Algorithms for the m-coverage Problem and k-connected m-Coverage Problem in Wireless Sensor Networks

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Abstract. An important issue in deploying a wireless sensor network （WSN） is to provide target coverage with high energy efficiency and fault-tolerance. In this paper, we study the problem of constructing energy-efficient and fault-tolerant target coverage with the minimal number of active nodes which form an *m*-coverage for targets and a *k*-connected communication subgraph. We propose two heuristic algorithms for *m*-coverage problem, and get the performance ratio of one heuristic. Then two heuristic algorithms are further proposed to solve the *k*-connected *m*-coverage problem. The simulation results demonstrate the desired efficiency of the proposed algorithms.

Key words: *k*-connected *m*-coverage; sensor networks; approximation algorithm; energy-efficient

1 Introduction

Monitoring a geographical region or a set of targets and collecting the relevant data are very important tasks in wireless sensor networks. Since sensor nodes are often deployed in an arbitrary manner, one of the fundamental issues in the task of target monitoring is target coverage which reflects how well the deployed sensor nodes can monitor a set of targets. Meanwhile, the energy-efficiency is another important issue in WSNs. In general, sensor nodes are powered by very limited battery resources. Recent research has found that significant energy savings can be achieved by elaborate managing the duty cycle of nodes in WSNs with high node density. In this approach, some nodes are scheduled to sleep (or enter a power saving mode) while the remaining active nodes keep working.

Sensing is only one responsibility of a sensor network. To operate successfully, most sensor networks must also remain connected, i.e., the active nodes should not be partitioned in any configured schedule of node duty cycles. A sensor network must provide satisfactory connectivity so that nodes can communicate for data fusion and reporting the results to base stations. Single connectivity often is not sufficient for many sensor networks because a single failure could disconnect the network, and single coverage is also not sufficient. Therefore, maintaining sufficient sensing coverage and network connectivity with minimal active nodes are critical requirements in WSNs.

In this paper, we study more general coverage problem--*k*-connected *m*-coverage problem: to find minimized number of active nodes to form a target *m*-coverage and meanwhile any pair of active nodes is connected by at least *k* disjoint paths. To solve the *k*-connected m-coverage problem, we first investigate an introductory problem, namely *m*-coverage problem that is to find the minimum number of active nodes ensuring that each target can be covered by at lease m distinct sensor nodes. We show that the *m*-coverage problem is NP-hard and then give one heuristic and an approximation algorithm accordingly. Next, based on the *k*-connected coverage problem [19], we propose two heuristic algorithms to solve the *k*-connected *m*-coverage problem.

The rest of the paper is organized as follows. In section 2 we present related works. Section 3 describes network model and problems studied in this paper. Section 4 and 5 propose two heuristics for *m*-coverage and *k*-connected *m*-coverage problem respectively . Section 6 describes the simulations and section 7 concludes the paper.

2 Related works

There are many studies on the coverage problem ([1-5 etc.]) in WSNs. Different formulations of the coverage problem have been proposed, depending on the subject to be covered (area versus discrete points) [4,5], the sensor deployment mechanism (random versus deterministic [6]), as well as other wireless sensor network properties (e.g. network connectivity and minimum energy consumption). For energy efficient area coverage, the works in [7] and [8] consider a large population of sensors, deployed randomly for area monitoring.

Zhang and Hou [9] prove an important, but intuitive result that if the communication range Rc is at least twice the sensing range Rs, a complete coverage of a convex area implies connectivity of the working nodes. They further discuss the case of Rc > Rs. Wang et al [10] generalize the result in [9]. Wu and Yang [11] propose two density control models for energy conserving protocol in sensor networks, using the adjustable sensing range of several levels. Zhou et al [12,13] address the problem of selecting a minimum size connected k-cover.

The energy-efficient target coverage problem deals with the problem of covering a set of targets with minimum energy cost [1,6,14]. Cardei and Du [1] address the target coverage problem where the disjoint sets are modeled as disjoint set covers, such that every cover completely monitors all the target points. Cardei et. [14] propose an approach different from [1] by not requiring the sensor sets to be disjoint and by allowing sensors to participate in multiple sets, and design two heuristics that efficiently compute the sets, using linear programming and a greedy approach.

