Adaptive Boolean Network Tomography for Link Failure Detection

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***Abstract*—In this paper, we consider boolean network tomography to identify link failures in a network. In boolean network tomography, the relationship between end-to-end measurements and link states are represented with a system of boolean equations, and failure links are identified by solving the equations. In order to establish measurement paths efficiently, we propose an adaptive boolean network tomography scheme, where measurement paths are established sequentially according to a candidate set of failure links. Here, to derive the candidate set, we extend *CBP (Combinatorial Basis Pursuit)*, a representative decoding algorithm in *Combinatorial Group Testing*, and utilize its property that it can identify failure links without false negative errors. We evaluate the performance of the proposed scheme in terms of the number of measurement paths and compare it with a non-adaptive boolean network tomography scheme. Furthermore, we propose *mobility-assisted boolean network tomography*, which can improve the *ambiguity problem* in boolean network tomography.**

I. Introduction

Network Tomography is an inference technique for internal network characteristics such as link delays and link loss rates from end-to-end measurements [1]. In this paper, we consider *boolean network tomography* to identify failure links [5]. In boolean network tomography schemes, the relationship between end-to-end measurements and link states are represented with a system of boolean equations, and failure links are identified by solving the equations.

Duffield [5] proposes the SCFS (smallest consistent failure set) inference algorithm for boolean network tomography, which identifies the smallest set of links to satisfy the relationship between the measurements and link states as failure links, and it can achieve a small false positive rate in tree topologies. In [3], [6], [7], *combinatorial group testing* [4] is applied to the boolean network tomography problem in general network topologies. Combinatorial group testing can be classified into *non-adaptive group testing* and *adaptive group testing*. When non-adaptive group testing is applied to the problem [3], [6], measurement paths are established before the link failure detection procedure. On the other hand, when adaptive group testing is applied to the problem [7], measurement paths are sequentially established according to the measurements that have already obtained, and as a result, we can reduce the number of measuremen paths.

In this paper, we propose a boolean network tomography scheme based on adaptive group testing. In [7], an adaptive network tomography is proposed under the assumption that many measurement nodes can be deployed within the network. On the other hand, we consider a network with only two measurement nodes, and measurement paths are established between them. The proposed scheme has a *coarse-to-fine* structure to identify failure links. Namely, it first establishes an initial set of measurement paths, and roughly estimates a set of failure links, which is referred to as a *candidate set* of failure links. The candidate set is then refined by adding measurement paths iteratively. In order to derive and update the candidate set, we extend *CBP (Combinatorial Basis Pursuit)* [2], a representative decoding algorithm in combinatorial group testing, and utilize its property that it can identify failure links without false negative errors.

The reminder of this paper is organized as follows. In section II, we formulate the boolean network tomography problem and explain the detection scheme of failure links based on CBP. In section III, we explain the procedure of the proposed scheme. In IV, we evaluate the performance of the proposed scheme with simulation experiments. Finally, we conclude the paper in section V.

II. Boolean Network Tomography

A. System Model

In this paper, we assume a general network model without intending any specific network model. From the viewepoint of implementation, however, we consider that multihop-based access networks such as wireless mesh networks are possible candidates to apply network tomography rather than core backbone networks, because source nodes need to explicitly determine intermediate nodes on each measurement path. We assume that source routing is implementd in all nodes.

Let $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ denote a directed network, where $\mathcal{V}$ and $\mathcal{E} = \{e_1, e_2, \ldots, e_L\} \subset \mathcal{V} \times \mathcal{V}$ denote the sets of all nodes and links, respectively. $L = |\mathcal{E}|$ denotes the number of links. There are two types of links in the network: *failure links* and *normal links*. We assume that normal links successfully transfer packets with probability 1 and failure links always discard them (i.e., drop them with probability 1). We define

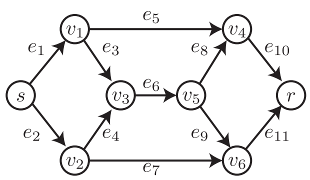


Fig. 1. Network topology

$\mathcal{E}_{\rm F} \subset \mathcal{E}$ and $\mathcal{E}_{\rm N} = \mathcal{E} \setminus \mathcal{E}_{\rm F}$ as the sets of failure links and normal links, respectively.

We assume that there are only two *measurement nodes* $s, r \in \mathcal{V}$, where s and r represent a source measurement node and a receiver measurement node, respectively. We simply refer to s and r as the source node and the receiver node, hereafter. The source node s transmits probe packets on *measurement paths* in order to deliver it to the receiver node r. We define $\mathcal{W}_{\rm all}$ as the set of all paths from the source node s to the receiver node r and $\mathcal{W} = \{w_m \in \mathcal{W}_{\rm all} \mid m = 1, 2, \ldots, M\}$ as a set of measurement paths, where M denotes the number of measurement paths.

