\star_{\text{LP}}$) has been reached, we solve exactly the last restricted master problem, i.e., the restricted master problem of the last iteration in the column generation solution process, using a branch-and-bound method, leading then to an ε-optimal ILP solution ($\text{OBJ}^{\text{LB}}_{\text{ILP}}$), where

$$\varepsilon = \frac{\text{OBJ}^\star_{\text{LP}} - \text{OBJ}^{\text{LB}}_{\text{ILP}}}{\text{OBJ}^{\text{LB}}_{\text{ILP}}},$$

where the optimal value of the linear relaxation ($\text{OBJ}^\star_{\text{LP}}$) provides an upper bound on the optimal value of the ILP ($z^\star_{\text{ILP}}$). Branch-and-price methods can be used in order to find optimal solutions, if the accuracy (ε) is not satisfactory, see, e.g., [14].

B. Nested Column Generation and Integer Solution

We propose here a nested decomposition solution scheme, meaning that the pricing problem itself is defined as a decomposition model in which the pricing problem generates a lightpath. In such a case, it becomes difficult to solve exactly the upper level pricing problem (i.e., it requires a branch-and-price algorithm, see [14]). Consequently, we do not have anymore the guarantee to solve exactly the linear programming relaxation of the upper master problem, and therefore need a way to compute an upper bound in order to assess the quality of the integer solutions.

We consider a Lagrangian bound, in order to compute an upper bound on the integer solution of the master problem, valid for any set of generated columns. At each iteration τ of the column generation, a Lagrangian relaxation bound LR can be calculated as follows, using Vanderbeck [15]:

$$\text{LR}^\tau(x,z,u) = \sum_{k \in K} d_k x_k + \sum_{\ell \in L} u_\ell^{(2)}(1 - \sum_{\gamma \in \Gamma_{\ell^\star}} z_\gamma) + \sum_{\ell \in L} \sum_{s \in S} u^{(3)}(1 - \sum_{\gamma \in \Gamma} a_{\ell s}^\gamma z_\gamma) + \sum_{k \in K} u^{(4)}(-x_k + \sum_{\gamma \in \Gamma} a_k^\gamma z_\gamma) \qquad (7)$$

subject to:

$$\sum_{\gamma \in \Gamma_\ell} z_\gamma \leq 1 \qquad \ell \in L \tag{8}$$

$$z_\gamma \geq 0 \qquad \gamma \in \Gamma \tag{9}$$

$$x_k \geq 0 \qquad k \in K, \tag{10}$$

where $u^{(2)}_\ell, u^{(3)}$, and $u^{(4)}$ are the values of the dual variables associated with constraints (2), (3), and (4).

Let us expand the expression of $\text{LR}^\tau(x, z, u)$:

$$\text{LR}^\tau(x,z,u) = \sum_{k \in K} d_k x_k + \underbrace{\sum_{\ell \in L} u^{(2)}_\ell + \sum_{\ell \in L} \sum_{s \in S} u^{(3)}_{\ell s}}_{ub} \tag{11}$$

$$- \sum_{\ell \in L} u^{(2)}_\ell \sum_{\gamma \in \Gamma_{\ell^\star}} z_\gamma - \sum_{\ell \in L} \sum_{s \in S} u^{(3)}_{\ell s} \sum_{\gamma \in \Gamma} a^\gamma_{\ell s} z_\gamma + \sum_{k \in K} u^{(4)}_k (-x_k + \sum_{\gamma \in \Gamma} a^\gamma_k z_\gamma) \tag{12}$$

$$\text{LR}^\tau(x,z,u) = \sum_{k \in K} \underbrace{(d_k - u^{(4)}_k)}_{\text{RCOST}(x_k) \leq 0} x_k + ub + \sum_{\ell \in L} \sum_{\gamma \in \Gamma_\ell} \underbrace{\left(-u^{(2)}_\ell - \sum_{s \in S} a^\gamma_{\ell s} u^{(3)}_{\ell s} + \sum_{k \in K} u^{(4)}_k a^\gamma_k \right)}_{\text{RCOST}^{\text{LP},\tau}_{\gamma,\ell}} z_\gamma$$

$$\leq ub + \sum_{\ell \in L} \sum_{\gamma \in \Gamma_\ell} \text{RCOST}^{\text{LP},\tau}_{\gamma,\ell}. \tag{13}$$