Alam[15] et al consider coverage and connectivity in 3-Dimensional networks. Liu [16,17,18] et al consider maximal lifetime scheduling for sensor surveillance systems with *k* sensors to 1 target. In these papers, they assume each sensor watch at most a target and each target is watched by at least *k* sensor.

In [19] we addressed the *k*-connected coverage problem for targets. In this paper we extend our work [19] to *k*-connected *m*-coverage problem. Our model is different from [16-18], in our model, a sensor may watch all targets in its sensing range.

3 Network Model and Problem Specification

In this section, we formulate the target *m*-coverage problem and the *k*-connected *m*-coverage problem addressed in this paper.

Let us assume that *n* sensors $v_1, v_2, ...v_n$ are deployed in a region to monitor *t* targets $I_1, I_2, ...I_t$. Each node v_i has a sensing region $S(v_i)$ and communication range R. Any target inside $S(v_i)$ is cover by v_i. v_i can directly communicate with v_j if their Euclidian distance is less than communication range R. Consequently, the sensor nodes in the communication network can form a undirected graph *G*=(*V*, *E*), where $V = \{v_1, v_2, ...v_n\}$ is a set of sensor nodes and *E* is a set of edges (*i*, *j*). Without loss of generality, assume $T = \{I_1, I_2, ...I_t\}$ to be a given set of targets. For each sensor $v \in V$, there is a subset T_v of $T = \{I_1, I_2, ...I_t\}$, which is covered by *v*. Note that the targets are different from the sensor nodes.

The graph is *k*-connected if there are *k* node-disjoint paths between any pair of nodes. A set of sensors $C \subseteq V$ is said to be *m*-coverage if each target in *T* is covered by at least *m* distinct sensor nodes in *C*.

In order to reduce the energy consumption, our work is to minimize the number of sensor nodes.

Thus, the problem studied in this paper can be now formally defined as follows:

***m*-coverage problem:** Given a graph *G*=(*V*, *E*) and a set of targets *T*, we want to find a minimal number of sensor nodes in *V*, where these nodes form a *m*-coverage for targets.

***k*-connected *m*-coverage problem:** Given a graph *G*=(*V*, *E*) and a set of targets *T*, we want to find a minimal number of sensor nodes in *V*, where these nodes form a *m*-coverage for targets and the subgraph induced by these nodes is *k*-connected.

4 Approximation algorithms to m-coverage problem

In this section, we will first investigate an introductory problem, namely *m*-coverage problem, of the *k*-connected *m*-coverage problem. This problem is NP-hard as it is a generalization of set cover, which is already known to be NP-hard. We present two heuristics for *m*-coverage problem. We first model the *m*-coverage problem as Integer Programming in section 4-A, and then use the relaxation technique to design a linear programming based heuristic in section 4-B. Next, we propose a heuristic based on duality rounding and give its approximation ratio in section 4-C.

A. Integer programming Formulation of the m-coverage problem

We formulate the *m*-coverage problem as follows:

Given:

n: The total number of sensor nodes; *t*: The total number of targets;

j: Indicator for sensor nodes, $j \in [1, n]$; *i*: Indicator for targets, $i \in [1, t]$.

$$a_{ij} = \begin{cases} 1 & \text{if target } i \text{ is covered by sensor } j; \\ 0 & \text{otherwise} \end{cases}$$

Variable:

$$x_j = \begin{cases} 1 & \text{if } j \text{ is selected for } m\text{-coverage;} \\ 0 & \text{otherwise} \end{cases}$$

The m-coverage problem then can be formulated as 0-1 programming as follows:

ILP:

$$Z_1 = Min \sum_{j=1}^{n} x_j \quad \textbf{(1)}$$

***subject to*:**

$$\sum_{j=1}^{n} a_{ij} x_j \geq m \qquad i \in [1, t] \quad \textbf{(2)}$$

$$x_j = 0 \text{ or } 1 \qquad j \in [1, n] \quad \textbf{(3)}$$

B. LP heuristic

This heuristic is a two-stage algorithm. At the first stage, an optimal solution for a linear programming (LP) relaxation of the ILP is computed. The obtained solution to LP may be fractional, so it may not satisfy the integer constraint (3). At the second stage, a greedy algorithm is employed to find an integral solution based on the optimal solution obtained at the first stage.