We define $\boldsymbol{x} = (x_1\ x_2\ \cdots\ x_L) \in \{0,1\}^L$ as a link state vector, where $x_l = 1$ if $e_l \in \mathcal{E}_{\rm N}$ $(l = 1, 2, \ldots, L)$ and $x_l = 0$ if $e_l \in \mathcal{E}_{\rm F}$. We define $\boldsymbol{y} = (y_1\ y_2\ \cdots\ y_M) \in \{0,1\}^M$ as a measurement vector, where $y_m = 0$ $(m = 1, 2, \ldots, M)$ if a packet transmitted on a measurement path $w_m \in \mathcal{W}$ is successfully received at the receiver node r, and $y_m = 1$ otherwise. Namely, y_m is given by

$$y_m = \begin{cases} 0 & \text{if } e \in \mathcal{E}_{\rm N} \text{ for } \forall e \in w_m, \\ 1 & \text{otherwise.} \end{cases}$$

We then obtain

$$\begin{aligned} y_m &= \bigvee_{l=1}^{L} a_{m,l} \wedge x_l \\ &= (a_{m,1} \wedge x_1) \vee (a_{m,2} \wedge x_2) \vee \cdots (a_{m,L} \wedge x_L), \end{aligned} \tag{1}$$

where $\vee$ and $\wedge$ denote OR and AND operations, respectively, and $a_{m,l} = 1$ $(m = 1, 2, \ldots, M,\ l = 1, 2, \ldots, L)$ if $e_l \in w_m$, and $a_{m,l} = 0$ otherwise.

Let $\boldsymbol{A} \in \{0,1\}^{M \times L}$ denote a *routing matrix*, where the (m,l)-th element $(m = 1, 2, \ldots, M,\ l = 1, 2, \ldots, L)$ of $\boldsymbol{A}$ corresponds to $a_{m,l}$. For example, consider the network topology shown in Fig. 1 and set $\mathcal{W} = \mathcal{W}_{\rm all} = \{w_i \mid i = 1, 2, 3, 4, 5, 6\}$, where $w_1 = \{e_1, e_5, e_{10}\}$, $w_2 = \{e_1, e_3, e_6, e_8, e_{10}\}$, $w_3 = \{e_1, e_3, e_6, e_9, e_{11}\}$, $w_4 = \{e_2, e_4, e_6, e_8, e_{10}\}$, $w_5 = \{e_2, e_4, e_6, e_9, e_{11}\}$, and $w_6 = \{e_2, e_7, e_{11}\}$. We then obtain

$$\boldsymbol{A} = \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}. \tag{2}$$

Algorithm 1 *CBP*$(\boldsymbol{y}, \mathcal{W})$

Require: $\boldsymbol{y} = (y_1\ y_2\ \cdots\ y_M)$, $\mathcal{W} = \{w_1\ w_2\ \cdots\ w_M\}$;
Ensure: $\mathcal{E}_{\rm C}$, $\mathcal{E}_{\rm I}$
1: $\mathcal{E}_{\rm C} := \mathcal{E}$, $\mathcal{E}_{\rm I} := \emptyset$;
2: **for** $m := 1$ **to** M **do**
3:   **if** $y_m = 0$ **then**
4:     **for all** $e \in w_m$ **do**
5:       $\mathcal{E}_{\rm C} := \mathcal{E}_{\rm C} \setminus \{e\}$;
6:     **end for**
7:   **end if**
8: **end for**
9: **for** $m := 1$ **to** M **do**
10:   **if** $|w_m \cap \mathcal{E}_{\rm C}| = 1$ **then**
11:     $\mathcal{E}_{\rm I} := \mathcal{E}_{\rm I} \cup (w_m \cap \mathcal{E}_{\rm C})$;
12:     $\mathcal{E}_{\rm C} := \mathcal{E}_{\rm C} \setminus (w_m \cap \mathcal{E}_{\rm C})$;
13:   **end if**
14: **end for**