Moreover, $\sum_{\ell \in L : \text{RCOST}^{\text{LP},\tau}_{\gamma,\ell} > 0} \text{RCOST}^{\text{LP},\tau}_{\gamma,\ell}$ is the summation of all the reduced cost of the pricing computed at iteration τ associated with each link ℓ of the network. In other words, we only consider non negative $\text{RCOST}^{\text{LP},\tau}_{\gamma,\ell}$, where $\text{RCOST}^{\text{LP},\tau}_{\gamma,\ell}$ is the optimal LP value.

The Lagrangian bound value is chosen by:

$$\text{LR}(x,z,u) = \min_\tau \text{LR}^\tau(x,z,u). \tag{14}$$

The resulting accuracy of the solution, ε, is then computed as follows:

$$\varepsilon = \frac{\min\{\text{Offered Load}, \text{LR}\} - \text{OBJ}^{\text{LB}}_{\text{ILP}}}{\text{OBJ}^{\text{LB}}_{\text{ILP}}}, \tag{15}$$

where the offered load is equal to $\sum_{k \in K} d_k$.

The solution process is summarized by the flowchart in Figure 3.

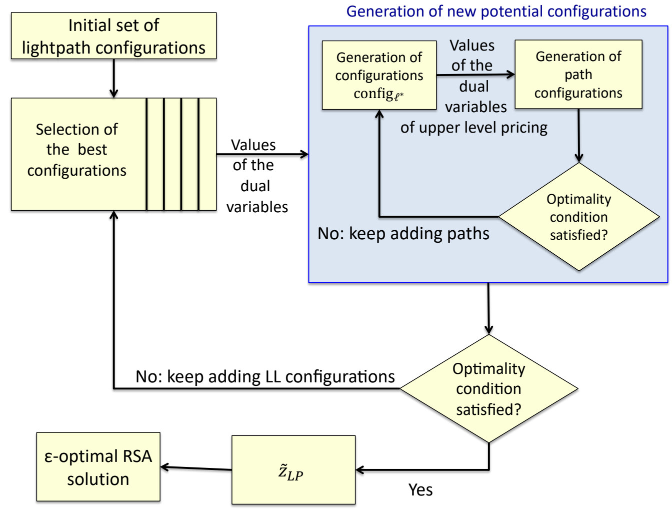


Fig. 3. Column Generation flowchart

C. Upper Level Pricing Problem

Each upper level pricing problem is dedicated to the search of a subset of lightpaths going through a specific link, denoted by $\ell^\star$. It has 5 sets of variables:

- y_p = 1 if path p is selected in LConfig$_{\ell^\star}$, 0 otherwise.
- a_k = 1 if request k is granted in the configuration under construction, 0 otherwise.
- $a_{\ell s}$ = 1 if for link ℓ, slot s is occupied, 0 otherwise.
- $a^p_{s\ell^\star}$ = 1 if slot s is assigned to p on $\ell^\star$, 0 otherwise.
- $b^p_{s\ell^\star}$ = 1 if slot s is the starting slot of p on $\ell^\star$, 0 otherwise.

In addition, there is parameter δ^p_ℓ = 1 if path p goes through link ℓ, 0 otherwise. We denote by P_k the set of paths for routing connection request k: we do not need to pre-compute it, thanks to the nested column generation framework, where paths are online computed only once as needed.

