Temporal Graph based Energy-limited Max-flow Routing over Satellite Networks

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***Abstract*—Nowadays satellite networks are playing an increasing role in earth observation, global communication, etc. Many space missions require to deliver large amounts of data to the ground system for different purposes, and analyzing the maximum throughput of the given satellite network is a prerequisite for efficient data transmission. However, satellite networks possess the time-varying topologies, dynamic bandwidth and limited on-board energy, which restricts the end-to-end capacity and poses challenges to the analysis. In this paper, we utilize temporal graphs for better solving the end-to-end max-flow problem over energy-limited satellite networks. An energy time-expanded graph (eTEG) is constructed to accurately represent the restriction of on-board limited energy on data transmission capability. Furthermore, to maximize flow delivery and energy utilization, we proposed an eTEG-based max-flow routing algorithm with time-dependent residual network update rules. Simulation results are also presented to verify the efficacy of our algorithm.**

***Index Terms*—Satellite networks, time-expanded graph, max-flow routing, energy utilization.**

I. Introduction

Due to the advantages in coverage, reliability and availability, satellite networks can provide various services to everywhere on the earth [1]. In many space missions, large amounts of data are required to be transmitted to the ground system for environment surveillance, disaster warning, etc [2]. In order to efficiently download massive spatial information, it is necessary to analyze the maximum throughput of given satellite network, i.e., the end-to-end maximum flow, which indicates the upper bound of the transmission capacity. For traditional static networks, numerous excellent works have been done on the max-flow problem solution [3]. Nevertheless, satellite networks possess the time-varying topologies, dynamic bandwidth and especially limited on-board energy, which pose challenges to the formulation and solution of the max-flow problem [4]. Considering an example in Fig. 1, traditional max-flow methods only involve link capacities and may select $s_1 \to r_1 \to r_2 \to g_1$ and $s_1 \to r_2 \to g_1$ to deliver the most data. However, since the available energy of r_1 and r_2 are both 0.1 kJ, only 2000 Mb can be forwarded and the end-to-end capacity is actually 2000 Mb. Theoretically, through more efficient energy utilization, that is, arranging data on both $s_1 \to r_1 \to g_1$ and $s_1 \to r_2 \to g_1$, to reach 4000 Mb.

In this paper, we design an enhanced temporal graph to effectively represent the heterogeneous resources and energy constraints of the satellite network, and propose a max-flow routing algorithm to jointly schedule the flow delivery and energy utilization for reaching the maximum throughput.

II. System Model and Problem Formulation

A satellite network with a source satellite s_1, multiple relay satellites $r_1, r_2, ..., r_i, ..., r_{N-1}$ and a ground station g_1 is considered, as shown in Fig. 1. s_1 is expected to deliver the most spatial data to g_1 during the planning time horizon $T = [t_0, t_k)$, either directly or via relays. We adopt a time slots system, i.e., dividing T into k time slots $\{\tau_1,...,\tau_h,...,\tau_k\}$, $\tau_h = [t_{h-1}, t_h)$, and model the network as a time-expanded graph [5] TEG= $(\mathcal{V}, \mathcal{A}, \mathcal{C}, \mathcal{S}, \mathcal{E})$ to accurately represent dynamic contact topology and heterogeneous network resources. As shown in Fig. 2, the constructed TEG includes

- Vertex set $\mathcal{V} = \mathcal{V}_s \bigcup \mathcal{V}_r \bigcup \mathcal{V}_g$, where $\mathcal{V}_s = \{s_1^h | 1 \le h \le k\}$, $\mathcal{V}_r = \{r_i^h | 1 \le i \le N, 1 \le h \le k\}$ and $\mathcal{V}_g = \{g_1^h | 1 \le h \le k\}$ denote the replicates of s_1, r_i and g_1, respectively.
- Arc set $\mathcal{A} = \mathcal{A}_t \bigcup \mathcal{A}_c$, with $\mathcal{A}_t = \{(v_i^h, v_j^h) | v_i^h, v_j^h \in \mathcal{V}, 1 \le h \le k\}$ indicting the communication link from any v_i to v_j during τ_h, and $\mathcal{A}_c = \{(v_i^h, v_i^{h+1}) | v_i^h, v_i^{h+1} \in \mathcal{V}, 1 \le h \le k-1\}$ representing the ability of any v_i to cache data across time slots.
- Capacity set $\mathcal{C}$. For any $(v_i^h, v_j^h) \in \mathcal{A}_t$, $\mathcal{C}_{v_i^h, v_j^h} = \int_{\tau_h} R_{v_i^h, v_j^h}(t)dt$ depicts its transmission capacity during τ_h, where $R_{v_i^h, v_j^h}(t)$ is the date rate at time $t \in \tau_h$.
- Storage set $\mathcal{S}$. For any *caching arc* (v_i^h, v_i^{h+1}), $\mathcal{S}_{v_i^h, v_i^{h+1}}$ denotes the available buffer of v_i in τ_h. Specially, since s_1 can send data at any time, and g_1 has sufficient storage resources, both $\mathcal{S}_{s_1^h, s_1^{h+1}}$ and $\mathcal{S}_{g_1^h, g_1^{h+1}}$ are set to infinity.
- Energy set $\mathcal{E}$, with each element $\mathcal{E}_{v_i}$ defining the total energy of v_i during T. Since the quantity and timing of energy input are known, $\mathcal{E}$ can be determined in advance. Moreover, due to the sufficient power supply to g_1, its energy limitation is dropped.

