Distributed Spectrum Management in TV White Space Cognitive Radio Networks

Jocelyne Elias
LIPADE
Université Paris Descartes
Email: jocelyne.elias@parisdescartes.fr

Marwan Krunz
ECE Department
University of Arizona
Email: krunz@email.arizona.edu

***Abstract*—In this paper, we investigate the spectrum management problem in TV White Space (TVWS) Cognitive Radio Networks using a game theoretical approach, accounting for adjacent-channel interference. TV Bands Devices (TVBDs) compete to access available TV channels and choose idle blocks that optimize some objective function. Specifically, the goal of each TVBD is to minimize the *price* paid to the Database operator and a *cost* function that depends on the interference between unlicensed devices. We show that the proposed TVWS management game admits a potential function under general conditions. Accordingly, we use a Best Response algorithm to converge in few iterations to the Nash Equilibrium (NE) points.**

We evaluate the performance of the proposed game, considering both static and dynamic TVWS scenarios and taking into account users' mobility. Our results show that at the NE, the game provides an interesting tradeoff between efficient TV spectrum use and reduction of interference between TVBDs.

Keywords*—*TV White Space, Database, TV bands devices, Guard bands, Spectrum management, Game theory, Nash equilibrium

I. Introduction

The Radio Frequency (RF) spectrum is a scarce resource in wireless communication systems, and the demand for this resource has been growing rapidly with the dramatic development of the mobile telecommunication industry in the last decades. For this reason, the Federal Communications Commission (FCC) has recently made the unused spectrum in the TV bands (also called "White Space") available for unlicensed broadband wireless devices. The opening of these bands for unlicensed use has significant benefits for both businesses and consumers, permitting considerable improvement in spectrum efficiency [1], [2].

TV White Space (TVWS) spectrum access is often designed without taking into account Adjacent-Channel Interference (ACI), which can occur between different transmissions of TV Bands Devices (TVBDs). Guard Bands (GBs) can be used to protect TVBDs' transmissions and mitigate the ACI problem. Two methods are discussed in the literature for managing ACI in classical channel assignment problems: (i) *new* guard bands are added to protect a transmission (this is referred to as *no GB reuse* model), or (ii) already assigned GBs can be reused to protect new arriving transmissions (*GB reuse* model). According to [3], the "GB reuse" model is suitable for Discontinuous Orthogonal Frequency Division Multiplexing (D-OFDM) systems, while the "no GB reuse" model is more appropriate for FDM-based ones.

We consider a Cognitive Radio Network (CRN) in which every TVBD is equipped with a single antenna that can be tuned to any combination of c consecutive licensed channels. This can be done by the OFDM technique with adaptive and selective allocation of OFDM subcarriers to utilize any subset of c licensed channels at the same time [4], [5]. In this work, we adopt the GB reuse model. Figure 1 illustrates an example of a channel assignment with GB reuse. Guard bands 4 and 6 are used to protect 3 independent transmissions that occupy channels 2 and 3, 5, and 7, respectively.

However, introducing GBs can significantly impact the effective use of the TV spectrum. This motivates our present work, which investigates the TV spectrum management problem while accounting for guard bands.

According to the FCC specifications [1], [2], each TVBD must contact the database to obtain the list of available TV channels and then decide which ones to use in order to maximize its own utility. This utility can be expressed as a function of interference or/and congestion. However, if TVBDs are located in the same area and are using the same TV channels, they may interfere with each other. Game theory is the natural framework to address the conflicts between such self-interested devices (or players), and the Nash Equilibrium (NE) is a well-suited concept to characterize system-wide equilibrium conditions. In this context, the NE is a set of devices' choices, such that none of the TVBDs has an incentive to deviate from unilaterally.

Therefore, in this work we address the TV spectrum management problem taking into account guard bands and considering a non-cooperative game theoretical approach, where unlicensed TVBDs (fixed or portable) choose the idle channel blocks that optimize their objective functions, which are expressed as a function of a price paid to the Database Operator

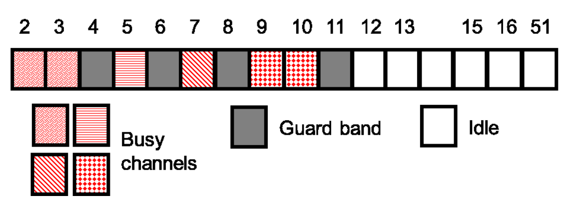


Fig. 1: An example of a channel assignment in the TV spectrum according to the GB reuse model. Guard bands 4 and 6 are used to protect 3 independent transmissions that are going on channels 2 and 3, 5, and 7, respectively.

ISBN 978-3-901882-68-5

(DO) for using TV bands and a congestion cost incurred by the TVBD for interfering with other coexisting devices that operate over the same blocks.

Based on a number of experimental studies previously conducted to highlight the benefits of channel bonding and aggregation (use of multiple contiguous and non-contiguous channels) in the context of cognitive radio networks (e.g., [3], [6], [7]), we assume that the Database operator will provide the TVBDs with the list of idle channels and guard bands, and then TVBDs will be able to implement some aggregation/bonding techniques on the TV available bands, choosing the idle blocks that optimize their objective functions.

The main contributions of this paper are as follows:

- We tackle the TV spectrum management problem taking into account unlicensed devices' characteristics and ACI to reduce interference among different devices' transmissions by means of guard bands.
- We formulate the TV spectrum management as a non cooperative guard band-aware spectrum management game, where TVBDs choose the idle blocks that optimize their objective functions, expressed as a function of a price set by the DO and a congestion cost.
- Under specific conditions, detailed in Section IV, we demonstrate that our game is potential, and hence admits at least one pure NE and that the Best Response algorithm converges to a NE.
- We perform thorough numerical evaluation of the proposed game, considering both static and dynamic TVWS scenarios, and show that our game provides a promising solution for managing TV resources in a distributed manner.

