A data aggregation methodology to assess the global production capacity of complex supply chains

Frederic Pereyrol[1], Jean-Christophe Deschamps[1], Julien François[1], Pascale Farthouat[1], Remy Dupas[1],

[1] University of Bordeaux, LAPS - IMS
351 Cours de la Liberation
33405 TALENCE cedex, France
{Frederic.Pereyrol, Jean-Christophe.Deschamps, Julien.François, Pascale.Farthouat, Remy.Dupas}@ims-bordeaux.fr

Abstract. Nowadays, no decisional tools allow to assess if an unforeseen customers demand variation should be accepted without creating material disruptions among a supply chain or not. The main difficulty consists in aggregating resources capacities, especially if resources perform different tasks with multiple items. This paper then proposes a data aggregation methodology based on graph analysis in order to assess the global production capacity of complex resources networks, like supply chains.

Keywords: aggregate capacity, networks flows, manufacturing processes

1 Introduction

In supply chains, decisions individually performed by each decision maker to elaborate its production plans guarantee a local optimal solution (i.e. the best compromise in terms of performances they intend to improve), but do not always ensure the most efficient coordination through the whole supply chain (SC). Many researchers have studied this problematic and have proposed different global solving approaches. Some studies proposed to develop a centralized middle-term planning process applied to the wide supply network. Others showed that performance is better when information (supply, production and delivery plans) is shared between industrial partners [2] [3] [5].

Since practical studies show that a centralized supply chain management represents today a non realistic approach for industrials, SC partners actually begin to share information to propose a common system of aggregated performance indicators. Assuming that this practice will be developed in the future, this paper proposes a data aggregation methodology starting from detailed technical data in order to assess the global production capacity of a supply chain. This study is motivated by the lack of tools allowing to evaluate if an unforeseen customers demand variation should be accepted without creating material disruptions among a SC or not.

In section 2, we will propose useful concepts and models in defining a new approach for assessing aggregated capacities in complex resources networks. Section 3 will describe the methodology used to reduce the complexity of graphs (and data) supporting the aggregate capacities calculation. A numerical example will be presented in section 4 to illustrate the considered approach before concluding.

2. Concepts and models

The originality of this paper consists in considering multi items performed on operating resources, their capacities being considered as dependant of the nature of the achieved products. The proposed approach consists in estimating an aggregated capacity to implement a global planning process in order to help managers to assess their own production loads in an aggregate way, according to customers' demands. A multi-level decomposition is applied to analyse the physical processes and graphs are used to model the supply chain structure. The proposed approach is defined, considering its application on a single time unit but should be generalized to a whole horizon time.

2.1 Modelling production processes by graphs

The proposed structured analysis framework is based on graph theory in order to assess the capacity of heterogeneous and interdependent production resources performing multiple items.

Let consider a detailed production process composed of a set $\underline{R}$ of operating resources R_r (where r is the index of resource – $\underline{R}=\{R_r \mid r\in[1..R]\}$). These ones perform a set $\underline{T}$ of tasks T_t - $\underline{T}=\{T_t \mid t\in[1..T]\}$. The relations between tasks and resources are expressed through the definition of a matrix Ω: each resource R_r fulfils one or several tasks, and its capacity is defined by a vector $\omega_r = [\omega_{1r, \ldots,}\omega_{tr, \ldots}\ \omega_{Tr}]^T$ $(t\in[1,\ldots,T])$ wherein each component ω_{tr} represents "the maximum number of tasks t performed per time unit when the resource r only executes this type of task". When a resource R_r has no technical competence to perform a task T_t, $\omega_{tr}=0$. The vectors set is synthesized through the matrix $\Omega=\{\omega_r \,/\, \forall r\in\underline{R}\}$ which represents the capacities set of all resources involved in production. The production process performs P items (components, intermediate and finished products). Those required for performing the different tasks are defined in matrix $B=[\beta_{pq} \mid p,q \in[1..P]\}$ where β_{pq} is the number of component P_p required to perform the product P_q, i.e matrix B characterizes the bill of materials (BoM) coefficients.

