Transmitting Conditions of Cable Tree

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Abstract. Research and development of Spectrum Administration for metallic access networks analyzed possibilities of matrix representation for transmission circumstances. Some interesting remarks were gained as a side output of this work.

Keywords: Cable tree, signal spectrum function, noise spectrum function, transmitting function, PWD.

1 Cable tree

Key term **Cable tree** is defined in the *last mile* topology. **Cable tree** itself consists of sequence coupled cable sections concentrated in site of Main distribution frame in common trunk cable, see Fig. 1.
Note: Cable tree may be equipped by optical network units ONU, where optical signal is transformed into electrical and vice versa respectively; and which is in ONU-MDF received or transmitted into/from pairs of tided metallic cable.
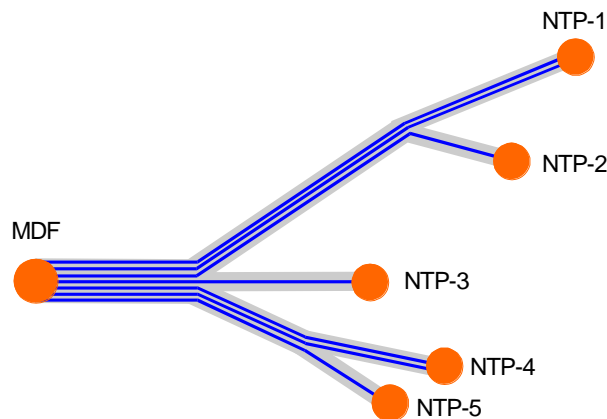

Fig. 1. Basic architecture of cable tree

2   Analytical Performance of Useful Signals & Disturbing Noises

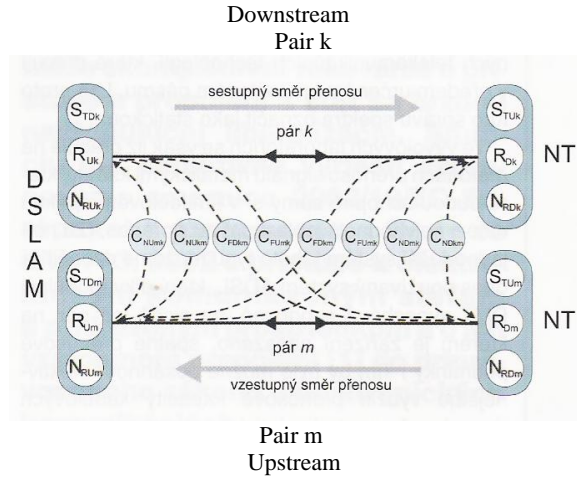Circumstances of mutual influences upon symmetric metallic cable are performed in Fig. 2:

Downstream
Pair k

Pair m
Upstream

Fig. 2: Signals a Couplings between two Pairs

The simplest generalizable case of bidirectional transmission alongside two symmetric pairs is performed here. These pairs are terminated in exchange site in v DSLAM, in subscriber site in two network terminations. Three transmission characteristics belong to each of four here performed terminating points:

- Transmitted spectrum of signals $S(\omega)$ or maybe $S(f)$
- Non wanted disturbing noise $N(\omega)$
- Receiver sensitivity $R(\omega)$

as well as inter pair crosstalk couplings

- Near end crosstalk (NEXT) $C_N(\omega)$
- Far end crosstalk (FEXT) $C_F(\omega)$

Indexes k and m define mutual position of pairs;
indexes D and U discriminate downstream and upstream transmission directions;
indexes T and R assign appropriate characteristics to transmission and receive *of* signal.

Transmitting circumstances in site of subscriber termination, *in Fig. 1 pointed NT* are expressed by following equation:

$$S_{RDk}(\omega) = T_{CTk}(\omega)\, S_{TDk}(\omega) \qquad\qquad (1a)$$

Equally in site of exchange termination *(DSLAM)* is valid:

$$S_{RUk}(\omega) = T_{CTk}(\omega)\, S_{TUk}(\omega) \qquad\qquad (1b)$$

where T_{CT} is a vector of cable tree transmitting functions

Noise circumstances in site of NT may be described by following equation:

$$N_{RDk}(\omega) = S_{TDm}(\omega)\, C_{FDmk}(\omega) + S_{TUm}(\omega)\, C_{NDmk}(\omega) \qquad (2a)$$

as well as dually in site exchange termination DSLAM

$$N_{RUk}(\omega) = S_{TDm}(\omega)\, C_{NUmk}(\omega) + S_{TUm}(\omega)\, C_{FUmk}(\omega) \qquad (2b)$$

Such circumstances are only exceptional in reality; maybe only in last cable section led from subscriber cabinet supposing one-quad subscriber cable and both pairs exploited by broadband service. All couplings in real cable tree are multiplied and combined many times. We are able to express these circumstances by matrixes. These matrixes will be of equal order to number of pairs in trunk cable of cable tree. This number of pairs may reach hundreds as well as exceed thousand in metropolitan networks. Composition of such matrix exceeds real possibility of calculation.