The remainder of this paper is organized as follows. Section II discusses related work. Section III introduces the system model as well as the notation and assumptions considered in our work. Section IV describes the game theoretical approach for the GB-aware TV spectrum management problem, while Section V illustrates and analyzes numerical results that show the effectiveness and validity of our approach. Finally, concluding remarks and future research directions are discussed in Section VI.

II. Related Work

In this section, we discuss the most relevant works that deal with the TVWS channel assignment problem [8], [9], [10], [11].

In [8], the authors conduct a game theoretic analysis of a distributed spectrum sharing scheme with a geo-location database. They consider a TVWS system composed of access points (APs) and secondary users (SUs). They first model the channel selection problem among the APs as a distributed AP channel selection game and then propose a state-based game framework to model the distributed association of SUs to APs taking into account the cost of mobility.

Two pricing schemes (i.e., registration and service plan) for TVWS database are proposed in [9]. The Database Operator (DO) offers two pricing schemes in order to maximize its benefit (payment received from all SUs $-$ the total cost). Then SUs access the available TV channels maximizing their utilities, which are expressed as a function of the Shannon capacity. Registered SUs pay a registration fee to the DO during registration and exclusively access the reserved bandwidth. Conversely, unregistered SUs query the database only when they are in need of TV spectrum, and the DO charges them according to the number of database queries they make. The authors in [9] model the competition among SUs as a non-cooperative game and prove the existence of the Nash equilibrium under both complete and incomplete information cases. Finally, they propose distributed algorithms to determine TV channel allocations and pricing parameters.

A system composed of primary (TV base stations) and secondary users is considered in [10]. The goal is to maximize the network utility subject to a set of weighted power budget constraints and individual interference temperature constraints. A game theoretical framework for TV spectrum allocation in a cognitive cellular network is proposed in [11]. White base stations should decide in a non-cooperative manner which TV channels to select in order to minimize the sum of the inverse of signal-to-interference-plus-noise ratios of their associated terminals. The distributed TV channel allocation problem is addressed using the notion of congestion game and virtual resources. Finally, in [12] the authors introduce a spectrum broker, which is an entity that builds on the concept of frequency coordinators. The operational goal of the broker is to achieve robust technical protection of the incumbent, quality of service provisioning to the players, and spectrum trading revenue maximization.

To the best of our knowledge, our work provides the first attempt to (i) introduce aggregation and bonding techniques in the TVWS spectrum management problem in order to assign idle blocks to TVBDs guaranteeing their rate demands, and (ii) to deal with such problem using a non-cooperative game among TVBDs, which aim to maximize their objective functions which are expressed as a function of a price set by the DO and an interference-based congestion function.

III. System Model

In this section, we describe the system model and the notation used in this paper. We consider the TVWS scenario illustrated in Figure 2, with one TV bands database which is administrated by a third party operator (called Database Operator, DO) and a set $\mathcal{N}$ of unlicensed devices called TV Bands Devices (TVBDs). The set of potentially-available TV channels consists of channels 2 to 51 (except channels 3, 4 and 37) in the case of fixed TV bands devices, while it contains TV channels 21 to 51 (except channel 37) for personal/portable ones.

According to FCC, TVBDs incorporate a geo-location capability and a means to access the database that provides information about TV channels [1], [2]. Based on such information, the TVBD can determine the current channels' status; the set of available (or idle) channels, guard bands and occupied channels. Nonetheless, the TVBD can also perform

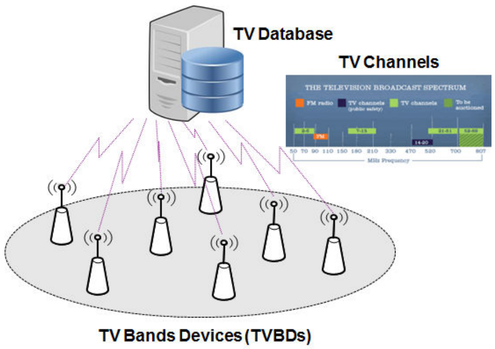


Fig. 2: TV White Space Scenario: A Database Operator (DO) providing the set of available TV channels for TV Bands Devices (TVBDs).

spectrum sensing to determine the spectrum utilization when there is no available database. Therefore, in the rest of this section, we assume that first the DO gives to all TVBDs the set of available, guard band and occupied channels, and then, based on such information, each TVBD i defines a set of *Idle frequency Blocks* (denoted as $\mathcal{IB}_i$), by using channel bonding and aggregation techniques. An idle frequency block consists of a set of contiguous idle TV channels.

In Figure 3, we illustrate an example with a subset of TV channels (from channel 5 to 22). We classify these channels into three subsets: idle, busy and guard band channels. In this particular example, since channels 8, 10, 16 and 17 are busy and channels 7, 9, 11, 15 and 18 are guard bands, we can build the set of idle blocks such that idle block IB_1 contains two idle channels 5 and 6, while IB_2 and IB_3 contain, respectively, idle channels $\{12, 13, 14\}$, and $\{19, 20, 21, 22\}$.

Let $\mathcal{M}$ and $\mathcal{G}$ denote the set of available TV channels and guard bands, respectively, and B the TV channel bandwidth which is the same for all channels. We denote by $\mathcal{M}_i$ ($\mathcal{M}_i \subseteq \mathcal{M}$) the set of available channels for TVBD i.

Let us denote by d_i the rate demand (in Mbps) of TVBD i, and by r the rate (in Mbps) supported by each TV channel. We assume here that all TV channels support the same rate.