B. Link Failure Detection

Boolean network tomography is a problem to infer $\boldsymbol{x}$ from a measurement vector $\boldsymbol{y}$. We assume that the routing matrix $\boldsymbol{A}$ is known in advance. In this paper, we use CBP (Combinatorial Basis Pursuit) [2], which is a decoding scheme for combinatorial group testing [4]. Algorithm 1 shows the procedure of CBP. In the procedure, $\mathcal{E}_{\rm C}$ and $\mathcal{E}_{\rm I}$ denote a *candidate set* and an *identified set* of failure links, respectively. $\mathcal{E}_{\rm C}$ contains links which have not been judged to be failure links or not, while $\mathcal{E}_{\rm I}$ contains links that have been judged to be failure links. Initially, $\mathcal{E}_{\rm C}$ is set to $\mathcal{E}_{\rm C} = \mathcal{E}$. From the definition of failure links described in the preceding section, we can judge all links on w_m $(m = 1, 2, \ldots, M)$ to be normal links if $y_m = 0$. Based on this idea, in lines 2–8, we remove links judged to be normal from $\mathcal{E}_{\rm C}$. If a measurement path w_m includes only one link in $\mathcal{E}_{\rm C}$, we can judge the link to be a failure link. Therefore, we update $\mathcal{E}_{\rm C}$ and $\mathcal{E}_{\rm I}$ in lines 9–14. We define $\hat{\mathcal{E}}_{\rm F} = \mathcal{E}_{\rm C} \cup \mathcal{E}_{\rm I}$ as an *estimated set* of failure links, and regard every link in $\hat{\mathcal{E}}_{\rm F}$ as a failure link.

In the link failure detection problem, we need to consider two types of detection errors: *false negative error* and *false positive error*. A false negative (positive) error occurs if a failure (normal) link is judged to be a normal (failure) link. It is obvious that false negative errors do not occur in CBP, that is, $\hat{\mathcal{E}}_{\rm F} \supseteq \mathcal{E}_{\rm F}$. False positive errors, however, can occur when $|\mathcal{E}_{\rm C}| > 0$. In general, a normal link $e \in \mathcal{E}_{\rm N}$ is judged to be a failure link, when one of the following conditions is satisfied:

(a) e is not included in any measurement paths, that is, $e \notin w_m$ for $\forall w_m \in \mathcal{W}$.
(b) Let $\mathcal{W}_e \subset \mathcal{W}$ denote a set of measurement paths including e. All measurement paths in $\mathcal{W}_e$ include at least one failure link, that is, $y_m = 1$ for $\forall w_m \in \mathcal{W}_e$.

Consider the failure link detection in Fig. 1. We set $\mathcal{E}_{\rm F} = \{e_1\}$, that is, $x_1 = 1$, which results in the measurement vector $\boldsymbol{y} = (1\ 1\ 1\ 0\ 0\ 0)$. By applying CBP to $\boldsymbol{y}$, we obtain $\hat{\mathcal{E}}_{\rm F} = \{e_1, e_3, e_5\}$, that is, normal links e_3 and e_5 are judged to be

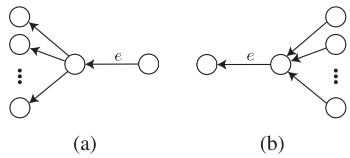


Fig. 2. Examples for false positive errors. When link e is a failure link, (a) all downstreaming links of e are judged to be a failure link, and (b) all upstreaming links of e are judged to be a failure link.

failure links. In general, when a link e is a failure link in Fig. 2(a), all downstreaming links of e are judged to be failure links. Moreover, when a link e is a failure link in Fig. 2(b), all upstreaming links of e are also judged to be failure links.

For $\forall \mathcal{E}_{\mathrm{F}} \subset \mathcal{E}$ and $s, r \in \mathcal{V}$, we refer to a set $\mathcal{W}$ of measurement paths as a *valuable* set of measurement paths when $\mathcal{W}$ minimizes the candidate set $\mathcal{E}_{\mathrm{C}}$. The objective of the paper is to find a valuable set $\mathcal{W}$ with a smaller cardinality $|\mathcal{W}|$. There are two approaches to find a set of paths: *non-adaptive* and *adaptive* methods. In the former approach, all measurement paths are established in advance, while in the latter approach, measurement paths are established adaptively. Although the non-adaptive method increases the number of measurement paths, its implementation is simpler. On the other hand, although the adaptive method increases the latency to detect failure links due to its sequential operation, it can reduce the number of measurement paths. The proposed scheme in this paper, which will be explained in the next section, is classified into the latter approach.

III. Adaptive Boolean Network Tomography

A. Overview of the Proposed Scheme

As mentioned in the preceding section, one of the advantages using CBP is that it does not cause false negative errors. This means that if $\mathcal{W}$ covers all links in the network, that is, $\bigcup_{w \in \mathcal{W}} w = \mathcal{E}$, the estimated set $\hat{\mathcal{E}}_{\mathrm{F}}$ obtained from $\mathcal{W}$ includes all failure links. The proposed scheme utilizes this property in order to construct an adaptive boolean network tomography scheme. Fig. 3 shows the overview of the proposed scheme. Initially, the candidate set $\mathcal{E}_{\mathrm{C}}$ of failure links contains all links as shown in Fig. 3(a). The proposed scheme first constructs an initial set $\mathcal{W}$ of measurement paths and then updates $\hat{\mathcal{E}}_{\mathrm{F}}$ by applying the initial set to CBP. An additional measurement path is established by utilizing $\hat{\mathcal{E}}_{\mathrm{F}}$ as a prior information for failure links, and updates $\hat{\mathcal{E}}_{\mathrm{F}}$ (Fig. 3(b)). By applying the procedure iteratively, measurement paths are added to $\mathcal{W}$ until the candidate set $\mathcal{E}_{\mathrm{C}}$ is minimized (Figs. 3(c), (d)).