Note: Variable ω will be again left out in following text

The equation 1a is able to be performed by product of diagonal matrix:

$$
\begin{bmatrix}
S_{RD1} & 0 & 0 & 0 \\
0 & S_{RD2} & \ldots & \ldots \\
\ldots & \ldots & \ldots & 0 \\
0 & \ldots & 0 & S_{RDp}
\end{bmatrix}
=
\begin{bmatrix}
T_{CT1} & 0 & \ldots & 0 \\
0 & T_{CTp} & \ldots & \ldots \\
\ldots & \ldots & \ldots & 0 \\
0 & \ldots & 0 & T_{CTp}
\end{bmatrix}
*
\begin{bmatrix}
S_{TD1} & 0 & \ldots & 0 \\
0 & S_{TD2} & \ldots & \ldots \\
\ldots & \ldots & \ldots & 0 \\
0 & 0 & 0 & S_{TDp}
\end{bmatrix}
\qquad (3a)
$$

Equally equation 1b:

$$
\begin{bmatrix}
S_{RU1} & 0 & 0 & 0 \\
0 & S_{RU2} & \ldots & \ldots \\
\ldots & \ldots & \ldots & 0 \\
0 & \ldots & 0 & S_{RUp}
\end{bmatrix}
=
\begin{bmatrix}
T_{CT1} & 0 & \ldots & 0 \\
0 & T_{CTp} & \ldots & \ldots \\
\ldots & \ldots & \ldots & 0 \\
0 & \ldots & 0 & T_{CTp}
\end{bmatrix}
*
\begin{bmatrix}
S_{TU1} & 0 & \ldots & 0 \\
0 & S_{TU2} & \ldots & \ldots \\
\ldots & \ldots & \ldots & 0 \\
0 & 0 & 0 & S_{TUp}
\end{bmatrix}
\qquad (3b)
$$

The equation 2a performing noise circumstances by matrixes of pth order for p pairs of cable tree trunk cable as a product of left diagonal signal matrix multiplied by right crosstalk matrix with zero diagonal. Resulting noise matrix is again with zero diagonal in accordance with multiplying rule for matrixes.

$$
\begin{bmatrix}
0 & N_{RD12} & \ldots & N_{RD1p} \\
N_{RD21} & 0 & \ldots & \ldots \\
\ldots & \ldots & 0 & N_{RD(p-1)p} \\
N_{RDp1} & \ldots & N_{RDp(p-1)} & 0
\end{bmatrix} =
$$

$$
\begin{bmatrix}
S_{TD1} & 0 & \ldots & 0 \\
0 & S_{TD2} & \ldots & \ldots \\
\ldots & \ldots & \ldots & 0 \\
0 & 0 & 0 & S_{TDp}
\end{bmatrix} *
\begin{bmatrix}
0 & G_{FD12} & \ldots & G_{FD1p} \\
G_{FD21} & 0 & \ldots & \ldots \\
\ldots & \ldots & 0 & G_{FD(p-1)p} \\
G_{FDp1} & \ldots & G_{FDp(p-1)} & 0
\end{bmatrix} +
$$

$$
+
\begin{bmatrix}
S_{TU1} & 0 & \ldots & 0 \\
0 & S_{TU2} & \ldots & \ldots \\
\ldots & \ldots & \ldots & 0 \\
0 & 0 & 0 & S_{TUp}
\end{bmatrix} *
\begin{bmatrix}
0 & G_{NU12} & \ldots & G_{NU1p} \\
G_{NU21} & 0 & \ldots & \ldots \\
\ldots & \ldots & 0 & G_{NU(p-1)p} \\
G_{NUp1} & \ldots & G_{NUp(p-1)} & 0
\end{bmatrix} \quad (4a)
$$

