A Simple Approach to Robust Optimal Pole Assignment of Decentralized Stochastic Singularly-Perturbed Computer Controlled Systems

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Abstract. This paper develops a simple algorithm for having robust optimal computer control in decentralized stochastic singularly-perturbed systems by poles assignment. This type of noise-disturbed system can be often seen in computer controlled large-scale systems such as electric power systems, communication networks, and aerospace systems. Due to that this computer controlled system possesses the fast response characteristics of the subsystems, the system analysis can be simplified by singularly perturbation methodology and the aggregation matrix is also applied to obtain faster calculation. Finally, the aggregation matrix is found out that will be an important intermediary to easily achieve the robust sub-optimal poles assignment. In the end, three steps are proposed to complete the robust sub-optimal pole assignment.

Key words: robust, computer, pole, decentralized, stochastic, singularly-perturbed, aggregation matrix.

1 Introduction

Pole placement of large-scale systems has been a difficult task due to the high dimension of the systems. How to simplify the process of placing optimal poles is the goal of this research. In this paper, the system is concerned with decentralized stochastic singularly-perturbed computer controlled systems. Such systems are two-time scale systems. Practically, computer controlled systems are this type of systems.

There are some similar researches related to this field. G. Enea, J. Duplaix, and M. Franceschi [1] use a recursive method to achieve optimal control with aggregative pole assignment in the discrete MIMO systems. A. R. Arar and M.E. Sawan [2] propose a design method for optimal control with eigenvalue placement in a specified region; in 1997, they present the work about the relation between pole-placement and linear quadratic regulator for discrete time systems [3]. In [4], Yao studied computer control of decentralized singularly-perturbed systems, but the noise disturbing factors, fast algorithms and robustness are not concerned.

Among all system performance requirements, robust stability is a paramount condition for designs of system control. Especially in [5]-[7], numerous approaches have been proposed and these systems concerned are singularly-perturbed systems. Yahli Narkis [8] developed a relation for direct calculation of the cost function for an optimally controller linear system with quadratic criteria, disturbed by a colored noise of any given spectral density distribution. Jianguo Wang; Guangyi Cao; Jin Zhou [9] study how the optimization methods can be used to deal with plant uncertainty. A weighed sensitivity error function is presented for an optimal robust controller design in a class of stochastic model errors. As observed by the previous work that has been done for the stability, enhancing performance, and cost minimization of decentralized stochastic systems, none had focus on the robust optimal pole assignment of decentralized stochastic singularly-perturbed computer controlled systems.

In this paper, the optimal poles found are based on a reduced-order system model. The optimal pole region of the close-loop system can be realized by adjusting the state weighting matrix and the input weighting matrix. After collecting and saving all the information of the relationship among the weighting matrices and the aggregation matrix, the optimal feedback gain of the system can be understood.

2 System Prescription

The mathematical model of the n-order decentralized stochastic singularly- perturbed system is shown as:

$$\begin{cases} \dot{x} = A_{00}x + A_{01}z_1 + A_{02}z_2 + A_{03}z_3 + \ldots\ldots A_{0m}z_m \\ \varepsilon\dot{z}_1 = A_{10}x + A_{11}z_1 + B_1u_1 + G_1w_1 \\ \varepsilon\dot{z}_2 = A_{20}x + A_{22}z_2 + B_2u_2 + G_2w_2 \\ \vdots \\ \varepsilon\dot{z}_m = A_{m0}x + A_{mm}z_m + B_mu_m + G_mw_m \end{cases} \tag{1a}$$

$$\begin{cases} y_1 = C_1z_1 + v_1 \\ y_2 = C_2z_2 + v_2 \\ \vdots \\ y_m = C_mz_m + v_m \end{cases} \tag{1b}$$

or

$$\dot{x} = A_{00}x + \sum_{i=1}^{m} A_{0i}z_i \tag{2a}$$

$$\varepsilon\dot{z}_i = A_{i0}x + A_{ii}z_i + B_iu_i + G_iw_i \tag{2b}$$

$$y_i = C_iz_i + v_i \tag{2c}$$

where i=1~m. The system is a linear time-invariant decentralized stochastic singularly-perturbed computer controlled system which has n-order and m independent inputs or m sub-systems. $x \in R^S$ and $z \in R^F$ are the slow and the fast state variables respectively; each sub-system z_i has its own order. $u_i \in R^{n_{1i}}$ and $y_i \in R^{n_{2i}}$ are the input vector of the i-th subsystem and the output vector of the i-th subsystem respectively. A_{00}, A_{0i}, A_{i0}, A_{ii}, C_i, and G_i are constant matrices with appropriate dimensions with i=1~m. $w_i \in R^S$; w_i and v_i are disturbing noises of inputs and outputs.