B. Adaptive Path Construction

Algorithm 2 shows the procedure of the proposed scheme. We assume that the set $\mathcal{W}_{\mathrm{all}}$ of all paths is obtained in advance by using a path enumeration algorithm such as in [10]. It is not always efficient to prepare the set of all paths because the number of paths drastically increases with the number of links. Therefore, we will develop a more scalable algorithm without the path enumeration algorithm in the future research.

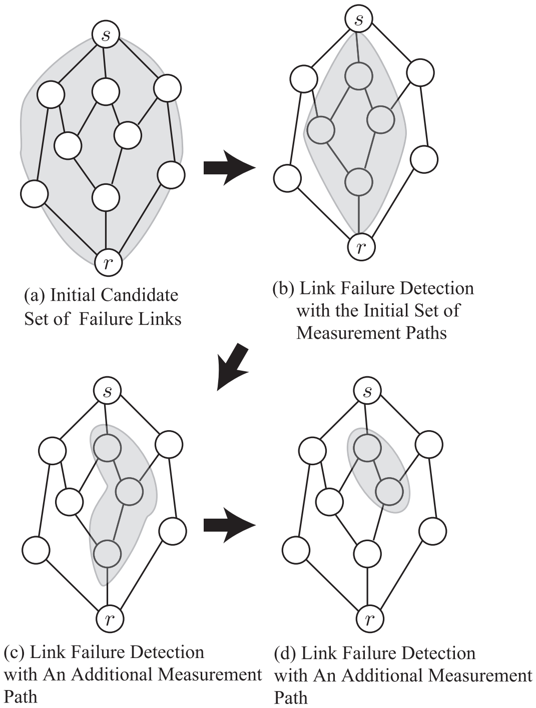


Fig. 3. Overview of the proposed scheme. The shaded area represents the candidate set of failure links.

Algorithm 2 Adaptive Path Construction.

Require: $\mathcal{W}_{\mathrm{all}}, \mathcal{E}$
Ensure: $\mathcal{E}_{\mathrm{C}}, \mathcal{E}_{\mathrm{I}}$
1: $\mathcal{W} := \mathit{InitPath}(\mathcal{W}_{\mathrm{all}}, \mathcal{E})$; $\boldsymbol{y} = \emptyset$;
2: **for** $m := 1$ **to** $|\mathcal{W}|$ **do**
3: $\quad y_m := \mathit{TxPacket}(w_m)$; $\boldsymbol{y} = \boldsymbol{y} \cup \{y_m\}$;
4: **end for**
5: $[\mathcal{E}_{\mathrm{C}}, \mathcal{E}_{\mathrm{I}}] := \mathit{CBP}(\boldsymbol{y}, \mathcal{W})$
6: **loop**
7: $\quad w_{m+1} := \mathit{SelectPath}(\mathcal{W}_{\mathrm{all}}, \mathcal{W}, \mathcal{E}_{\mathrm{C}}, \mathcal{E}_{\mathrm{I}})$;
8: $\quad$ **if** $w_{m+1} =$ NULL **then**
9: $\quad\quad$ **return** $\mathcal{E}_{\mathrm{C}}$, $\mathcal{E}_{\mathrm{I}}$;
10: $\quad$ **end if**
11: $\quad y_{m+1} := \mathit{TxPacket}(w_{m+1})$; $\boldsymbol{y} := \boldsymbol{y} \cup \{y_{m+1}\}$;
12: $\quad \mathcal{W} := \mathcal{W} \cup \{w_{m+1}\}$;
13: $\quad [\mathcal{E}_{\mathrm{C}}, \mathcal{E}_{\mathrm{I}}] := \mathit{CBP}(\boldsymbol{y}, \mathcal{W})$;
14: $\quad m := m + 1$;
15: **end loop**

An initial set $\mathcal{W}$ of measurement paths is established with $\mathit{InitPath}(\mathcal{W}_{\mathrm{all}}, \mathcal{E})$ (line 1), and probe packets are transmitted on the paths (lines 2, 3, 4). $\mathcal{E}_{\mathrm{C}}$ and $\mathcal{E}_{\mathrm{I}}$ are constructed with CBP (line 5). We describe the procedure for constructing the initial set of measurement paths in section III-B1.