Finally, to determine the size of an idle block (the number of idle TV channels belonging to the block), it suffices to define a $|\mathcal{M}| \times |\mathcal{IB}_i|$ matrix $\boldsymbol{A}$, whose element $a_{cb} = 1$ states that channel $c \in \mathcal{M}$ belongs to idle block $b \in \mathcal{IB}_i$, and 0 otherwise. Then, the size of idle block b (denoted as s_b) is: $s_b = \sum_{c\in\mathcal{M}} a_{cb}$.

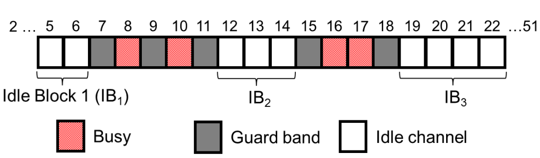


Fig. 3: An example illustrating a set of idle frequency blocks in the TV spectrum. Idle block IB_1 contains idle channels 5 and 6, while IB_2 and IB_3 contain, respectively, idle channels $\{19, 20, 21, 22\}$ and $\{12, 13, 14\}$, and $\{19, 20, 21, 22\}$.

IV. Game Theoretical TVWS Spectrum Management Approach

Given the above definitions and notation, in this section, we address the *guard band-aware TVWS* spectrum management problem in a fully distributed fashion using a game theoretical approach.

We introduce binary decision variables $x_{ib}, \forall i \in \mathcal{N}, b \in \mathcal{IB}_i$ defined as follows:

$$x_{ib} = \begin{cases} 1 & \text{if TVBD } i\text{'s transmission is assigned to} \\ & \text{idle block } b \\ 0 & \text{otherwise} \end{cases}$$

These variables represent the set of spectrum access strategies of TVBD i, i.e., $\boldsymbol{x}_i = \{x_{i1}, x_{i2}, \ldots, x_{i|\mathcal{IB}_i|}\}$.

We denote by E_b (which needs not be symmetric) the interference matrix associated with idle block b, and by $e_{ik,b}$, element of E_b, the interference parameter between TVBDs i and k on idle block b. More specifically, $e_{ik,b}$, with $i, k \in \mathcal{N}, b \in \mathcal{IB}_i \cap \mathcal{IB}_k$ is defined as follows:

$$e_{ik,b} = \begin{cases} 1 & \text{if TVBD } i \text{ interferes with TVBD } k \text{ on} \\ & \text{idle block } b \\ 0 & \text{otherwise} \end{cases}$$

We propose that the database operator sets a pricing scheme for TV channels' usage to provide incentives for TVBDs to share these channels in an efficient manner. Therefore, we define a (per user) price p_i, which is expressed as a function of both the *number* and *size* of frequency blocks chosen by the i-th TVBD: $p_i = a_i \cdot (\sum_{b\in\mathcal{IB}_i} r s_b x_{ib})^{\tau_i} + c_i$, where a_i, τ_i and c_i are TVBD i-dependent parameters. Note that if TVBDs are homogeneous, then these parameters (or the pricing scheme) are the same for all TVBDs. This pricing scheme extends the basic one proposed in [13] for spectrum sharing in cognitive radio networks.

The basic notation used throughout the paper is summarized in Table I.

TABLE I: Basic Notation

$\mathcal{N}$	set of TV Bands Devices (TVBDs)
$\mathcal{M}$	set of available (or idle) TV channels
$\mathcal{G}$	set of guard bands
$\mathcal{M}_i$	set of available TV channels for TVBD i; $\mathcal{M}_i \subseteq \mathcal{M}$
$\mathcal{IB}_i$	set of Idle frequency Blocks of TVBD i
s_b	size of (or number of TV channels belonging to) idle block b
p_i	Price charged by DO to TVBD i for acquiring idle blocks
B	Bandwidth of each TV channel
r	Rate (Mbps) supported by each TV channel
d_i	Rate demand (Mbps) of TVBD i
x_{ib}	0-1 variable that indicates if TVBD i is assigned to idle block b

A. Objective function of the TVBD

Having defined above the pricing scheme, in this section we introduce the objective function of the TVBD. Let us begin by illustrating the *cost function* J_i of TVBD $i, \forall i \in \mathcal{N}$.

The first component of the cost function is the *total price* that TVBD i must pay when transmitting over a subset of idle blocks, and is given by:

$$a_i \cdot (\sum_{b \in \mathcal{IB}_i} rs_b x_{ib})^{\tau_i} + c_i,$$

while the second component of the *cost function* represents a congestion cost that the device incurs due to the *interference* with other devices on the same block b.

$$\sum_{b \in \mathcal{IB}_i} rs_b x_{ib} \cdot [\alpha_b \cdot (\sum_{j \in \mathcal{N}} rs_b e_{ji,b} x_{jb})^{\beta_b} + \gamma_b],$$

In this case, all TVBDs need to exchange between each other their choices (x_{jb}) in order to compute the current value of the total interference.

The second component of TVBD i's cost function depends on the total number of its neighboring TVBDs sharing the same idle block b, where α_b, β_b and γ_b are frequency block-dependent positive real parameters. This cost represents a penalty due to interference, it is increasing in the number of TVBDs sharing the same block, and is used to incite them to choose free or underutilized blocks.

On one hand, the price p_i is introduced to ensure an efficient utilization of the spectrum (with the GB reuse model) by encouraging TVBDs to choose simultaneously a limited number and small size idle blocks. On the other hand, the congestion cost is proposed to discourage TVBDs from choosing "crowded" frequency blocks, thus reducing the interference. It is worth noting that there is a tradeoff between minimizing the number of chosen idle blocks (paying a lower p_i), and minimizing the interference with other TVBDs, searching for extra idle blocks.