Based on TEG, we formulate the EMF problem as

$$\textbf{P1:} \quad \max \ f_m = \sum_{h=1}^{k} \sum_{v_i^h \in \mathcal{V} - \mathcal{V}_g} f_{v_i^h, g_1^h}, \tag{1}$$

$$\text{s.t.} \quad 0 \le f_{v_i^h, v_j^h} \le \mathcal{C}_{v_i^h, v_j^h}, \forall (v_i^h, v_j^h) \in \mathcal{V}_t, \tag{2}$$

$$0 \le f_{v_i^h, v_i^{h+1}} \le \mathcal{S}_{v_i^h, v_i^{h+1}}, \forall (v_i^h, v_i^{h+1}) \in \mathcal{V}_c, \tag{3}$$

$$\sum_{h=1}^{k} \sum_{v_i^h \in \mathcal{V}} f_{v_i^h, v_j^h} = \sum_{h=1}^{k} \sum_{v_i^h \in \mathcal{V}} f_{v_j^h, v_i^h}, \quad \forall v_j^h \in \mathcal{V} - \{\mathcal{V}_s, \mathcal{V}_g\}, \tag{4}$$

$$E_{v_i}^{\Delta} + E_{v_i}^{*} \le \mathcal{E}_{v_i}, \tag{5}$$

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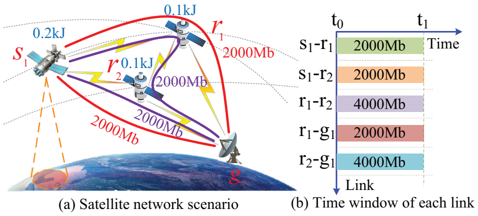


Fig. 1: The impact of satellite's energy on flow transmission.

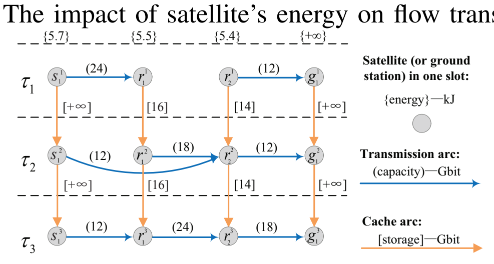


Fig. 2: The satellite network modeled by TEG.

where f_m is the expected maximum flow and $f_{v_i^h, v_j^h}$ denotes the network flow delivered through (v_i^h, v_j^h). In particular, (2) restricts the flow through any communication link to be smaller than link's capacity, and (3) restricts the flow cached in any satellite cannot exceed the remaining buffer. (4) is the flow conservation constraint, that is, the incoming flow of any relay satellite during T is equivalent to its outgoing flow. (5) describes that the sum of the data transmission $E_{v_i}^{\Delta}$ and inherent $E_{v_i}^*$ energy consumption in any satellite v_i should be no more than its total energy $\mathcal{E}_{v_i}$. With reference to [4], $E_{v_i}^*$ indicates the energy consumed by the electronic circuit, which can be assumed proportional to T at the given power. $E_{v_i}^{\Delta}$ is dependent on the transmitted network flow, and can be represented as a linear function of flow size. More precisely, we have

$$E_{s_1}^{\Delta} = \rho_s \cdot \sum\nolimits_{h=1}^{k} \sum\nolimits_{v_j^h \in \mathcal{V} - \mathcal{V}_s} f_{s_1^h, v_j^h}, \tag{6}$$

for the source satellite and

$$E_{r_i}^{\Delta} = (\rho_s + \rho_r) \cdot \sum\nolimits_{h=1}^{k} \sum\nolimits_{v_j^h \in \mathcal{V} - \mathcal{V}_{r_i}} f_{r_i^h, v_j^h}, \tag{7}$$

for each relay satellite, where constants ρ_s and ρ_r denotes the energy consumed for sending and receiving unit bit network flow respectively, as determined by carrier frequency, transmission rate, bit error rate, etc [6].