Observe that:

$$a_{\ell s} = \sum_{k \in K} \sum_{p \in P_k} \delta^p_\ell a^p_{s\tilde{\ell}} \qquad \ell \in L : \ell \neq \tilde{\ell}, s \in S.$$

Maximize the reduced cost, i.e.,

$$\text{RCOST}_\gamma = -u^{(2)}_{\ell^\star} - \sum_{s \in S} \sum_{\ell \in L} u^{(3)}_{s\ell} \underbrace{\sum_{k \in K} \sum_{p \in P_k} \delta^p_\ell a^p_{s\ell^\star}}_{a_{\ell s}} + \sum_{k \in K} u^{(4)}_k a_k. \tag{16}$$

subject to:

$$\sum_{p \in P_k} y_p = a_k \qquad k \in K \tag{17}$$

$$a^p_{s\ell^\star} \leq y_p \qquad p \in P_k, k \in K, s \in S \tag{18}$$

$$\sum_{p \in P_k} \frac{1}{n_p} \sum_{s \in S} a^p_{s\ell^\star} = a_k k \in K \tag{19}$$

TABLE I
COMPUTATIONAL COMPARISON ON SPAIN NETWORK

Data Instances			$\tilde{z}_{\text{LP}}$	$\text{OBJ}^{\text{LB}}_{\text{ILP}}$	LR	ε	CPU (sec.)	from [11]		from [12]
Offered Load (Tbps)	$\|\mathcal{SD}\|$	$\|S\|$						$\text{OBJ}^{\text{LB}}_{\text{ILP}}$	CPU (sec.)	$\text{OBJ}^{\text{LB 1}}_{\text{ILP}}$
3.675	35	50	3.675	3.675	3.875	0	3.6	3.17	50	3.675
4.750	45	60	4.750	4.750	5.750	0	3.1	4.15	86	4.750
6.775	60	75	6.738	6.725	6.825	0.007	8.4	5.75	147	6.775
7.450	64	85	7.450	7.450	9.775	0	5.9	6.00	176	7.450
7.375	70	100	7.375	7.375	7.450	0	11.6	6.17	263	7.375
9.675	80	120	9.675	9.675	9.775	0	45.4	8.15	323	9.675
7.450	35	80	7.050	7.050	9.100	0.057	3.7	6.70	134	7.450
9.750	45	110	9.750	9.750	11.900	0	5.5	8.80	177	9.750
10.700	60	156	10.700	10.700	10.850	0	18.8	9.45	261	10.700
15.500	64	170	15.500	15.500	15.550	0	16.1	12.95	630	15.500
15.100	70	236	15.025	14.950	15.014	0.004	86.8	13.10	1342	15.100
16.850	80	256	16.700	16.600	16.800	0.012	61.3	14.45	1419	16.850
$\|\mathcal{SD}\|$ denotes the number of requests, $\|S\|$ designates the number of frequency slots										

$$\sum_{p \in P_k} \sum_{s \in [1,|S|-n_p+1]} b^p_{s\tilde{\ell}} = a_k \qquad k \in K \tag{20}$$

$$\sum_{i=0}^{n_p-1} a^p_{t+i,\ell^\star} \geq n_p b^p_{t\ell^\star} \qquad t \in [1, |S| - n_p + 1], \quad k \in K, p \in P_k \tag{21}$$

$$\sum_{k \in K} \sum_{p \in P_k} a^p_{s\ell^\star} \leq 1 \qquad s \in S \tag{22}$$

$$y_p \in \{0,1\} \qquad p \in P_k, k \in K \tag{23}$$

$$a_k \in \{0,1\} \qquad k \in K \tag{24}$$

$$a^p_{s\ell^\star} \in \{0,1\} \qquad p \in P_k, k \in K, s \in S. \tag{25}$$

$$b^p_{s\ell^\star} \in \{0,1\} \qquad p \in P_k, k \in K, s \in S. \tag{26}$$

Constraints (17) ensure that we select at most one path (routing) for request k if it is granted in the configuration under construction. Constraints (18) force variable $y_p = 1$ if provisioning of path p uses any slot s on link $\ell^\star$. Constraints (19) make sure the total number of slots for p matches n_p. Constraints (20) ensure a unique starting slot for each request. Constraints (21) express the contiguity constraints on link $\ell^\star$. Constraints (22) ensure that each slot is used at most once in the overall set of connection requests.