Digraphs (directed graphs) are used to depict two production representations: the resources dependencies and tasks sequencing. Let $\underline{V}$ and $\underline{E}$ be respectively the vertex and edge set of a acyclic graph $G=(\underline{V},\underline{E})$. Depicting the resources network by a graph G allows us to have a flow-oriented representation. Each vertex (or node) $v \in \underline{V}$ is associated to an operating resource R_r and a task T_t (fig 1) according to matrices:

- $D=[d_{vr} \mid v\in\underline{V}, r\in\underline{R}]$ with $d_{vr}=1$ if resource R_r is related to node v, 0 otherwise,

- $L=[l_{vt} \mid v\in \underline{V},\ t\in \underline{T}]$ with $l_{vt}=1$ if task T_t is related to node v, 0 otherwise,

Vr should be deduced from matrix D, expressing the vertices set associated to the same resource R_r. Directed edges $e\in \underline{E}$ (also called arcs and noted $e=(u,v)\ \forall u,v\in \underline{V}$) between nodes model the operating activities sequencing. Fig 1 proposes an example of graph G. Notice that, in this case, task T_2 is performed by resources R_3 or R_4, resource R_2 (respectively R_4) performs tasks T_4 (respectively T_2 and T_5).

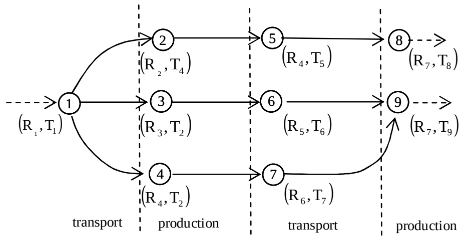


Fig. 1 Production process modelling by graph

In the following, we assume that, in graph G, a transportation activity always separates two production activities; divergence of materials flows is only due to transportation: a transportation task is represented by a node with one incoming arc and various outgoing arcs. Convergence of materials flow is only made in entrance of a production task T_t, which is then modelled by a vertex v with various incoming edges and one outgoing edge (each operation performs one item (one outgoing product) resulting from the production/assembly of one or various components with different quantities).

We also make the hypothesis that a product P_p and the task T_t which accomplishes this one should be assimilated, so that p=t (ex: task T_2 performs a product P_2).Based on this hypothesis, we define a matrix $A=[a_{uv} \mid u,v\in \underline{V}]$ similar to a classical adjacency matrix except that the value a_{uv} associated to each edge (when it exits) is determined as the coefficient β_{pq} such as it is known at the customer's who orders the product p (performed through the task T_t associated to vertex u).

2.2 Calculation of network capacities based on classical solving approaches

The Max-Flow determination in the graph G representing flow interactions between operating resources is one way to assess the network capacity, when each resource r is dedicated to one type of tasks (only one component of vector ω_r is not equal to zero).

In this case, the capacity matrix $C=[c_{uv} \mid u,v\in \underline{V}]$ of the graph G can be easily deduced from the matrix Ω. C is derived from the adjacency matrix of the graph G in which c_{uv} is a weight associated to an edge e outgoing from vertex u and incoming to vertex v. In order to simplify the graph analysis, we assume that bill of materials (BoM) coefficients β_{pq} are taken into account in the definition of capacities c_{uv}

expressed on the graph G (through the use of matrix A), so that the unit of measure is "the number of lots":

$$c_{uv}=(\sum_r \sum_t l_{ut}.d_{ur}.\omega_{tr})/(a_{uv}) \qquad \forall u\in \underline{V},\ v\in Out(u) \qquad (1)$$