3 Main Results

Finding a easy algorithm of robust sub-optimal control is the goal of this study. A system performances based on system uncertainties is necessarily investigated and tested. Uncertainties of systems are caused by the inevitable errors in system modeling due to inexact and incomplete data, simplifying approximations, neglected high frequency dynamics, and unpredicted disturbances from the environment. In this research, robust control is defined that if the desired performance still exists after using the reduced-order controllers in the full-order systems.

In this type of particular system the major uncertainty would be the fast state variables of the subsystems. Because the overall system is a decentralized computer controlled system, the responses of computer-based subsystems are a lot faster than the main plant. The responses of the fast state variables will die out pretty fast in the very initial time period. Therefore, the overall structure is potentially a singularly perturbed system. When the system reaches Quasi-steady state, the parameter ε can be assumed as zero. Due to this phenomenon, the fast state variables can be ignored and the order of the system can be reduced. This also rises the idea that the state model of the system can be approximated.

In Eq. (2b), the sub-station station variables, z_1, z_2, z_3 ... have reached quasi-steady state. Hence, the system order is reduced to the order of the main station which is equal to the dimension of the slow state variable x. Eq. (2b) can be shown as:

$$\begin{cases} z_1 = A_{11}^{-1}(-B_1 u_1 - A_{10}x - G_1 w_1) = -A_{11}^{-1}B_1 u_1 - A_{11}^{-1}A_{10}x - A_{11}^{-1}G_1 w_1 \\ z_2 = A_{22}^{-1}(-B_2 u_2 - A_{20}x - G_2 w_2) = -A_{22}^{-1}B_2 u_2 - A_{22}^{-1}A_{20}x - A_{22}^{-1}G_2 w_2 \\ \vdots \\ z_m = A_{mm}^{-1}(-B_m u_m - A_{m0}x - G_m w_m) = -A_{mm}^{-1}B_m u_m - A_{mm}^{-1}A_{m0}x - A_{mm}^{-1}G_m w_m \end{cases} \tag{3a}$$

Then,

$$z_i = -A_{ii}^{-1}B_i u_i - A_{ii}^{-1}A_{i0}x - A_{ii}^{-1}G_i w_i \tag{3b}$$

where i=1~m; $A_{11} \sim A_{mm}$ are nonsingular matrices. Next, recall the state equation of the slow state variable in (2a). We can obtain new representations for the equation of slow state variables by using Eq. (3b):

$$\dot{x} = [A_{00} - \sum_{i=1}^{m} A_{0i}A_{ii}^{-1}A_{i0}]x + \left[-A_{01}A_{11}^{-1}B_1 u_1 - A_{02}A_{22}^{-1}B_2 u_2 ..... - A_{0m}A_{mm}^{-1}B_m u_m\right] \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_m \end{bmatrix} +$$

$$[\sum_{i=1}^{m} - A_{0i}A_{ii}^{-1}G_i w_i] \tag{4}$$

Now, define $G_r = [\sum_{i=1}^{m} - A_{0i} A_{ii}^{-1} G_i w_i]$ and Let $G_r = Hw$ where H is a non-square matrix and $w = \begin{bmatrix} w_1 \\ w_2 \\ : \\ w_m \end{bmatrix}$. Then, $H = [\sum_{i=1}^{m} - A_{0i} A_{ii}^{-1} G_i w_i] \, w^R = [\sum_{i=1}^{m} - A_{0i} A_{ii}^{-1} G_i w_i] \, [(w^T w)^{-1} w^T]$ where $w^R = [(w^T w)^{-1} w^T]$ is pseudo right inverse of w. An n-order multi-input decentralized stochastic singularly-perturbed computer controlled system is reduced into an S=(n-F)-order multi-input time-invariant system. The state model can be revised as

$$\begin{cases} \dot{x}_r = A_r x_r + B_r u + G_r = A_r x_r + B_r u + Hw & \text{(5a)} \\ y_i = C_i z_i + v_i = C_{ri} x_r + D_{ri} u_i + v_i & \text{(5b)} \end{cases}$$