In each iteration of the loop 6–15, an additional measurement path is constructed from $\mathcal{E}_{\mathrm{C}}$ and $\mathcal{E}_{\mathrm{I}}$ by using $\mathit{SelectPath}(\mathcal{W}_{\mathrm{all}}, \mathcal{W}, \mathcal{E}_{\mathrm{C}}, \mathcal{E}_{\mathrm{I}})$, and a probe packet is transmitted on the path (line 11). $\mathcal{E}_{\mathrm{C}}$ and $\mathcal{E}_{\mathrm{I}}$ are then updated with CBP (line 13). If no measurement path is constructed, which

Algorithm 3 Construction Procedure $InitPath(\mathcal{W}_{\rm all}, \mathcal{E})$ for an Initial Set of Measurement Paths

Require: $\mathcal{W}_{\rm all}$, $\mathcal{E}$
Ensure: $\mathcal{W}_0$
1: $\mathcal{L} := \emptyset$; $\mathcal{W}_0 := \emptyset$;
2: **while** $\mathcal{L} \neq \mathcal{E}$ **do**
3: $\quad w := \displaystyle\mathop{\arg\max}_{w' \in \mathcal{W}_{\rm all} \setminus \mathcal{W}_0} |w' \cap (\mathcal{E} \setminus \mathcal{L})|$;
4: $\quad \mathcal{W}_0 := \mathcal{W}_0 \cup \{w\}$;
5: $\quad \mathcal{L} := \mathcal{L} \cup w$;
6: **end while**
7: **return** $\mathcal{W}_0$

means that $\mathcal{E}_{\rm C}$ is minimized, the procedure is finished (lines 8–10). We describe the procedure for constructing additional measurement paths in section III-B2.

1) Constructing an Initial Set of Measurement Paths: Algorithm 3 shows the procedure for constructing an initial set $\mathcal{W}_0$ of measurement paths. In order to avoid false positive errors, the initial set must satisfy a condition that every link in the network is included at least one measurement path in the initial set. Therefore, we can consider the following optimization problem to reduce the number of measurement paths in the initial set:

$$\min_{\mathcal{W} \subset \mathcal{W}_{\rm all}} |\mathcal{W}| \quad \text{subject to} \bigcup_{w_i \in \mathcal{W}} w_i = \mathcal{E}.$$

Instead of considering algorithms to solve the problem, we use a Greedy algorithm as shown in lines 2–5. In the algorithm, $\mathcal{L}$ denotes a set of links which have already been included in at least one measurement path in $\mathcal{W}_0$. In each iteration, a measurement path covering the maximum number of links in $\mathcal{E} \backslash \mathcal{L}$ is selected. We leave algorithms to solve the optimization problem as a future research.

2) Constructing Additional Measurement Paths: As mentioned in section II-B, CBP judges all links on a measurement path to be normal when a probe packet is successfully transmitted on the path. When an additional measurement path can reduce the number of elements in $\mathcal{E}_{\rm C}$, the path is referred to as an *effective* measurement path. When a measurement path $w \subset \mathcal{W}_{\rm all} \backslash \mathcal{W}$ satisfies at least one of the following conditions, the path is not effective:

(i) w includes at least one element of $\mathcal{E}_{\rm I}$.
(ii) w does not include any elements of $\mathcal{E}_{\rm C}$.
(iii) $w \cap \mathcal{E}_{\rm C} \supseteq w_i \cap \mathcal{E}_{\rm C}$ for $\exists w_i \in \{w_j \in \mathcal{W} \mid y_j = 1\}$.

For given $\mathcal{E}_{\rm C}$ and $\mathcal{E}_{\rm I}$, let $\mathcal{W}_{\rm eff} \subseteq \mathcal{W}_{\rm all} \setminus \mathcal{W}$ denote the set of all effective measurement paths. In the procedure $SelectPath(\mathcal{W}_{\rm all}, \mathcal{W}, \mathcal{E}_{\rm C}, \mathcal{E}_{\rm I})$ in algorithm 2, an additional measurement path $w_{\rm add} \in \mathcal{W}_{\rm eff}$ is given by

$$w_{\rm add} = \mathop{\arg\min}_{w \in \mathcal{W}_{\rm eff}} \left| |w \cap \mathcal{E}_{\rm C}| - \frac{|\mathcal{E}_{\rm C}|}{2} \right|, \tag{3}$$

where the number of links in $w_{\rm add} \cap \mathcal{E}_{\rm C}$ is closest to $|\mathcal{E}_{\rm C}|/2$. If there are several measurement paths satisfying (3), a measurement path is randomly chosen. Suppose that $|w_{\rm add} \cap \mathcal{E}_{\rm C}| = |\mathcal{E}_{\rm C}|/2$. In this case, if a probe packet is successfully transmitted on $w_{\rm add}$, a half of links in $\mathcal{E}_{\rm C}$ is judged to be normal.