The weight $c_{uv}\neq 0$ of every edge e is completed by a flow f_{uv} such as flows satisfy the classical constraints (2) and (3):

$$f_{uv}\leq c_{uv} \qquad \forall u,v\in \underline{V} \qquad (2)$$

$$\sum_{u\in In(v)} f_{uv} = \sum_{w\in Out(v)} f_{vw} \qquad \forall v\in \underline{V} \qquad (3)$$

with In(v) set of vertices which are the origin of all edges incoming to vertex v
Out(v) set of vertices which are the destination of all edges outgoing from v

Expression (3) verifies the Kirchhoff law and is applied only if flows converging to a vertex v can be interpreted as the same product flow: the various incoming edges of vertex v represent the various possibilities for delivering the product to the resource associated to the vertex. In this context, the calculation of the maximum flow through the graph G (or maximum throughput) is obtained by applying the famous Ford-Fulkerson algorithm [1] [5] or others analytical approaches [6].

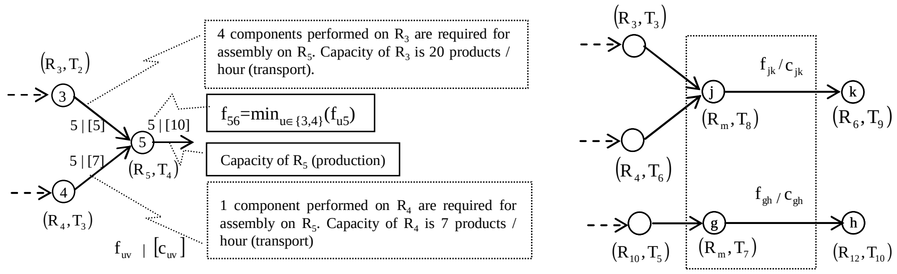
2.3 New fundaments for assessing the network capacity

Nevertheless, this calculus becomes hard when graphs represent more complex phenomena as operating resources which assemble components in intermediate products (i.e. several flows of components converge on the resources), or operating resources which perform multiple items during the same time interval, i.e. the resources capacity is dependent on the product.

The first quoted problem refers to production processes based on bill of materials with tree structures representing that different components types are required to perform assemblies and multiple material flows income to resources. Basing the calculation of the maximum throughput on the respect of the Kirchhoff law, graphs do not model these situations. In case of flows convergence representing different components (fig. 2a), the considered sub graph can be simplified to one vertex with one outgoing weighted arc (u,v). Expression (4) replaces expression (3):

$$f_{vw}=\min_{u\in In(v)}(f_{uv}) \qquad \forall v\in \underline{V},\ w\in Out(v) \qquad (4)$$

Besides, complex production processes should contain resources that (not simultaneously) perform multi items, represented by multiple vertices $v\in \underline{V}_r$ ($\underline{V}_r\subset\underline{V}$ is defined as the vertices subset associated to resource R_r, i.e. $v\in \underline{V}$ | $d_{vr}=1$). We assume that a constant capacity expressed on the edge outgoing from each $v \in \underline{V}_r$ is not representative of real phenomena when production load on resource R_r relative to a task completion reduces the remaining capacity useful in others tasks accomplishment.



(a) Convergence of different material flows (b) Resources performing multi items

Fig. 2 Complex phenomena modeled in graphs

Detailed information concerning the coupling of capacities associated to any edge modelling an output flow from a resource R_r should be kept on graphs to assess in the best way the maximum throughput. Considering that capacities on graph G are not always constants, depending on partial loads due to the various tasks execution on a same resource R_r, these variable capacities are dynamically calculated in accordance with any flow on graph describing an output material flow of this resource (fig. 2b). Capacities are updated every time the flows vary, according to the following relation:

$$c_{uv} = \sum_t l_{ut}.\omega_{rt}/a_{uv} - \sum_{g \in V_r - \{u\}} \sum_{h \in Out(g)} \sum_t \frac{l_{ut}.\omega_{rt}/a_{uv}}{l_{gt}.\omega_{rt}/a_{gh}} f_{gh} \qquad \forall u \in \underline{V},\ v \in Out(u),\ r \in \{R \mid d_{ur}=1\} \qquad (6)$$

