Estimation of mean response time of multi–agent systems

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Abstract. The following analytical approaches: queuing network models, stochastic automata networks, stochastic timed Petri nets, stochastic process algebra, Markov chains can be used in performance evaluation of multi–agent systems. In this paper, new approach which is based on PERT networks is presented. This approach is applied in performance evaluation of layered multi–agent system. Time-out mechanisms are used in communication between agents. Our method is based on approximation using Erlang distribution. Accuracy of our approximation method is verified using simulation experiments.

1 Introduction

In this paper, an analytical approach, which is based on stochastic PERT networks, is developed. The approach is applied in performance evaluation of layered multi–agent system. These layers are associated with the following types of agents: manager, bidder, and searcher ones. Our method is based on approximation using Erlang distribution. Erlang distribution is one of probability distributions that are used in evaluation of completion times in stochastic PERT networks. In the paper [4], an approximation method which is based on Erlang distribution has been applied for the above layered multi–agent system. In this paper, there was no bounds for time of waiting for messages from the agents. In present paper, time–out mechanisms are used in communication between the agents. Accuracy of our approximation method is verified using simulator. This simulator has been previously used in simulation experiments with the following multi–agent systems: personalized information system [1], industrial system [2], system with static agents and system with mobile agent [3]. These systems have been expressed in standard FIPA [5] which the JADE technology [6] is complied with.

In section 2, the multi–agent system is described. Then our approximation method is presented. In section 4, accuracy of our approximation method is verified by comparison with simulation results. Finally, there are conclusions.

2 Layered multi–agent system

We consider layered multi–agent information retrieval (*MAS*) system given at Fig. 1.

The *MAS* includes: one manager type agent (*MTA*) as Fat Agent, two bidder type agents(*BTAs*) as Thin Agents, Searcher type agents (*STAs*) as Thin Agents. One *BTA* co-operates with a number of *STAs*.

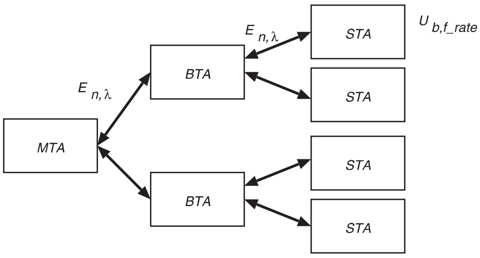

Fig. 1. Layered multi–agent information retrieval system

After receiving a request from an user, the *MTA* sends messages to the *BTAs* in order to inform them about the user request. Then the timer of the *MTA* is started, and the *MTA* is waiting for two responses from the *BTAs*. The waiting time is limited by the termination time *tm*. Having two responses from the *BTAs*, the *MTA* prepares the response for the user. If the maximal waiting time *tm* has elapsed then the *MTA* prepares the response for the user having information received from the *BTAs* until the *tm* has elapsed.

After receiving a request from the *MTA*, the *BTA* sends messages to all *STAs* co-operating with this *BTA*. Then the timer of the *BTA* is started, and the *BTA* is waiting for responses from all its *STAs* but no longer than the termination time *tb*. Having responses from the *STAs*, the *BTA* prepares the response for the *MTA*. If the maximal waiting time tb has been elapsed then the *BTA* prepares the response for the *MTA* having information received from the *STAs* until the *tb* has elapsed.

The *STA* prepares the response by searching in Data Base (*DB*). Each *STA* is associated with one *DB*. The probability of finding the response in the *DB* is denoted by *f_rate*. Time unit is second, and it will be omitted. Searching time is expressed by uniform distribution over the time interval $[0, b)$. Hence, the expected searching time, provided there is the required information in the *DB*, is equal to $b/2$. Searching time is equal to b with the probability $1 - f\_rate$.

Message transmition times between the *MTA* and the *BTA*, and between the *BTA* and the *STA* are given by *n* stage Erlang distributions with parameter λ for each stage.

3 Erlang distribution based approximation method

We will explain how the expected value of time of receiving of a response by the user is approximated. Because of the lack of space some derivations will be omitted. Probability distributions of times are approximated by Erlang ones [7].

Random variable (*RV*) with this distribution will be denoted by $E_{n,\lambda}$. This *RV* can be interpreted as a sum of *n RVs* with exponential distribution and each with parameter λ. Expected value and variance for this *RV* are equal to $E(E_{n,\lambda}) = n/\lambda$ and $Var(E_{n,\lambda}) = n/\lambda^2$, respectively. For the *RV T*, the squared coefficient of variation (*SCV*) of the *T* is defined by the formula:

$SCV(T) = Var(T)/E(T)^2$ where: $E(T)$ is the expected value of T, $Var(T)$ is the variance of T. The *SCV* for the $E_{n,\lambda}$ is equal to $SCV(E_{n,\lambda}) = 1/n$.

The *RV* of the *STA* searching time in the *DB* will be denoted by U_{b,f_rate}. This *RV* has the probability density function:

$$f_{U_{b,f\_rate}}(t) = \begin{cases} f\_rate \cdot 1/b & \text{for } t \in [0, b) \\ (1 - f\_rate) \cdot \delta(t-b) & \text{for } t = b \\ 0 & \text{otherwise} \end{cases}$$

Expected value, variance, and *SCV* for this *RV* can be found in [4].

Let us consider the approximation of the probability distribution of the *RV X* of the length of the time interval between the time instant when the *BTA* sends the request to given *STA* and the time instant when the *BTA* receives the response from this STA. This *RV* is given by the expression: $X = E_{n,\lambda} + U_{b,f\_rate} + E_{n,\lambda}$. We suppose that *RVs* of transmission times between agents and *RVs* of searching processes in the *DBs* are independent. The formulae for expected value, variance, and *SCV* of *RV X* can be found in [4].