Let $n_{\rm C}(e)$ ($e \in \mathcal{E}_{\rm C}$) denote the number of measurement paths which include link e, that is, $n_{\rm C}(e)$ is given by

$$n_{\rm C}(e) = \sum_{w \in \mathcal{W}} |w \cap \{e\}|.$$

Intuitively, a link $e \in \mathcal{E}_{\rm C}$ with larger $n_{\rm C}(e)$ is likely to be a failure link. Therefore, we can consider an extension of (3):

$$w_{\rm add} = \begin{cases} \displaystyle\mathop{\arg\min}_{w \in \mathcal{W}_{\rm eff}^{(e_{\max})}} \left| |w \cap \mathcal{E}_{\rm C}| - \frac{|\mathcal{E}_{\rm C}|}{2} \right| & \text{if } \mathcal{W}_{\rm eff}^{(e_{\max})} \neq \emptyset, \\ \displaystyle\mathop{\arg\min}_{w \in \mathcal{W}_{\rm eff}} \left| |w \cap \mathcal{E}_{\rm C}| - \frac{|\mathcal{E}_{\rm C}|}{2} \right| & \text{if } \mathcal{W}_{\rm eff}^{(e_{\max})} = \emptyset, \end{cases} \tag{4}$$

where $e_{\max}$ and $\mathcal{W}_{\rm eff}^{(e_{\max})}$ are defined as

$$\begin{aligned} e_{\max} &= \mathop{\arg\max}_{e \in \mathcal{E}_{\rm C}} \{n_{\rm C}(e)\}, \\ W_{\rm eff}^{(e_{\max})} &= \{w \in \mathcal{W}_{\rm eff} \mid w \cap \{e_{\max}\} = \emptyset\}, \end{aligned}$$

respectively. We refer to the proposed scheme based on (3) as the *proposed scheme 1* and the proposed scheme based on (4) as the *proposed scheme 2*.

IV. Performance Evaluation

A. Simulation Environment

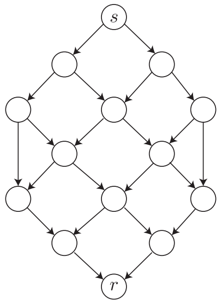
In this section, we evaluate the performance of the proposed scheme with simulation experiments. Figs. 4(a), 4(b), and 4(c) show the network topologies used in the simulation experiments. In each topology, we assign k links as a failure link, that is, $|\mathcal{E}_{\rm F}| = k$. Let $\mathcal{F}_k$ denote the family of all the possible sets of failure links for a given network topology, that is, $\mathcal{F}_k = \{\mathcal{E}_{\rm F} \mid |\mathcal{E}_{\rm F}| = k\}$. We define $\mathcal{E}_{\rm F}^{(k,i)} \in \mathcal{F}_k$ ($i = 1, 2 \ldots, |\mathcal{F}_k|$) as the i-th set of failure links in $\mathcal{F}_k$. Note that $|\mathcal{F}_k| = {}_L\mathrm{C}_k$. Let $\mathcal{W}_{\rm val}^{(k,i)}$ denote a valuable set of measurement paths for given $\mathcal{E}_{\rm F}^{(k,i)}$. Finally, we define $N_{\rm path}^{(k,i)} = |\mathcal{W}_{\rm val}^{(k,i)}|$.

We evaluate the proposed scheme with three metrics for the number of measurement paths: the average $N_{\rm avg}^{(k)}$, the variance $N_{\rm var}^{(k)}$, and the maximum $N_{\max}^{(k)}$ of the number of measurement paths. $N_{\rm avg}^{(k)}$, $N_{\rm var}^{(k)}$, and $N_{\max}^{(k)}$ are obtained by

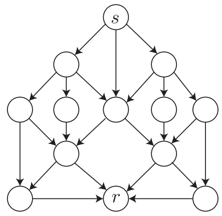
$$\begin{aligned} N_{\rm avg}^{(k)} &= \frac{1}{|\mathcal{F}_k|} \sum_{i \in \{j | \mathcal{E}_{\rm F}^{(k,j)} \in \mathcal{F}_k\}} N_{\rm path}^{(k,i)}, \\ N_{\rm var}^{(k)} &= \frac{1}{|\mathcal{F}_k|} \sum_{i \in \{j | \mathcal{E}_{\rm F}^{(k,j)} \in \mathcal{F}_k\}} \left(N_{\rm path}^{(k,i)} - N_{\rm avg}^{(k)} \right)^2, \\ N_{\max}^{(k)} &= \max_{i \in \{j | \mathcal{E}_{\rm F}^{(k,j)} \in \mathcal{F}_k\}} \left\{ N_{\rm path}^{(k,i)} \right\}, \end{aligned}$$

respectively.