Hence, the goal of TVBD i is to minimize the following cost function:

$$\begin{aligned} J_i = a_i \cdot (\sum_{b \in \mathcal{IB}_i} rs_b x_{ib})^{\tau_i} + c_i \quad (1) \\ + \sum_{b \in \mathcal{IB}_i} rs_b x_{ib} \cdot [\alpha_b \cdot (\sum_{j \in \mathcal{N}} rs_b e_{ji,b} x_{jb})^{\beta_b} + \gamma_b] \end{aligned}$$

According to the used application's QoS requirements, we can have two types of TV bands devices:

- TVBDs with given traffic demands: the goal of the TVBD is to minimize its own cost J_i.
- Devices with elastic traffic: the goal of the devices is to maximize the difference between their utilities and costs.

In this work, we focus on *elastic* TVBDs and we introduce in the following their utility functions, which are affine as a function of TVBDs' strategies.

$$U_i = \sum_{b \in \mathcal{IB}_i} \delta_{ib} rs_b x_{ib}, \quad (2)$$

where δ_{ib} is a positive parameter that represents the worth for TVBD i for using block b. Hence, the objective function that (elastic) TVBD i aims to maximize is given by $OF_i = U_i - J_i$, subject to
data rate constraint:

$$\sum_{b \in \mathcal{IB}_i} rs_b x_{ib} \geq d_i, \quad (3)$$

and integrality constraint:

$$x_{ib} \in \{0, 1\}, \forall b \in \mathcal{IB}_i \quad (4)$$

B. Potential Function

Hereafter we demonstrate that our TVWS spectrum management game admits a potential function Φ (under some conditions to be specified later). Indeed, if a potential function exists, then the game is *potential* [14], and possesses at least one pure Nash equilibrium. Hence, a Best Response algorithm can be used to achieve a NE.

Proposition IV.1. *The Guard Band-aware TVWS spectrum management game admits a potential function Φ which is given by the following expression:*

$$\begin{aligned} \Phi = \sum_{i \in \mathcal{N}} \big[U_i - \frac{1}{2} J_i - \frac{a_i}{2} \cdot (\sum_{b \in \mathcal{IB}_i} rs_b x_{ib})^{\tau_i}\big] \quad (5) \\ - \frac{1}{2} \sum_{i \in \mathcal{N}} \sum_{b \in \mathcal{IB}_i} \big[\alpha_b r^2 s_b^2 x_{ib}^2 + rs_b \gamma_b x_{ib}\big] \end{aligned}$$

Proof: Let us assume that $c_i = 0$, $\beta_b = 1$, and $e_{ki,b} = e_{ik,b}$, $\forall i, k \in \mathcal{N}, b \in \mathcal{IB}_i \cap \mathcal{IB}_k$, i.e., the interference between TVBDs is symmetric. Furthermore, we assume that all TVBDs have the same set of idle blocks ($\mathcal{IB}_i = \mathcal{IB}_k, \forall i, k \in \mathcal{N}$).

We recall that the function Φ is a *potential function* if it satisfies (for each player i, each multi-strategy $x = \{x_1, \ldots, x_i, \ldots, x_{|\mathcal{N}|}\} = \{x_i\} \cup \{x_{-i}\}$ and each strategy y_i) the following condition:

$$\Phi(x_i, x_{-i}) - \Phi(y_i, x_{-i}) = OF_i(x_i, x_{-i}) - OF_i(y_i, x_{-i}) \quad (6)$$

The function Φ has the following expression:

$$\begin{aligned} &\Phi(x_{ib}, x_{-ib}) \quad (7) \\ &= \sum_{k \in \mathcal{N}: k \neq i} U_k(x_{kb}) + U_i(x_{ib}) - \sum_{k \in \mathcal{N}} a_k \cdot (\sum_{b \in \mathcal{IB}_k} rs_b x_{kb})^{\tau_k} \\ &- \frac{1}{2} \sum_{k \in \mathcal{N}} \sum_{b \in \mathcal{IB}_k} rs_b x_{kb} \cdot [\alpha_b \cdot (\sum_{j \in \mathcal{N}} rs_b e_{jk,b} x_{jb}) + \gamma_b] \\ &- \frac{1}{2} \sum_{k \in \mathcal{N}} [\alpha_b (rs_b x_{kb})^2 + rs_b \gamma_b x_{kb}] \end{aligned}$$

$$\begin{aligned}
&\Phi(x_{ib}, x_{-ib}) \quad (8)\\
&= \sum_{k\in\mathcal{N}:k\neq i} U_k(x_{kb}) + U_i(x_{ib})\\
&\quad - \sum_{b\in\mathcal{IB}_k} \frac{\alpha_b r^2 s_b^2}{2}\Big\{ \sum_{k,j\in\mathcal{N}:k,j\neq i} e_{jk,b} x_{jb} x_{kb}\\
&\quad + \sum_{k\in\mathcal{N}:k\neq i} e_{ik,b} x_{ib} x_{kb}\\
&\quad + \sum_{j\in\mathcal{N}:j\neq i} e_{ji,b} x_{jb} x_{ib} + \sum_{k\in\mathcal{N}:k\neq i} 2x_{kb}^2 + 2x_{ib}^2 \quad \Big\}\\
&\quad - \sum_{k\in\mathcal{N}:k\neq i} \sum_{b\in\mathcal{IB}_k} r s_b \gamma_b x_{kb} - \sum_{b\in\mathcal{IB}_i} r s_b \gamma_b x_{ib}
\end{aligned}$$