LP Heuristic:

Input: n sensor nodes and t targets

Output: m-coverage for targets

Formulate m-coverage problem as ILP, and relax ILP to LP

Compute an optimal solution $\{ x_j^* \}$ and make an decreasing order $x_1^* \geq x_2^* \geq .... \geq x_n^*$;

For j=1 to n

$\quad x_j = 0$

For j=1 to n

$\quad$ If $\{v_1, v_2, ... v_{j-1}\}$ is not m-coverage, then

$\quad\quad x_j = 1$

$\quad\quad j = j+1$

We have seen that in the above heuristic, the optimal solution to a linear programming relaxation is employed to find out the priority of variables being assigned with 1. There is a disadvantage with this approach: Computing the optimal solution takes $O(n^{3.5})$ time for LP of n variables [20]. It is the main portion of the total computation time for this heuristic. In order to reduce the computational time and improve the quality of output solution, we will design another heuristic by applying rounding by duality.

C. Heuristic based on rounding by duality

To simplify the description of this heuristic, we consider the primal form of linear programming (PLP).

PLP

$$Z_2 = Min \sum_{j=1}^{n} c_j x_j \tag{4}$$

subject to

$$\sum_{j=1}^{n} a_{ij} x_j \geq b_i \qquad i \in [1,t] \tag{5}$$

$$0 \leq x_j \leq 1 \qquad j \in [1,n] \tag{6}$$

Where a_{ij} is 1 or 0, $b = (m,m,....,m)^T$ in which there are t's m. $c = (1,1,....,1)$ in which there is n's 1. $x^T = (x_1, x_2,....,x_n)$ is variable vector. The dual of the above linear programming is DLP, and $y^T = (y_1, y_2,....,y_t)$, $z^T = (z_1, z_2,....,z_n)$ are dual variable vector.

We use a two-stage algorithm to find an approximation solution for it. First, a feasible solution is obtained for the Dual Linear Programming problem (DLP). The corresponding solution for the Primal form (PLP) is obtained by the rounding procedure:

The formal description of the algorithm is given below:

Heuristic based on rounding by duality:

Initially, set x^0=0, (y^0, z^0)=(0,0), k=0 // x^0 , y^0, z^0 are vectors

While x^k is not prime feasible **do begin**

Set $J(k) = \{j \mid x_j^k = 0\}$;

Set $I(k) = \{i \mid \sum_{j=1}^{n} a_{ij} x_j^k \leq b_i - 1\}$;

Choose $r \in J(k)$ such that

$$\frac{c_r - \sum_{i=1}^{t} a_{ir} y_i^r}{\sum_{i \in I(k)} a_{ir}} = \alpha = \min_{j \in J} \left\{ \frac{c_j - \sum_{i=1}^{t} a_{ij} y_i}{\sum_{i \in I(k)} a_{ij}} \mid c_j - \sum_{i=1}^{t} a_{ij} y_i \geq 0 \right\}$$

Set $x_j^{k+1} = x_j^k$ if $j \neq r$ and $x_r^{k+1} = 1$;

Set $y_i^{k+1} = y_i^k + \alpha$ if $i \in I(k)$ and $y_i^{k+1} = y_i^k$ if $i \notin I(k)$

Set $z_j^{k+1} = \max(\sum_{i=1}^{t} a_{ij} y_i^{k+1} - c_j, 0)$;

$k \leftarrow k+1$;

end-while

Output x^k *with* $Z_{1A} = \sum_{j=1}^{n} c_j x_j^k$

Theorem The performance ratio of heuristic based on rounding by duality is $f = \max_{1 \leq i \leq t} \sum_{j=1}^{n} a_{ij}$, and the time complexity is $O(n^2)$

5 Approximation algorithm for *k*-connected *m*-coverage problem

In this section, we address the *k*-connected *m*-coverage problem which is NP-hard because *m*-coverage problem is NP-hard. We will design two heuristic algorithms. One is called as *km*TS algorithm, the other is called *km*Reverse algorithm.