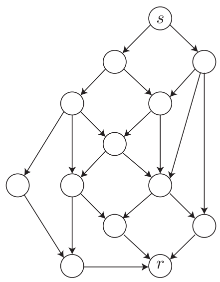
There have been several works for boolean network tomography [6]–[9]. We leave the performance comparison of these works and the proposed scheme as a future work. In this paper, we compare the performance of the proposed scheme with that of a non-adaptive boolean network tomography scheme,



(a) Network topology 1



(b) Network topology 2



(c) Network topology 3

Fig. 4. Network topologies used in simulation experiments.

where all measurements paths are established in advance, as described in section II-B. The non-adaptive scheme is required to identify all the possible sets of failure links in $\mathcal{F}_k$. Let $\mathcal{Z}_{\mathrm{all}}^{(k)}$ denote a family of all the possible sets of measurement paths to satisfy the requirement. We define the minimum set $\mathcal{Z}_{\mathrm{min}}^{(k)}$ of measurement paths and the minimum number $N_{\mathrm{non}}^{(k)}$ of measurement paths as $\mathcal{Z}_{\mathrm{min}}^{(k)} = \arg\min_{\mathcal{Z}\in\mathcal{Z}_{\mathrm{all}}^{(k)}} |\mathcal{Z}|$ and $N_{\mathrm{non}}^{(k)} = |\mathcal{Z}_{\mathrm{min}}^{(k)}|$, respectively.

B. Simulation Results

Tables I, II, III shows the performance of the proposed schemes and the non-adaptive scheme for network topology 1, 2, and 3, respectively. In the tables, "proposal 1", "proposal 2", and "non-adaptive" correspond to the performance of the proposed scheme 1, the proposed scheme 2, and the non-adaptive scheme, respectively.

We first compare the performance of the proposed schemes with that of the non-adaptive scheme. We observe $N_{\mathrm{avg}}^{(k)} < N_{\mathrm{non}}^{(k)}$ for $k = 1, 2$, that is, the propose schemes 1 and 2 can achieve the smaller average number of measurement paths than the non-adaptive scheme. We also observe that when $k = 1$, $N_{\mathrm{max}}^{(k)}$ is smaller than $N_{\mathrm{non}}^{(k)}$. When $k = 2$, however, $N_{\mathrm{max}}^{(k)}$ in the network topologies 1 and 3 are higher than $N_{\mathrm{non}}^{(k)}$, which results in a larger variance of the number of measurement paths. The reason is that in the network topology 1, both source and receiver nodes have only two output links and two input links, respectively, and in the network topology 3, source node has only two output links. For example, suppose that in the network topology 1, both the output links of the source node are failure links. In this case, all links in the network are judged to be failure links because all measurement paths include one of the failure links. We refer to this problem as the *ambiguity problem* of boolean network tomography.

We next compare the performance of the proposed schemes 1 and 2. From the tables, we can observe that the proposed scheme 2 provides better or equal performance than the proposed scheme 1, which indicates that our strategy based on (4) is effective to reduce the number of measurement paths. Further, we observe that the proposed scheme 2 obtains a significant performance gain especially in the network topology 3. The reason is that the network topology 3 is asymmetric while the network topologies 1 and 2 are symmetric. In asymmetric network topologies, $n_{\mathrm{C}}(e)$ $(e \in \mathcal{E})$ are unbalanced as compared with those in symmetric network topologies. Namely, some specific links are included on many measurement paths. Therefore, if these links are a failure link, the proposed scheme 2 avoids selecting a measurement path including the links.

TABLE I
THE NUMBER OF MEASUREMENT PATHS IN THE PROPOSED SCHEME FOR THE NETWORK TOPOLOGY 1.

		$k=1$	$k=2$
the number of all measurement paths	$\|\mathcal{W}_{\mathrm{all}}\|$	20	20
proposal 1	$N_{\mathrm{avg}}^{(k)}$	8.41	10.75
	$N_{\mathrm{var}}^{(k)}$	3.15	9.23
	$N_{\mathrm{max}}^{(k)}$	14	20
proposal 2	$N_{\mathrm{avg}}^{(k)}$	8.36	10.54
	$N_{\mathrm{var}}^{(k)}$	1.77	5.03
	$N_{\mathrm{max}}^{(k)}$	11	20
non-adaptive	$N_{\mathrm{non}}^{(k)}$	12	18

TABLE II
THE NUMBER OF MEASUREMENT PATHS IN THE PROPOSED SCHEME FOR THE NETWORK TOPOLOGY 2.