According to the energy constraint (5), the total energy of each satellite could be mapped to the maximum network flow that can be delivered. For s_1, we can obtain

$$\sum\nolimits_{h=1}^{k} \sum\nolimits_{v_j^h \in \mathcal{V} - \mathcal{V}_s} f_{s_1^h, v_j^h} \leq (\mathcal{E}_{s_1} - E_{s_1}^*)/\rho_s, \tag{8}$$

while the flow forwarded by any r_i should meet

$$\sum\nolimits_{h=1}^{k} \sum\nolimits_{v_j^h \in \mathcal{V} - \mathcal{V}_{r_i}} f_{r_i^h, v_j^h} \leq (\mathcal{E}_{r_i} - E_{r_i}^*)/(\rho_s + \rho_r). \tag{9}$$

III. Temporal Graph Based Max-flow Routing

A. Energy Time-expanded Graph

To overcome the bottleneck that the original TEG fails to depict the restriction of each satellite's limited energy on its flow transmission capability, we design the eTEG = $(\mathcal{V}^*, \mathcal{A}^*, \mathcal{C}^*, \mathcal{S}, \mathcal{E}^*)$ by introducing some energy-related virtual vertices and arcs, as shown in Fig. 3. The construction process is summarized as follows.

- **Temporal graph representation of energy constraints**. For each $s_1^h \in \mathcal{V}_s$, $\widehat{s}_1^h$ and $(s_1^h, \widehat{s}_1^h)$ are introduced to represent sending energy constraints, with the initial energy-to-capacity set as $\mathcal{E}_{s_1^h, \widehat{s}_1^h}^* = (\mathcal{E}_{s_1} - E_{s_1}^*)/\rho_s$. With respect to any $r_i^h \in \mathcal{V}_r$, not only $\widehat{r}_i^h$ and $(r_i^h, \widehat{r}_i^h)$, but also $\widetilde{r}_i^h$ and $(\widetilde{r}_i^h, r_i^h)$ are brought in. From (9), it follows that $\mathcal{E}_{r_i^h, \widehat{r}_i^h}^* = \mathcal{E}_{\widetilde{r}_i^h, r_i^h} = (\mathcal{E}_{r_i} - E_{r_i}^*)/(\rho_s + \rho_r)$ holds. For simplicity, sets $\mathcal{V}_t = \{\widehat{v}_i^h | v_i^h \in \mathcal{V}_s \bigcup \mathcal{V}_r\}$, $\mathcal{V}_a = \{\widetilde{v}_i^h | v_i^h \in \mathcal{V}_r\}$, $\mathcal{A}_s = \{(v_i^h, \widehat{v}_i^h) | v_i^h \in \mathcal{V}_s \bigcup \mathcal{V}_r, \widehat{v}_i^h \in \mathcal{V}_t\}$ and $\mathcal{A}_r = \{(\widetilde{v}_i^h, v_i^h) | v_i^h \in \mathcal{V}_r, \widetilde{v}_i^h \in \mathcal{V}_a\}$ are constructed, satisfying $\mathcal{V}^* = \mathcal{V}_s \bigcup \mathcal{V}_r \bigcup \mathcal{V}_g \bigcup \mathcal{V}_t \bigcup \mathcal{V}_a$.
- **Re-characterization of communication links**. We remove each $(v_i^h, v_j^h) \in \mathcal{A}_t$ and introduce $(\widehat{v}_i^h, \widetilde{v}_j^h)$ to represent the communication link between satellite v_i and v_j, whose capacity meets $\mathcal{C}_{\widehat{v}_i^h, \widetilde{v}_j^h}^* = \mathcal{C}_{v_i^h, v_j^h}$. Without loss of generality, the arc set $\mathcal{A}_e = \{(\widehat{v}_i^h, \widetilde{v}_j^h) | 1 \leq h \leq k\}$ is defined and $\mathcal{A}^* = \mathcal{A}_c \bigcup \mathcal{A}_s \bigcup \mathcal{A}_r \bigcup \mathcal{A}_e$ is obtained.