3. A data aggregation methodology to assess a global production capacity

The proposed methodology based on graphs analysis should be applied recursively at an abstraction level n-1 in order to obtain an aggregate representation at level n. We introduce the notations R_r^n (respectively T_p^n) which reference any resource r (respectively any task t) at the level n. The methodology uses graphs properties and solving algorithms previously mentioned to simplify the graph structure, when it is required to aggregate at the level n (graph G^n), any detailed network defined by a graph G^{n-1} at the level n-1. The aggregation process is decomposed in several steps:

- *Step 1*: any arc e=(u,v) is deleted if $\sum_{w \in In(u)} c_{wu} < c_{uv}$ or $\sum_{w \in Out(v)} c_{vw} < c_{uv}$.
- *Step 2*: based on the detailed graph G^{n-1} describing the resource and task network at the lower abstraction level, a vertices subset $\underline{V}_r$ is defined,

considering that the subset $\underline{V}_r$ represents one tasks subsequence (i.e a unique tree/road exists and connects the various nodes $v \in \underline{V}_r$).

- *Step 3*: we define a resource $R_{r'}^{n}$ (respectively $T_{p'}^{n}$) which is the encapsulation of resources R_r^{n-1} (respectively tasks T_p^{n-1}) associated to vertices $v \in \underline{V}_r$. $\underline{V}_r$ is then modelled by one vertex u in graph G^n with one weighted outgoing arc e=(u,v) such as its capacity is :

$$c_{uv} = \min_{x,y \in \underline{V}_r}(c_{xy}) \tag{6}$$

  Notice that expression (6) is deduced from expression (4). $\underline{V}_r$ can define graph structures representing parts of a production process with operating resources which assemble different components. Go to step 2 until no new set $\underline{V}_r$ can be identified (any task subsequence is aggregated).

- *Step 5*: on the graph G^{n-1}, we search a vertices subset $\underline{V}_{r'}$ such as this one represents alternative tasks subsequence (i.e $\underline{V}_{r'}$ defines a acyclic sub graph G_s in which multiple roads connect the source to the sink).
- *Step 6*: resources and tasks are aggregated as defined in step 2 and G^n is partially completed. The capacity c_{uv} is deduced from the application of the Ford-Fulkerson algorithm to the sub graph G_s. Go to step 5 until all sets $V_{r'}$ are treated.

Graph G^n results from the reduction and simplification of the graph G^{n-1}, according to the calculation of aggregate capacities. The remaining structure of the graph is then mainly composed of vertices associated to resources performing multi items and should undergo a last reduction in order to keep one vertex associated to each performed product type. The global capacity of the network is then deduced from the graph G^n by calculating the maximum throughputs according to capacities associated to the edges set. The aggregation process should be applied recursively.

4. Illustrative example

In this section, a SC example is defined to concretely apply the data aggregation methodology.

4.1 Description / modeling

The studied SC instance is composed of a two coupled multi-stage supply chains structure. Three products are performed: tables (product 1), small-sized shelves (product 2) and large-sized shelves (product 3). For reducing the complexity, transport capacities are supposed infinite, so that only production activities impact the global capacity assessment. Some details concerning the case study from which this numerical example is extracted should be found in [3] [4]. The following graph G^0 models the SC at the detailed level (Fig. 3a).