The number of variables in the pricing problem can be further reduced by limiting the set of requests to $K_{\ell^\star}$, with $K_{\ell^\star}$ only made up of requests whose origin is equal to the node source of $\ell^\star$.

D. Nested Pricing Problem

The lowest level pricing problem consists in computing weighted paths, and can be formulated as follows. Maximize the reduced cost:

$$\text{RCOST}_p = -\sum_{s \in S} \sum_{\ell \in L} u^{(3)}_{s\ell} \qquad p \in P_k, k \in K \tag{27}$$

which is equivalent to solving a weighted shortest path problem from v^k_s to v^k_d (source and destination node of request k, respectively) with weight $\text{WEIGHT}_\ell = \sum_{s \in S} u^{(3)}_{s\ell}$ for link ℓ.

VI. Computational Results

The model and algorithm described in the previous sections was implemented on a 3.6-4.0 GHz 4-cores machine with 32 GB of RAM, with the use of CPLEX (version 12.8.0.0) for solving the (integer) linear programs.

A. Computational Comparisons on Spain Network

In a first set of experiments, we conducted experiments in order to assess the scalability of our solution process, and the accuracy of the RSA solutions that were output, in comparison with previous works. We used the same set of data instances as [11], i.e., the Spain network with 21 nodes and 35 links ([6]) and the same demand sets. Results are shown in Table I, and include a comparison with those of [11].

In Table I we run the algorithms on many instances of the Spain network and compare it with the previous results of [11], improved in [12]. The performance of our new model and algorithm is better in terms of the quality of the solutions and of computational times.

B. Computational Results on Larger Datasets

We consider here larger instances, both on the Spain network of the previous section, and the USA network [16] with 24 nodes and 86 links. For all the experiments with the USA network, we used $|S| = 380$. While computational times are increasing, they remain reasonable for a planning problem,

TABLE II
COMPUTATIONAL RESULTS WITH LARGER INSTANCES ON SPAIN NETWORK

Data Instances			$\tilde{z}_{\text{LP}}$	$\text{OBJ}_{\text{ILP}}^{\text{LB}}$	LR	ε	CPU (sec.)
Offered Load (Tbps)	$\|\mathcal{SD}\|$	$\|S\|$					
8.075	100	300	8.075	8.075	8.263	0	82.5
9.625	120	300	9.600	9.600	9.635	0.003	102.7
11.225	140	380	11.225	11.225	11.255	0	147.4
13.300	160	380	13.188	13.150	13.225	0.006	228.1
21.925	100	380	21.925	21.925	21.958	0	89.9
25.600	120	380	25.413	25.275	25.479	0.008	182.7
29.675	140	380	29.525	29.525	29.525	0	405.5
33.675	160	380	32.875	32.875	32.925	0.002	364.8

TABLE III
NUMERICAL EXPERIMENTS ON DIFFERENT TRAFFIC INSTANCES ON USA NETWORK

Data Instances		$\text{OBJ}_{\text{ILP}}^{\text{LB}}$	LR	ε	CPU (Sec.)
Offered Load (Tbps)	$\|\mathcal{SD}\|$				
21.925	100	21.925	21.925	0	305.9
25.600	120	25.350	25.875	0.010	138.7
29.675	140	29.675	29.700	0	215.4
33.675	160	33.550	33.725	0.004	242.5
43.075	160	41.650	41.950	0.007	589.3
49.250	180	47.150	47.575	0.009	834.3
54.675	200	53.425	53.635	0.004	1,458.6

i.e., less than 1 hour. Note that the accuracy of the solutions is always smaller than 1%.

VII. CONCLUSIONS

In this paper we proposed a new decomposition model for the RSA problem, which can be solved using a nested column generation technique. The advantage of such a link-based decomposition is that the number of pricing problems is significantly less than the number of pricing problems in a slot-based decomposition as in [11], as the number of links is less than the number of frequency slots. In addition, pricing problems are less complex to solve, and therefore can be solved faster, and in parallel. It therefore offers a promising solution scheme, with an enhanced scalability in comparison with the previous RSA decomposition schemes of the literature.

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