Since the interference between TVBDs is symmetric ($e_{ki,b} = e_{ik,b}$, $\forall i,k \in \mathcal{N}, b \in \mathcal{IB}_i \cap \mathcal{IB}_k$), we can further simplify the expression of the function Φ:

$$\begin{aligned}
&\Phi(x_{ib}, x_{-ib}) \quad (9)\\
&= \sum_{k\in\mathcal{N}:k\neq i} U_k(x_{kb}) + U_i(x_{ib})\\
&\quad - \sum_{b\in\mathcal{IB}_k} \frac{\alpha_b r^2 s_b^2}{2}\Big\{ \sum_{k,j\in\mathcal{N}:k,j\neq i} e_{jk,b} x_{jb} x_{kb}\\
&\quad + 2 \sum_{k\in\mathcal{N}:k\neq i} e_{ki,b} x_{ib} x_{kb} + \sum_{k\in\mathcal{N}:k\neq i} 2x_{kb}^2 + 2x_{ib}^2 \quad \Big\}\\
&\quad - \sum_{k\in\mathcal{N}:k\neq i} \sum_{b\in\mathcal{IB}_k} r s_b \gamma_b x_{kb} - \sum_{b\in\mathcal{IB}_i} r s_b \gamma_b x_{ib}
\end{aligned}$$

We can verify that any unilateral deviation of TVBD i on a block b is exactly equal to the difference in function Φ. In fact, the following equality holds:

$$\begin{aligned}
&\Phi(x_{ib}, x_{-ib}) - \Phi(y_{ib}, x_{-ib}) \quad (10)\\
&= U_i(x_{ib}) - U_i(y_{ib})\\
&\quad - a_i \cdot (\sum_{b\in\mathcal{IB}_i} r s_b x_{ib})^{\tau_i} + a_i \cdot (\sum_{b\in\mathcal{IB}_i} r s_b y_{ib})^{\tau_i}\\
&\quad - \sum_{b\in\mathcal{IB}_i} \left[\alpha_b r^2 s_b^2 (x_{ib}^2 - y_{ib}^2) - r s_b \gamma_b (x_{ib} - y_{ib})\right]\\
&\quad - \sum_{b\in\mathcal{IB}_i} \Big(\sum_{k\in\mathcal{N},k\neq i} \alpha_b r^2 s_b^2 e_{ki,b} x_{kb} \quad \Big)(x_{ib} - y_{ib})\\
&= OF_i(x_{ib}, x_{-ib}) - OF_i(y_{ib}, x_{-ib}).
\end{aligned}$$

Hence we prove that Equations (5) and (6) hold, and that the guard band-aware TVWS spectrum management game admits Φ as potential function. □

C. Best Response Algorithm

The best response of a player (or a TVBD) is an action that maximizes its objective function for a given action tuple of the other players, subject to the constraints (3)-(4).

Definition IV.1. *BR_i is a best response by player i to x_{-i} if*

$$BR_i = argmax_{x_i \in X_i} OF_i(x_i, x_{-i}), \quad (11)$$
s.t. constraints (3)-(4)

The same procedure is repeated for all users in the network, and such procedure converges to a Nash equilibrium of our spectrum management game.

V. Numerical Results

In this section, we measure the sensitivity of our approach to different parameters, like the number of TVBDs and TV channels, the interference between TVBDs as well as the rate demands in several network scenarios. We first describe the simulation setup, and then we analyze and discuss the numerical results.

The performance metrics we consider are (1) the TVBD objective function ($OF_i = U_i - J_i$), and (2) the Price of Anarchy (PoA), which is defined in our context as the ratio between the utility of the socially optimal solution and that of the worst Nash equilibrium [15].

A. Simulation Setup

In our simulations, we consider a TV white space system composed of M TV channels and N TVBDs randomly scattered over a 1000 meter × 1000 meter area. The bandwidth of each TV channel is 8 MHz, and the rate r supported by each TV channel is 10 Mb/s.

We simulate both static and dynamic TVWS scenarios. For static scenarios, we assume that TVBDs are fixed, with a maximum transmission power P_{max}=0.1 Watt, while for the dynamic case, we assume that TVBDs are mobile (i.e., personal or portable unlicensed devices), with P_{max}= 40 mWatt. For the sake of comparison, we assume that fixed and portable unlicensed devices have the same TV spectrum bands. Therefore, the set of TV channels is $\{21, \ldots, 51\} \setminus \{37\}$, unless otherwise stated. Figure 4 illustrates an example of the TV spectrum that we consider in our numerical analysis. We further assume a free-space path loss model between any two unlicensed devices.

All the results reported hereafter are the Nash equilibria and optimal solutions of the considered scenarios obtained, respectively, by formalizing the TVWS management algorithm in OPL, and solving them with CPLEX [16].

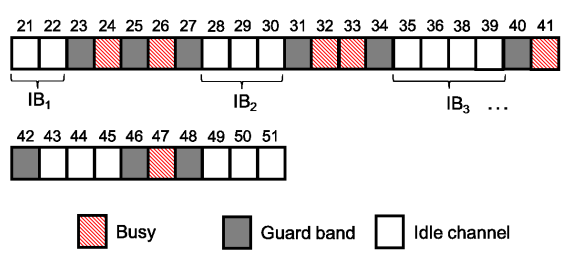


Fig. 4: Set of TV channels ($\{21, \ldots, 51\} \setminus \{37\}$) and set of idle blocks considered in the numerical analysis.

B. Performance Evaluation

In the following, we measure the effect of the number of TVBDs, the number of available TV channels as well as the rate demands on the performance of the proposed TVWS spectrum management approach. We first discuss the results obtained in the static scenario and then those of the dynamic one, where TVBDs are personal or portable.

1) ***Static TVWS scenario****:* In this subsection we consider a static TVWS scenario. We fix the transmission power to 20 dBm, the TVBDs' rate demands d_i are homogeneous and equal to 20 Mb/s. Parameters a_i, τ_i and c_i are set to 1, 2 and 0, respectively, for all players, while α_b, β_b and γ_b are set to 1, 1 and 0, respectively, for all $b \in \mathcal{IB}_i$.