B. eTEG-based Max-flow Routing Algorithm

We propose an eTEG-based max-flow routing algorithm to jointly schedule the flow transmission and energy utilization. Specifically, some definitions are given first.

*1) **Residual Network**:* For any eTEG with a network flow $f > 0$ transmitting through, the residual network $G_f = \{\mathcal{V}^*, \mathcal{A}_f^*, \mathcal{C}_f^*, \mathcal{S}_f, \mathcal{E}_f^*\}$ can be constructed by following steps:

- **Add reverse arcs**. For any $(v_i^h, v_j^l) \in \mathcal{A}^*$ delivering flow $f_{v_i^h, v_j^l} = f$, both (v_i^h, v_j^l) and (v_j^l, v_i^h) are added into $\mathcal{A}_f^*$ to support the adjustment of $f_{v_i^h, v_j^l}$.
- **Update the capacities**. For any $(v_i^h, v_j^l) \in \mathcal{A}^*$ with flow $f_{v_i^h, v_j^l} = f$, the residual capacities on (v_i^h, v_j^l) and the reverse counterpart (v_j^l, v_i^h) are updated as follows:

$$\begin{cases} \mathcal{C}_{f(v_i^h, v_j^l)}^* = \mathcal{C}_{v_i^h, v_j^l}^* - f, & (v_i^h, v_j^l) \in \mathcal{A}_a \bigcup \mathcal{A}_e, \\ \mathcal{C}_{f(v_j^l, v_i^h)}^* = \mathcal{C}_{v_j^l, v_i^h}^* + f, & (v_i^h, v_j^l) \in \mathcal{A}_a \bigcup \mathcal{A}_e, \\ \mathcal{S}_{f(v_i^h, v_j^l)} = \mathcal{S}_{v_i^h, v_j^l} - f, & (v_i^h, v_j^l) \in \mathcal{A}_c, \\ \mathcal{S}_{f(v_j^l, v_i^h)} = \mathcal{S}_{v_j^l, v_i^h} + f, & (v_i^h, v_j^l) \in \mathcal{A}_c, \\ \mathcal{E}_{f(v_i^h, v_j^l)}^* = \mathcal{E}_{v_i^h, v_j^l}^* - f, & (v_i^h, v_j^l) \in \mathcal{A}_s \bigcup \mathcal{A}_r, \\ & \forall\, 1 \leq h = l \leq k, \\ \mathcal{E}_{f(v_j^l, v_i^h)}^* = \mathcal{E}_{v_j^l, v_i^h}^* + f, & (v_i^h, v_j^l) \in \mathcal{A}_s \bigcup \mathcal{A}_r. \end{cases}$$

Note that since all the replicates v_i^h $(1 \leq h \leq k)$ correspond to one satellite v_i, their energy resources are shared, that is, the energy consumed by any replicate would reduce the available energy of others. Hence, once any $(v_i^h, \widehat{v}_i^h) \in \mathcal{A}_s$ (or $(\widetilde{v}_i^h, v_i^h) \in \mathcal{A}_r$) delivers flow, the capacities on $\{(v_i^l, \widehat{v}_i^l) | 1 \leq l \leq k\}$ (or $\{(\widetilde{v}_i^l, v_i^l) | 1 \leq l \leq k\}$) should also be modified.

Definition 1: For a given G_f, a temporal augmenting path $\mathcal{P}$ is an acyclic path from s_1^1 to g_1^k with the capacity on each arc greater than 0.

Different from static paths, $\mathcal{P}$ can support the across-time flow transmission with vertices' caching ability. To measure

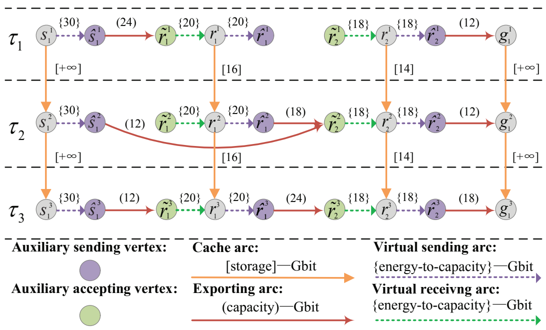


Fig. 3: eTEG for modeling energy constraints.

the maximum flow size being delivered through $\mathcal{P}$, the feasible flow metric is also defined.

Definition 2: For given $\mathcal{P}$, its feasible flow $f(\mathcal{P})$ is the minimum value of the capacities on all arcs in $\mathcal{P}$.