		$k=1$	$k=2$
The number of all measurement paths	$\|\mathcal{W}_{\mathrm{all}}\|$	22	22
proposal 1	$N_{\mathrm{avg}}^{(k)}$	9.83	11.04
	$N_{\mathrm{var}}^{(k)}$	1.36	3.66
	$N_{\mathrm{max}}^{(k)}$	13	17
proposal 2	$N_{\mathrm{avg}}^{(k)}$	10.13	11.32
	$N_{\mathrm{var}}^{(k)}$	0.98	2.18
	$N_{\mathrm{max}}^{(k)}$	12	16
non-adaptive	$N_{\mathrm{non}}^{(k)}$	16	22

C. Mobility-Assisted Boolean Network Tomography

The ambiguity problem described in the preceding section is the most critical problem in boolean network tomography. Namely, when all output links of the source node or all input links of the receiver node are failure links, all links are judged to be failure links. These localized link failures can happen such as in a situation that a wide-scale disaster destroys

TABLE III
THE NUMBER OF MEASUREMENT PATHS IN THE PROPOSED SCHEME FOR THE NETWORK TOPOLOGY 3.

		$k=1$	$k=2$
The number of all measurement paths	$\lvert\mathcal{W}_{\rm all}\rvert$	22	22
proposal 1	$N_{\rm avg}^{(k)}$	9.39	11.55
	$N_{\rm var}^{(k)}$	6.67	10.76
	$N_{\rm max}^{(k)}$	18	22
proposal 2	$N_{\rm avg}^{(k)}$	8.77	10.6
	$N_{\rm var}^{(k)}$	1.27	4.16
	$N_{\rm max}^{(k)}$	11	22
non-adaptive	$N_{\rm non}^{(k)}$	14	21

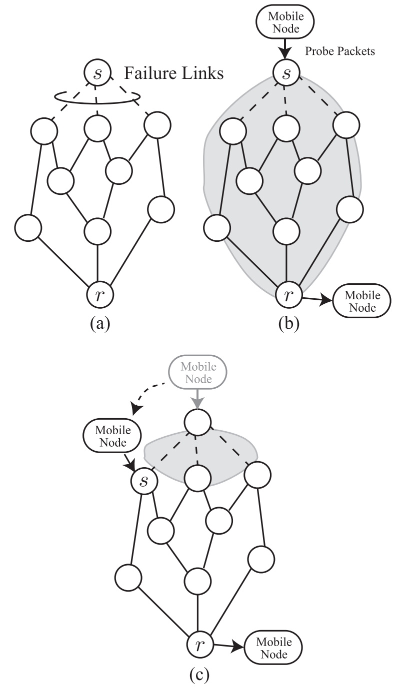


Fig. 5. Mobility-Assisted Boolean Network Tomography. The mobile node injects a probe packet as a virtual source node. (a) A network topology and a set of failure links. Although the source node at the initial position cannot reduce the candidate set of failure links ((b)), the mobility of the mobile node improves the identifiability of the boolean network tomography ((c)).

communication infrastructures. Although we can solve this problem by using multiple source and receiver nodes, this solution is not practical in such an emergent situation.

We propose a *mobility-assisted boolean network tomography* scheme to solve this problem. Fig. 5 shows its basic idea. As shown in this figure, we use two mobile nodes as *virtual source and receiver nodes*. One mobile node connects with a node as the virtual source node and injects probe packets, and the other mobile node connects with another node as the virtual receiver node and receives probe packets. The boolean network tomography scheme, proposed in this paper, is then applied in order to obtain a candidate set of failure links. When it is found that there are no more valuable measurement paths, the mobile nodes move and connect to other nodes. Although we need to consider a movement algorithm for mobile nodes, we leave it as a future work.

V. CONCLUSION

In this paper, we proposed an adaptive boolean network tomography scheme for identifying failure links in a network. The proposed scheme establishes an initial set of measurement paths and then adds measurement paths adaptively according to the candidate set and the identified set of failure links. Simulation experiments showed that the proposed scheme can reduce the number of measurement paths as compared with the non-adaptive boolean network tomography.

We have some remaining issues in the performance evaluation of the proposed scheme. In the future research, we will evaluate some important metrics such as the false positive rate and how some properties of network topologies such as the degree of nodes and the size of networks afect these metrics. Further, we will consider the procedure of the mobility-assisted boolean network tomography and evaluate its performance.

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