We begin by showing the convergence of the Best Response algorithm for small-size scenario (with 10 TVBDs) in Figure 5(a) and for large-size TVWS scenario (with 40 TVBDs) in Figure 5(b). It can be seen that for small size TVWS scenarios, all TVBDs converge, under the best response dynamics, to a high value of the objective function, while for large size scenarios, some TVBDs (i.e., TVBD 1 and TVBD 19) suffer from a high interference and get an objective function value close to zero. Indeed, this trend is expected since the number of idle blocks is limited (5 available idle blocks) with respect to the total number of TVBDs. Furthermore, the convergence speed of the algorithm is fast independently of the number of players/TVBDs; specifically, it is equal to 5 iterations for 10 TVBDs and 7 iterations in the case of 40 TVBDs.

a) Effect of the number of TVBDs: We vary the number of TVBDs in the range $[10, 50]$ in order to show the impact of this parameter on the interference among the devices. Figure 6(a) shows the average value of the objective function as a function of the total number of players (unlicensed devices). Let us focus on the curve (in red color) corresponding to 5 available idle blocks previously depicted in Figure 4; as expected, it can be seen that the objective function decreases when increasing the number of players, and this is in fact due to the increase in the interference between TVBDs.

b) Effect of the number of available TV channels: We now vary the number of available idle blocks (and as a consequence the number of available TV channels) in the range $[3, 5]$. Figure 6(a) shows the average value of the objective function with respect to the number of TVBDs for different configurations of the idle block set. In fact, the behavior observed in the case of 5 available idle blocks is confirmed for 4 and 3 available blocks; the decreasing trend, however, becomes more and more accentuated with the reduction of available blocks, especially when the number of players exceeds 20.

c) Effect of rate demands: To evaluate the effect of the rate demand on the objective function value, we compare the results previously discussed (for d_i=20 Mb/s) to those obtained for d_i=30 Mb/s. From Figure 6(b) it can be seen that the increase in the rate demand highly impacts the player congestion cost (compared to the number of TVBDs parameter), since this cost (Equation (1)) varies quadratically while the utility term (Equation (2)) is linear with the players' strategies.

2) ***Dynamic TVWS scenario****:* In the *dynamic* TVWS scenario, we simulate TVBDs' mobility using the random way-point model [17], [18], as commonly assumed in the literature. In particular, we divide the operating time of the system into 10 consecutive time epochs, and for each time epoch we compute the random displacements of all mobile devices according to a displacement vector. The mobile device speed is set to 1 m/s, while the time epoch duration is fixed to 60 s. Since we consider here mobile TVBDs, we fix the transmission power to 16 dBm.

Figure 7 illustrates an example of a dynamic TVWS scenario where the positions of 20 mobile TVBDs are generated, according to the random way-point model, on a square area of 500×500 m^2 at time epochs 1, 5 and 10. For the sake of clarity, in this figure, we show only 20 devices and 3 out of 10 time epochs.

Figures 8(a)-8(b) show, respectively, the Cumulative Distribution Function (CDF) of the objective function of all devices for 40 and 50 TVBDs, assuming that 5 idle blocks are available and the rate demand is fixed to 20 Mb/s. It can be observed that the percentage of devices that have high objective functions' values in the case of 40 TVBDs is higher than the one corresponding to the case of 50 TVBDs. Indeed, this trend is expected, since the interference between the TVBDs increases with the number of mobile devices.

3) ***Price of Anarchy****:* We now study the efficiency of the Nash equilibria reached in our proposed game by comparing them to the socially optimal solutions, through the determination of the PoA.

Socially optimal solutions maximize the sum of all TVBDs' utilities, i.e., they maximize $\sum_{i \in \mathcal{N}} \{OF_i = U_i - J_i\}$, subject to constraints (3)-(4), $\forall i \in \mathcal{N}$, where J_i and U_i are given in Equations (1) and (2), respectively. The PoA is defined as the ratio between the utility of this solution and that of the worst NE.

We determine hereafter the PoA for static and dynamic TVWS scenarios. The parameters settings are as follows: the transmission power is equal to 20 dBm (16 dBm) for fixed (mobile) TVBDs, rate demands d_i are homogeneous and equal to 20 Mb/s, and the rest of the parameters are set as in the previous sections.

Figures 9(a)-9(b) show the average value of the PoA for static and dynamic TVWS scenarios varying the number of