A. kmTS algorithm: The main idea of *km*TS algorithm is that the algorithm includes two steps: the first step is to construct a *m*-coverage of targets; The second step is to increase small size nodes to this *m*-coverage such that the subgraph by these increased nodes and nodes of *m*-coverage is *k*-connected. For the first step, we may use the algorithms in section 4 to get an approximation for *m*-coverage problem. For the second step, we may use our algorithms [19] to get solution for *k*-connected *m*-coverage problem.

kmTS Algorithm: Construct an approximate solution for *k*-connected *m*-coverage

Input: Given $G=(V, E)$, a set T of targets, and $T_v, \forall v \in V$, which is a subset of T covered by v

Output: k-connected m-coverage for T

(1) Construct m-coverage C for T using m-coverage heuristic.

(2) Connect set C into k-connected subgraph, i.e. finding a subset X of $V-C$ to C such that $G[C \cup X]$ is k-connected subgraph and $|X|$ is minimized.

B. kmReverse algorithm: In the following, we will give another algorithm--reverse algorithm which directly apply Lemma[19]. The main idea of kmReverse algorithm is as follows: initially, each sensor node in the sensor network is active, then, change one active node to inactive node each time if it satisfies two conditions (1) deleting the node, the remain nodes also form a *m*-coverage (2) any two neighbours of the node has *k*-node disjoint paths in remain graph after deleting the node.

kmReverse algorithm: Construct an approximate solution

Input: Given $G=(V, E)$, a set T of targets, and $T_v, \forall v \in V$, which is a subset of T covered by v.

Output: k-connected m-coverage for T

1. $V^k := V$;

2. Sort all nodes in V in an increasing order of degree in T as $v_1, v_2, \ldots v_n$ such that $D_T(v_1) \le D_T(v_2) \le \ldots \le D_T(v_n)$, where $D_T(v) = |\{r_j \mid r_j \text{ is covered by } v\}|$

3. For i=1 to n,

   if $\forall u_1, u_2 \in N(v_i)$, u_1 is k-connected to u_2 in $G[V^k - \{v_i\}]$, and $V^k - \{v_i\}$ is a m-coverage for T, then

   $V^k = V^k - \{v_i\}$

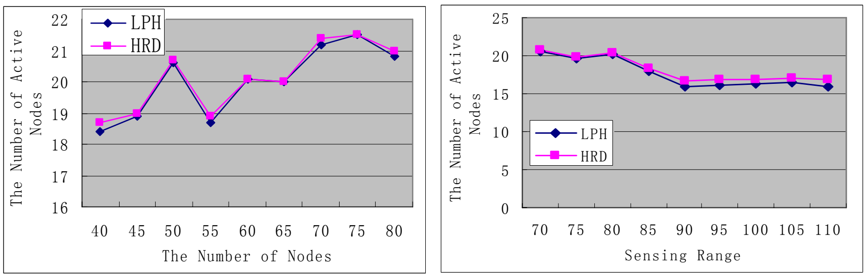
   $i := i + 1$

6 Performance Evaluation

In this section we evaluate the performance of proposed algorithms. We simulate a stationary network with sensor nodes and target points randomly located in a 500×500 area. We assume the sensing range is equal for all the sensors in the network, and the communicating range is also equal for all the sensors in the network. In the simulation we consider the following tunable parameters:

- *N*, the number of sensor nodes, which varies between 40 and 80.
- *M*, the number of targets to be covered, which varies between 10 and 26.
- *R*, the communicating range which varies between 120 to 200.
- *S*, the sensing range,which varies between 70 and 110.

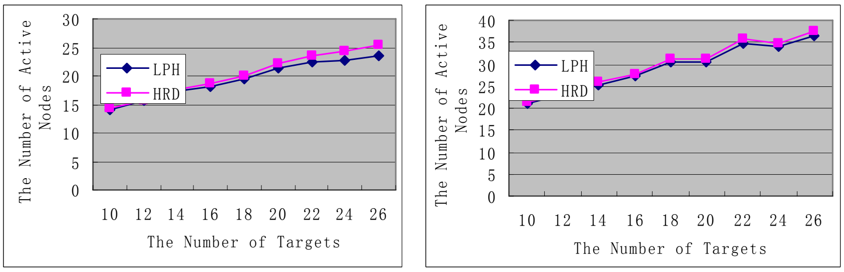
The simulation is conducted in a 500×500 2-D free-space by independently and uniformly allocating *N* nodes and *M* targets. All nodes have the same transmission range *R*. And all nodes have the same sensing range *S*. For any pair of nodes, if the distance between the two nodes is no more than the value of transmission range *R*, there exists an edge between the two nodes. For any sensor node and any target, if the distance between the sensor and the target is no more than the value of sensing range, the target is covered by the sensor node. We present averages of 100 separate runs for each result shown in figures. In each run of the simulations, for given N and M, we randomly place N nodes in the square, and randomly place M nodes as targets. Any topology which is not connected or targets are not covered by all sensor nodes is discarded.