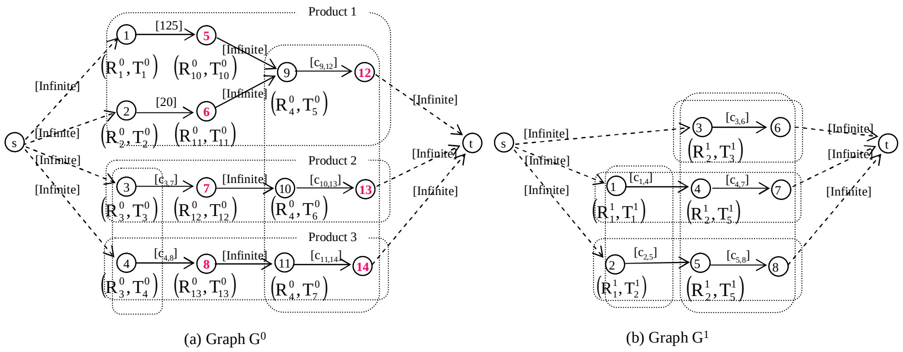


Fig. 3 SC modeling by graphs

Arc capacities c_{uv} mentioned on the graph G^0 result from equation (6) such as:

$c_{3,7}$ =15–1,5$f_{4,8}$; $c_{4,8}$=10–0,67$f_{3,7}$
$c_{9,12}$=20- f $_{10,13}$–1,67$f_{11,14;}$ $c_{10,13}$=20 -$f_{9,12}$–1,67$f_{11,14}$; $c_{11,14}$ = 12–0,6$f_{9,12}$–0.6$f_{10,13}$

According to the proposed methodology, graph G^0 is aggregated leading to the elaboration of graph G^1 (fig. 3b) from which we can deduce the matrix C^1 defining the various capacities of arcs, assimilated to those of the supply chain (represented by a single resource) in relation with the different aggregate tasks (one task for one performed product). Considering a workload vector $W=(w_1\ w_2\ w_3)^T$ in which w_p is the number of products P_p to be performed in the supply chain, we assume that $(f_{1,4}\ f_{2,5})^T = (w_2\ w_3)^T$ represents the order book for the resource R_1^1, and $(f_{3,6}\ f_{4,7}\ f_{5,8})^T = (w_1\ w_2\ w_3)^T$ represents the order book for the resource R_2^1. The capacity of the aggregate resource (supply chain R) is then a vector, so that:

$$CAP(R)=\begin{bmatrix} 20\text{-}w_2\text{-}1{,}67w_3 \\ \min(15\text{-}1{,}5w_3,\ 20\text{-}w_1-1{,}67w_3) \\ \min(10\text{-}0{,}67w_2,\ 12\text{-}0{,}6w_1\text{-}0{,}6w_2) \end{bmatrix} \qquad (10)$$

4.2 Exploitation of the aggregate capacity

Each component of CAP(R) is interpreted as the maximum number of each product the supply chain performs, if loads induced by others productions is known. The comparison of CAP(R) with values defined in W allows to verify if orders book W should be performed or not, during the considered time unit. Suppose that the workload vector is $W=(4\ 4\ 2)^T$. Expression (10) leads to assess the supply chain capacity as $CAP(R)=(12.67\ \ 12.00\ \ 7.20)^T$. We notice that W<CAP(R) (i.e. the value of every element of W is lower than the value of the corresponding element in

CAP(R)), showing that the supply chain may perform the orders book. If we now consider a vector W=(10 5 4)^T, the vector CAP(R) becomes CAP(R)=(8.33 3.33 3)^T. We can observe that 10 products P_1 are required to be performed and only 8 can be completed during the considered time interval (one time unit). If necessary, overloaded resources should then be defined by analysing the capacities on resources R_1^1 and R_2^1: CAP(R_1^1)=(9 6.67)^T and CAP(R_2^1)=(8.33 3.33 3)^T shows that overload is only due to the second resource.

5. Conclusion

The assessment of aggregate capacities in complex resources networks is today a great challenge in developing rapid computational approaches able to show if new orders to perform in a supply chain may cause disruption or not. In this context, we propose first fundaments of a data aggregation methodology, referring to graph theory. Nevertheless, the numerical example being simple, the approach feasibility must be proved through its application to more complex networks. Experiments must also be performed to determinate the inaccuracy of production loads calculations based on aggregated capacities knowledge, relative to results issue from a detailed complex planning process.

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