The equation 2b may be performed dually as follows:

$$
\begin{bmatrix}
0 & N_{RU12} & \ldots & N_{RU1p} \\
N_{RU21} & 0 & \ldots & \ldots \\
\ldots & \ldots & 0 & N_{RU(p-1)p} \\
N_{RUp1} & \ldots & N_{RUp(p-1)} & 0
\end{bmatrix} =
$$

$$
\begin{bmatrix}
S_{TD1} & 0 & \ldots & 0 \\
0 & S_{TD2} & \ldots & \ldots \\
\ldots & \ldots & \ldots & 0 \\
0 & 0 & 0 & S_{TDp}
\end{bmatrix} *
\begin{bmatrix}
0 & G_{NU12} & \ldots & G_{NU1p} \\
G_{NU21} & 0 & \ldots & \ldots \\
\ldots & \ldots & 0 & G_{NU(p-1)p} \\
G_{NUp1} & \ldots & G_{NUp(p-1)} & 0
\end{bmatrix} +
$$

$$
\begin{bmatrix}
S_{TU1} & 0 & \ldots & 0 \\
0 & S_{TU2} & \ldots & \ldots \\
\ldots & \ldots & \ldots & 0 \\
0 & 0 & 0 & S_{TUp}
\end{bmatrix} *
\begin{bmatrix}
0 & G_{FU12} & \ldots & G_{FU1p} \\
G_{FU21} & 0 & \ldots & \ldots \\
\ldots & \ldots & 0 & G_{FU(p-1)p} \\
G_{FUp1} & \ldots & G_{FUp(p-1)} & 0
\end{bmatrix} \quad (4b)
$$

*Note: Due to the fact of invalidity commutative law for multiplying of matrixes, this sequence should be abided. On the other hand **change of this obligatory sequence leads to same phenomenon as by changing disturbing and disturbed lines, called as crosstalk measurement „straight" and „across" or marked = and x**.*

3   The Condition for Information Recoverability

The couple of equations 1 and 2 is simplified for next consideration, again missing variable ω for useful signal in downstream as well as upstream:

$$S_{Ri} \;=\; T_{CT}\,S_{Ti} \qquad \text{where } i = D,\, U \tag{5}$$

and analogous noise circumstances in site of subscriber exchange:

$$N_{RD} = S_{TD}\,G_{FD} + S_{TU}\,G_{ND} \tag{6a}$$

and again dually in site of network termination.

$$N_{RU} = S_{TD}\,G_{NU} + S_{TU}\,G_{FU} \tag{6b}$$

The condition for information transmission secured in the simplest configuration as to Fig. 2 in sufficient quality may be performed for down(up)stream as follows:

$$N_{RD(U)k}(\omega) \le R_{D(U)k}(\omega) \qquad \text{for } \; \omega \ni \Omega \tag{7}$$

These inequalities may be overwritten for cable tree generally:

$$N_{RD} \le R_D \tag{8a}$$

$$N_{RU} \le R_U \tag{8b}$$

Induction of both inequalities 8 into couple of equations 6 enables determination for highest assigned signals, there define masks of Power spectral density of transmitted signals:

$$S_{TD} \le \frac{1}{G_{NU} - \dfrac{G_{FU}G_{FD}}{G_{FU}}}\left(R_U - \frac{G_{NU}R_D}{G_{ND}} \right) \text{ for } \omega \ni \Omega \tag{9a}$$

and

$$S_{TU} \leq \frac{1}{G_{ND} - \dfrac{G_{FU}G_{FD}}{G_{NU}}} \left(R_D - \frac{G_{FD}R_U}{G_{NU}} \right) \text{ for } \boldsymbol{\omega} \ni \boldsymbol{\Omega} \qquad (9b)$$

Inequality system 9 may be altered by other constants produced by legislative measures of non-technical character. Matrixes of higher order are proposed to be calculated generally; therefore the volume of necessary calculations may exceed realizable measures.

List of symbols:

G_{FD} - matrix of transmitting functions of FEXT in downstream

G_{FU} - matrix of transmitting functions of FEXT in upstream

G_{ND} - matrix of transmitting functions of NEXT in downstream

G_{NU} - matrix of transmitting functions of NEXT in upstream

N_{RD} - noise spectrum vector in site of receivers in downstream

N_{RU} - noise spectrum vector in site of receivers in upstream

R_D - receiver sensitivity vector in downstream

R_U - receiver sensitivity vector in upstream

S_{RD} - signal spectrum vector in site of receivers in downstream

S_{RU} - signal spectrum vector in site of receivers in upstream

S_{TD} - signal spectrum vector in site of transmitters in downstream

S_{TU} - signal spectrum vector in site of transmitters in upstream

T_{CT} - transmitting functions vector of cable tree

ω - radian frequency

Ω - frequency interval (transmitted band)

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