For multi–agent system described in section 2, the *RVs* of transmission times between agents are two stage Erlang distributions with parameter $\lambda = 1$ for each stage, and will be denoted by $E_{2,1}$.

The *RV X* is approximated by the *RV* $E_{n,\lambda}$, and with the $SCV = 1/n$ such that $|SCV(X) - 1/n|$ is minimal. The expected values of the *RVs X* and $E_{n,\lambda}$ are equal. Hence, the parameter λ is selected according to the equality $\lambda = n/E(X)$.

Let *m* be the number of *STAs* associated with one *BTA*. Let $E_{n,\lambda}(i)$ be such a *RV* $E_{n,\lambda}$ that approximates the length of the time interval between the time instant when the *BTA* sends the request to i^{th} *STA* and the time instant when the *BTA* receives the response from this *STA*. In this case, the *RV Y* of the *BTA* waiting time for all responses from *STAs* is $Y = \max_{i \in \{1,\ldots,m\}} E_{n,\lambda}(i)$. The cumulated distribution function of the *RV Y* is given by the expression: $F_Y(t) = \left(F_{E_{n,\lambda}}(t)\right)^m$. The k^{th} moment (noncentral) of the *RV Y* is obtained by numeric integration of the following formula:

$$\mu^{(k)}(Y) = k \int_0^{\infty} t^{k-1} \, (1 - F_Y(t)) \, dt$$

Then the *RV Y* is approximated by *RV* E_{n_Y,λ_Y} in the same way as the *RV X* has been approximated by the *RV* $E_{n,\lambda}$.

Now, let us suppose that the *BTA* waits for the responses from the *m STAs* not longer than for the termination time *tb*. Therefore, we analyse the *RV* E_{n_Y,λ_Y} truncated in the *tb*. This *RV* will be denoted by *W*. The *CDF* and the *k*-th moment of the *RV W* are given by the expressions:

$$F_W(t) = \begin{cases} 0 & \text{for } t \le 0 \\ F'_W(t) & \text{for } 0 \le t \le tb \\ 1 & \text{otherwise} \end{cases}$$

$$\text{where} \quad F'_W(t) = \frac{\gamma\,(n_Y + 1, \lambda_Y\, t) + {\lambda_Y}^{n_Y} t^{n_Y} e^{-\lambda_Y\, t}}{n_Y!} \tag{1}$$

$$\mu_W^{(k)} = \frac{\gamma\,(n_Y + k, \lambda_Y \cdot tb)}{(n_Y - 1)!\, {\lambda_Y}^k} + \frac{tb^k\, \Gamma\,(n_Y, \lambda_Y \cdot tb)}{(n_Y - 1)!}$$

$$E(W) = \mu_W^{(1)}; \quad Var(W) = \mu_W^{(2)} - (\mu_W^{(1)})^2$$

The *RV* W is not approximated. The *RV* of the length of the time interval between the time instant when the *MTA* sends the request to given *BTA* and the time instant when the *MTA* receives the response from this *BTA* is approximated by the *RV*: $Z = E_{2,1} + W + E_{2,1}$.

The expected value of time of receiving of a response by the *MTA* (or user), i.e. response time, is approximated in the similar way as the expected value of the *RV Y* has been approximated.

4 Accuracy of the approximation method

In order to evaluate the accuracy of the approximation method, the simulation for: the *MAS* containing *m STAs* for each *BTA*, where $m = 3,\ 10$, have been performed. For each *MAS*, the following values of $f\_rate = 0.1,\ 0.3,\ 0.6,$ and 0.9 have been considered. The transmission time between agents is given by *RV* $E_{2,1}$. Hence, the mean transmission time between the agents is equal to $E(E_{2,1}) = 2$. In table 1, the percentage errors of

Tab. 1. Percentage errors of mean response time

b	tb	tm	m \ f_rate	0.1	0.3	0.6	0.9
16	20	27	3	0.7%	−0.4%	−2.5%	−2.6%
			10	1.4%	1.3%	0.7%	−0.1%
32	38	45	3	0.5%	−0.7%	−3.1%	−2.5%
			10	0.9%	0.7%	0.6%	1.0%
32	380	450	3	17.2%	11.9%	9.8%	7.6%
			10	28.2%	22.3%	24.6%	21.0%

mean response time for choosen values of b, tb and tm are given. In the case when the maximal searching time $b = 16$ and the termination times $tb = 20$, $tm = 27$, we have $b/E(E_{2,1}) = 8$. The approximation results are very good, errors are below 3%. When the maximal searching time $b = 32$ and the termination times $tb = 38$, $tm = 45$, then $b/E(E_{2,1}) = 16$. Even in this case, when the uniform distribution of *RV* of searching time is strongly dominating the Erlang distribution of *RV* of transmission times, the Erlang distribution based approximation is very good. In the third group of results the maximal searching time $b = 32$, the termination times $tb = 380$, $tm = 450$. In this case, $b/E\,(E_{2,1}) = 16$. Now, the approximation errors are much greater than previously. However, it is not realistic choice of parameters, because the termination time *tb* is more than 10 times greater than the mean time of the $RV\ \ X = E_{2,1} + U_{32,f\_rate} + E_{2,1}$.

5 Conclusions

In the approximation method, the *RV* with *n* stage Erlang distribution is used. It has been obtained from the simulation, that the sum of the *RV* of the Erlang distribution (representing the transmission time) and the *RV* of searching time with uniform distribution can be approximated by the other *RV* of Erlang distribution with suitable number of stages.

Many multi–agent systems have layered structure with the following agents: client assistant, brokers, execution agents. The presented performance approximation method can be used for finding the mean time of response on client request for this class of systems. In the future, we will try to get a better approximation using the general phase type distribution instead of the Erlang one.

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