(a) m=3, M=10, S=70 (b) m=3, M=10, N=50

Fig. 1. The number of active nodes with the number of nodes, sensing range

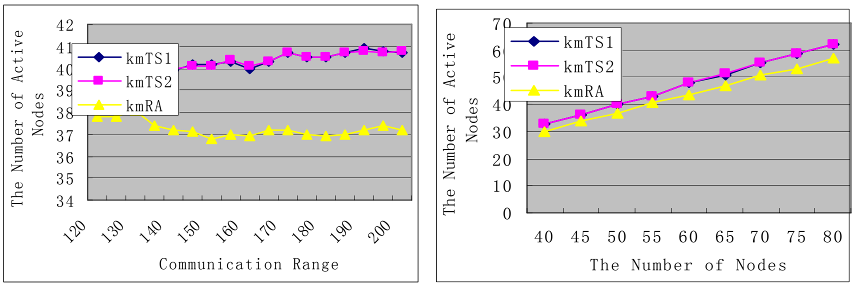
In the first experiment, we simulate the proposed LP Heuristic (LPH) and Heuristic based on rounding by duality(HRD) for m-coverage problem. The results show the performances of the two heuristics are close from Fig.1 and Fig. 2.



(a) m=2, S=70, N=70 (b) m=3, S=70, N=70

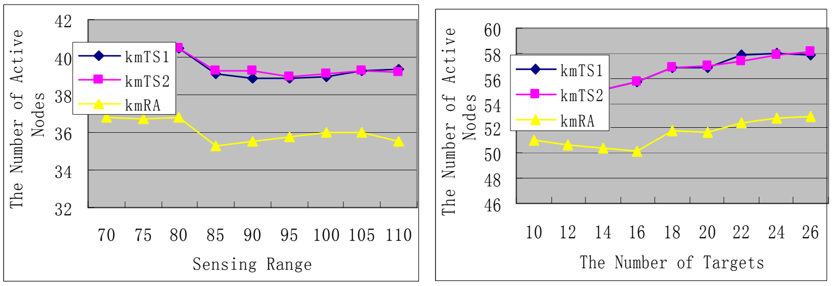
Fig. 2. The number of active nodes with number of targets

In the second experiment, we simulate the proposed kmTS and kmRA and compare their performances. We call $kmTS_1$ and $kmTS_2$ when kmTS using LPH and HRD respectively. The simulation results are shown in Fig.3 and Fig.4. The number of active nodes increases with the number of sensors and the number of targets, as more sensors need to participate so that each active pairs communicate with k-disjoint paths and more targets needs to be covered. The number of active sensors is not increased with increasing sensing range, because when sensing range is larger each target is covered by more sensors.



(a) k=3, m=3, M=10, N=50, S=70 (b) k=3, m=3, M=10, R=150, S=70

Fig. 3. The number of active nodes with comm. range, the number of nodes



(a) k=3, m=3, M=10, R=150, N=50 (b) k=3, m=3, N=70, R=150, S=70

Fig. 4. The number of active nodes with sensing range, the number of targets

7 Conclusions

In this paper, we study how to construct k-connected m-coverage with minimized number of active sensors for targets in wireless sensor networks. We first discuss the m-coverage problem in WSNs, we propose two heuristics and get performance ration of one. Then based on the discussion of the m-coverage and [19], we propose two heuristics to construct k-connected m-coverage. We also carry out extensive simulations for our algorithms and the obtained simulation results have demonstrated the high effectiveness of our algorithms.

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