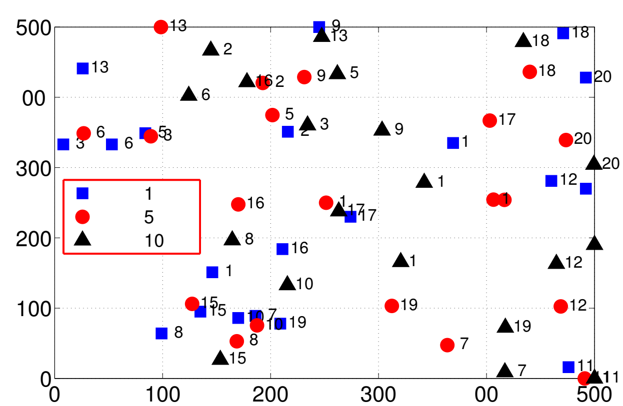
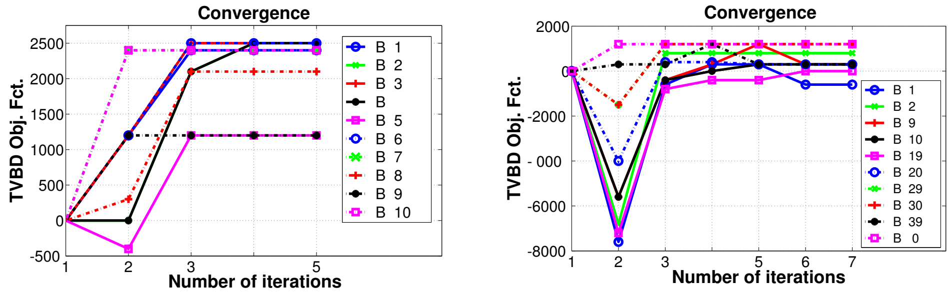
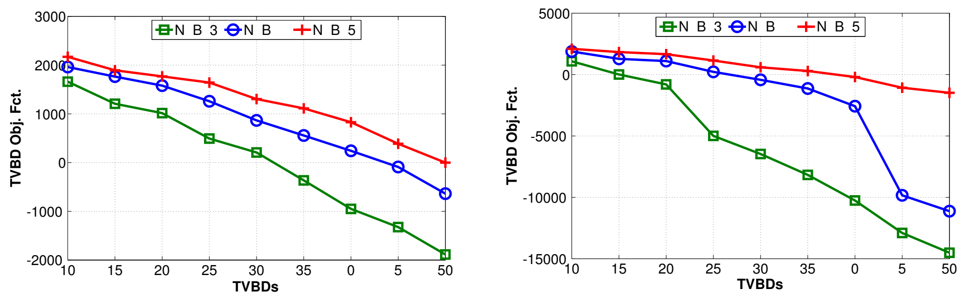


Fig. 7: An example of a dynamic TVWS scenario where the positions of 20 mobile TVBDs are generated on a square area of 500×500 m^2 for time epochs 1, 5 and 10.



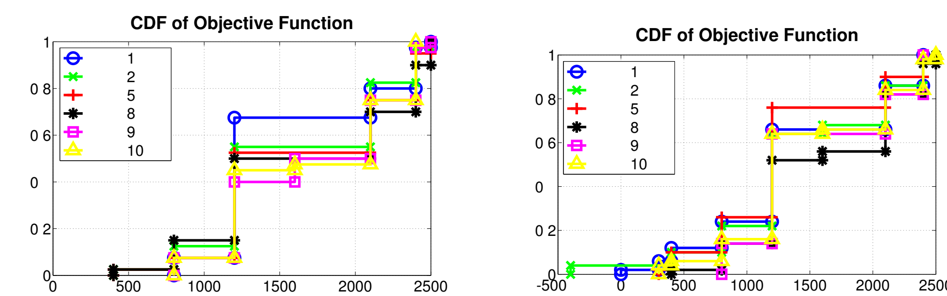
(a) 10 TVBDs (b) 40 TVBDs

Fig. 5: Static TVWS scenario: Convergence of the Best Response algorithm for (a) 10 TVBDs and (b) 40 TVBDs, with 5 available idle blocks and a rate demand equal to 20 Mb/s.



(a) Rate Demand 20 Mb/s (b) Rate Demand 30 Mb/s

Fig. 6: Static TVWS scenario: Objective functions values as a function of the number of TVBDs ($[10, 50]$), available idle blocks ($[3, 5]$) and for two different rate demands (viz., 20 Mb/s and 30 Mb/s).



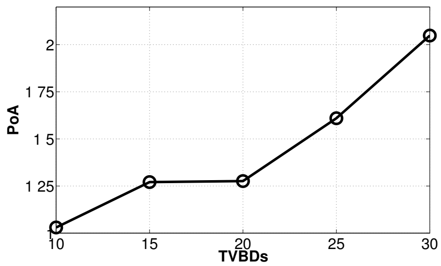
(a) 40 TVBDs (b) 50 TVBDs

Fig. 8: Dynamic TVWS scenario: CDF of the objective function value for two different numbers of TVBDs (viz., 40 and 50), 5 available idle blocks and rate demand equal to 20 Mb/s.

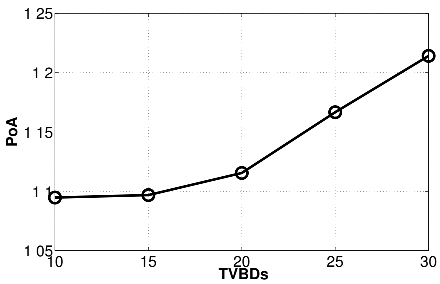
(fixed/mobile) devices in the range $[10, 30]$, assuming that 3 idle blocks are available. Note that in the case of mobile scenarios we compute the average PoA value over all ten time epochs.

For the static case, it can be observed that the PoA increases with the number of TVBDs; we obtain the PoA value of $\approx$2.1 for 30 devices which is two times higher than that achieved with 10 devices. Conversely, for dynamic scenarios, the PoA slightly increases with the total number of TVBDs. However, the average PoA value is small and varies in the interval $[1.05, \approx 1.22]$. Furthermore, when the number of devices is quite small (equal to 10) the PoA achieved in both the static and dynamic scenarios is similar, since the (fixed/mobile) devices are far away from each other and the interference is low even if fixed devices (in the static scenario) use a higher transmission power than mobile ones. However, when the number of devices is higher or equal to 15, the PoA calculated in the static scenario is always higher than that of the dynamic scenario since in the static case the interference between devices becomes higher due to both the increasing number of devices and a higher transmission power (20 dBm for a fixed TVBD vs. 16 dBm for a mobile TVBD).

Finally, in all the considered scenarios, the PoA remains low ($\leq\approx 2$). This trend is indeed due to the good properties provided by the proposed TVBD's objective function and confirms that our distributed approach can achieve good results that are close to the optimum.



(a) Static TVWS scenario



(b) Dynamic TVWS scenario

Fig. 9: Average PoA in the static and dynamic TVWS scenarios as a function of the number of TVBDs, assuming that 3 idle blocks are available and the rate demand is fixed to 20 Mb/s.