*2) **eTEG-based Max-flow Algorithm**:* With the defined residual network and temporal augmenting path, an eTEG-based max-flow algorithm is proposed to acquire the energy-limited maximum flow. The basic idea behind is continually seeking the temporal augmenting paths from s_1^1 to g_1^k and accumulating the feasible flow of those paths to the maximum, also with a time-dependent residual network update process. The detailed algorithm procedure is listed in Algorithm 1.

Algorithm 1 eTEG-based max-flow algorithm

1: **Input:** eTEG = $\{\mathcal{V}^*, \mathcal{A}^*, \mathcal{C}^*, \mathcal{S}, \mathcal{E}^*\}$.
2: **Output:** The maximum flow f_m from s_1^1 to g_1^k.
3: Initialize $n=1$, $f_m=0$, $f_{v_i^h, v_j^l}=0$ for $\forall\ (v_i^h, v_j^l) \in \mathcal{A}^*$, and $G_f^{(n)}$ = eTEG;
4: **repeat**
5: **Apply** the depth-first-search algorithm in $G_f^{(n)}$ to obtain a temporal augmenting path $\mathcal{P}^{(n)}$;
6: **Calculate** the feasible flow $f(\mathcal{P}^{(n)})$ and add it to f_m;
7: **Update** $G_f^{(n)}$ to $G_f^{(n+1)}$ through the residual network update process in subsection III-B;
8: **Modify** $n \leftarrow n+1$;
9: **until** no temporal augmenting path exists in $G_f^{(n)}$.

Remark 1: For the given eTEG with $|\mathcal{V}^*|$ vertices and $|\mathcal{A}^*|$ arcs, the complexity of the eTEG-based max-flow algorithm is $O((|\mathcal{V}^*|+|\mathcal{A}^*|)f_m)$, where f_m is the maximum flow.

IV. NUMERICAL RESULTS

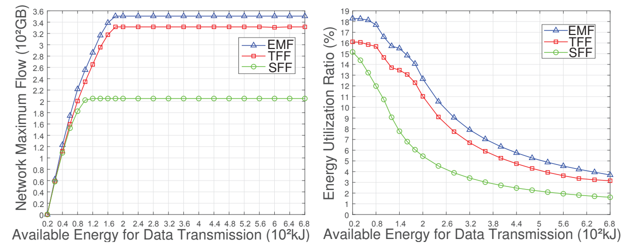
In this section, we evaluate the performance of our proposed algorithm. Simulations are conducted over Telesats constellation [7], selecting 6 polar orbits with 2 satellites on each orbital plane. A ground station at Kashi (39.5°N, 76°E) is specified to collect data. Simulation parameters are listed in Table.I, and the performance is evaluated by the following two metrics:

- *Network Maximum Flow* : the maximum flow transmitted from the source satellite to the ground station, i.e., f_m.
- *Energy Utilization Ratio* : the ratio of energy consumed by satellites for flow delivery to the total available energy for data transmission, i.e., $\alpha = \sum_{v_i} E_{v_i}^{\Delta} / \sum_{v_i} (\mathcal{E}_{v_i} - E_{v_i}^*)$.

We compare our algorithm with the method (marked as TFF) based on both original TEG and static Ford-Fulkerson algorithm in [5], and the method (marked as SFF) based on snapshots graph [8]. The simulation results in Fig. 4 shows that with the increase of energy, f_m obtained from each method rises and and gradually saturates, while α keeps decreasing. The reason is that more energy resources enable satellites to transmit more data, but link capacities restrict the upper bound of throughput. Specifically, simulation reveals that EMF is superior in both flow transmission and energy utilization.

TABLE I

Parameter	Value
planning time horizon	4 hours (2020/12/15 4:00 - 8:00)
time slot duration	30 seconds
date rate	400, 600 and 800 Mbps
buffer size in each satellite	10 GB
power of OS and circuit	50 Watt
ρ_s and ρ_r	0.04 Joule/Mb and 0.01 Joule/Mb
available energy of each satellite	0 - 680 kJ



(a) Network maximum flow. (b) Energy utilization ratio.

Fig. 4: The curves of the f_m and α versus the available energy.

V. CONCLUSION

We studied the end-to-end max-flow problem over energy-limited satellite networks. The TEG was first adopted to represent the heterogeneous network resources, followed by the problem formulation. Then, we extended the original TEG to eTEG, mapping each satellite's energy constraint to the capacity upper bound of the introduced virtual arc. Moreover, an eTEG-based max-flow routing algorithm with the idea of temporal augmenting path was proposed to achieve the maximum throughput. Simulation results demonstrate the efficacy of the proposed algorithm.

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