VI. Conclusion

In this paper, we have addressed the TV spectrum management problem considering a non cooperative game among TV bands devices (fixed and portable devices), taking accurate account of the Adjacent-Channel Interference between different devices' transmissions. To obtain efficient Nash equilibrium solutions, we introduced a *pricing* scheme for the Database operator and a congestion cost function that aims at reducing interference between unlicensed devices. We demonstrated under specific conditions on cost function parameters that the guard band-aware TVWS management game admits a potential function, and therefore we used a Best Response algorithm to converge fast to Nash equilibrium points.

We evaluated the performance of the proposed game considering both static and dynamic TVWS scenarios (characterized by users' mobility), illustrating its sensitivity to different parameters, including the number of TVBDs, the number of available idle blocks, and the rate demands. Numerical results showed that the proposed game theoretical approach performs well when the available TV resources are limited and the number of TVBDs is high. Finally, the proposed cost function provides an interesting tradeoff between an effective use of the TV spectrum and the interference reduction between unlicensed devices.

Finally, we note that in this work we have focused on a setting in which TVBDs act assuming that the set of idle channels and guard bands are given by the database operator. A natural extension of this work is to address the TV spectrum management problem by further considering the interaction of TVBDs with the database operator who can set idle blocks' prices (or determine the set of idle blocks to communicate to devices) anticipating the actions of the TVBDs to maximize energy efficiency, through a two-stage/Stackelberg game.

Acknowledgement

M. Krunz was supported in part by the National Science Foundation (grants # IIP-1265960, IIP-1432880, and CNS-1409172), the Army Research Office (grant W911NF-13-1-0302) and Raytheon Corporation. Any opinions, findings, conclusions, or recommendations expressed in this paper are those of the author(s) and do not necessarily reflect the views of the NSF or ARO.

References

[1] Federal Communications Commission et al. Second memorandum opinion and order. *FCC 10-174*, Washington, September 23, 2010.

[2] Federal Communications Commission et al. Third memorandum opinion and order. *FCC 12-36*, 36, Washington, April 5, 2012.

[3] H. Salameh, M. Krunz, and D. Manzi. Spectrum bonding and aggregation with guard-band awareness in cognitive radio networks. *IEEE Transactions on Mobile Computing*, 13(3):569–581, 2013.

[4] R. Rajbanshi, Q. Chen, A.M. Wyglinski, G.J. Minden, and J.B. Evans. Quantitative Comparison of Agile Modulation Techniques for Cognitive Radio Transceivers. *Proceedings of the fourth IEEE Consumer Comm. and Networking Conf. Workshop Cognitive Radio Networks*, pages 1144–1148, January 2007.

[5] H. Kim and K.G. Shin. Efficient Discovery of Spectrum Opportunities with MAC-Layer Sensing in Cognitive Radio Networks. *IEEE Transactions on Mobile Computing*, 7(5):533–545, 2008.

[6] S. Joshi, P. Pawelczak, D. Cabric, and J. Villasenor. When Channel Bonding is Beneficial for Opportunistic Spectrum Access Networks. *IEEE Trans. on Wireless Communications*, 11(11):3942–3956, 2012.

[7] R. Chandra, R. Mahajan, T. Moscibroda, R. Raghavendra, and P. Bahl. A Case for Adapting Channel Width in Wireless Networks. *Proceedings of the ACM SIGCOMM 2008 Conference*, pages 135–146, 2008.

[8] X. Chen and J. Huang. Game Theoretic Analysis of Distributed Spectrum Sharing With Database. *Proceedings of the IEEE 32nd International Conference on Distributed Computing Systems (ICDCS)*, pages 255–264, Macau, China, June 18-21, 2012.

[9] X. Feng, Q. Zhang, and J. Zhang. A Hybrid Pricing Framework for TV White Space Database. *IEEE Trans. on Wireless Communications*, 13(5):2626–2635, 2014.

[10] L. Zheng and C.W. Tan. Smart Spectrum Access Algorithms in Mobile TV White Space Networks for Utility Maximization. *IEEE INFOCOM Workshops*, pages 175–176, Toronto, Canada, April 27-May 2, 2014.

[11] D. Li and J. Gross. Distributed TV Spectrum Allocation for Cognitive Cellular Network under Game Theoretical Framework. *Proceedings of IEEE DYSPAN 2012*, pages 327–338, Bellevue, Washington, USA, 2012.

[12] H. Bogucka, M. Parzy, P. Marques, J.W. Mwangoka, and T. Forde. Secondary Spectrum Trading in TV White Spaces. *IEEE Communications Magazine*, 50(11):121–129, 2012.

[13] D. Niyato and E. Hossain. Competitive spectrum sharing in cognitive radio networks: a dynamic game approach. *IEEE Transactions on Wireless Communications*, 7(7):2651–2660, 2008.

[14] D. Monderer and L.S. Shapley. Potential games. *Games and economic behavior*, 14:124–143, 1996.

[15] E. Koutsoupias and C. H. Papadimitriou. Worst-Case Equilibria. *Proceedings of the 16th Annual Symposium on Theoretical Aspects of Computer Science*, pages 404–413, Trier, Germany, March 4-6, 1999.

[16] IBM ILOG CPLEX. 12.2 user's manual, 2010.

[17] D.B. Johnson and D.A. Maltz. Dynamic Source Routing in Ad-Hoc Wireless Networks. *Mobile computing*, 353:153–181, 1996.

[18] J. Elias, S. Paris, and M. Krunz. Cross Technology Interference Mitigation in Body Area Networks: an Optimization Approach. *IEEE Trans. on Vehicular Technology*, PP(99):1–14, 1 October 2014.