where $z_i = -A_{ii}^{-1} B_i u_i - A_{ii}^{-1} A_{i0} x_r$; $C_{ri} = -C_i A_{ii}^{-1} A_{i0}$; $D_{ri} = -C_i A_{ii}^{-1} B_i$; $A_r = [A_{00} - \sum_{i=1}^{m} A_{0i} A_{ii}^{-1} A_{i0}]$; $B_r = \begin{bmatrix} -A_{01} A_{11}^{-1} B_1 & -A_{02} A_{22}^{-1} B_2 & \ldots\ldots\ldots & -A_{0m} A_{mm}^{-1} B_m \end{bmatrix}$; $u = \begin{bmatrix} u_1 \\ u_2 \\ : \\ u_m \end{bmatrix}$; $G_r = [\sum_{i=1}^{m} - A_{0i} A_{ii}^{-1} G_i w_i]$; $w = \begin{bmatrix} w_1 \\ w_2 \\ : \\ w_m \end{bmatrix}$; $i = 1\ldots\ldots m$. Therefore, when the system data is processed by computer, the above equations will be transformed into discrete-time model as [10]

$$x_r((k+1)h) = \Phi x_r(kh) + \Gamma u(kh) + G_r(kh) \qquad \text{(6a)}$$

where $\Phi = \phi(h) = e^{A_r h}$; $\Gamma = \int_0^h \phi(\lambda) d\lambda B$. h denotes the sampling rate.

$$y_i(kh) = C_i z_i(kh) = C_{ri} x_r(kh) + D_{ri} u_i(kh) + v_i(kh) \qquad \text{(6b)}$$

Now we define another non-square matrix $\overline{T}$, which

$$x_r = \overline{T} x_f \qquad \text{(7)}$$

where the full-order state vector $x_f = \begin{bmatrix} x \\ z_i \end{bmatrix}$; $x_f \in R^{S+F}$ with $x \in R^S$ and $z \in R^F$.

By the state transformation, the non-square matrix $\overline{T}$ that is called the fast aggregation matrix here that is used as an intermediary to have transformation between the full order model (2) and the reduced order model (5). This non-square matrix will help to shorten the derivation process.

$$A_r = \overline{T} \begin{bmatrix} A_{00} & \sum_{i=1}^{m} A_{0i} \\ A_{i0}/\varepsilon & A_{ii}/\varepsilon \end{bmatrix} \overline{T}^R = \overline{T} \begin{bmatrix} A_{00} & \sum_{i=1}^{m} A_{0i} \\ A_{i0}/\varepsilon & A_{ii}/\varepsilon \end{bmatrix} \overline{T}^T (\overline{T}\,\overline{T}^T)^{-1} \qquad \text{(8)}$$

$$B_r = \overline{T}\begin{bmatrix} \Phi \\ B_i \end{bmatrix} \tag{9}$$

$$C_{ri} = [\Delta \quad C_i]\overline{T}^R = [\Delta \quad C_i]\overline{T}^T(\overline{T}\,\overline{T}^T)^{-1} \tag{10}$$

where R denotes the pseudo right inverse and Δ matrix has all the elements equal to zero with appropriate size. i's of Eq. (8) to Eq. (10) indicate the controlling subsystem. By applying Eq. (10), Eq. (8) can be revised as

$$A_r = \overline{T}\begin{bmatrix} A_{00} & \sum_{i=1}^{m} A_{0i} \\ A_{i0}/\varepsilon & A_{ii}/\varepsilon \end{bmatrix}[\Delta \quad C_i]^L C_{r1} \tag{11}$$

Theoretically, $[\Delta \quad C_i]^L$ is a singular matrix and this matrix will be crucial item to find the fast aggregation matrix, $\overline{T}$. Next, based on the reduced-order state model (6), Eq. (6a) can be shown as

$$x_r(k+1) = \Phi x_r(k) + \Gamma_1 u_1(k) + \Gamma_2 u_2(k) + \ldots\ldots + \Gamma_m u_m(k) + G_r(kh) \tag{12}$$

where Γ_1 is the first column of Γ; Γ_2 is the second column of Γ and so on. $u_1(k) \sim u_m(k)$ are the inputs of the subsystem one to the subsystem m. In the close-loop control systems as Fig. 1, we know

$$\begin{cases} u_1(k) = d_1(k) + K_1 x_r(k) + G_r \\ u_2(k) = d_2(k) + K_2 x_r(k) + G_r \quad \text{and so on.} \end{cases} \tag{13}$$

where $d_1(k)$ and $d_2(k)$ are additional inputs; G_{r1} and G_{r2} are disturbing signals.

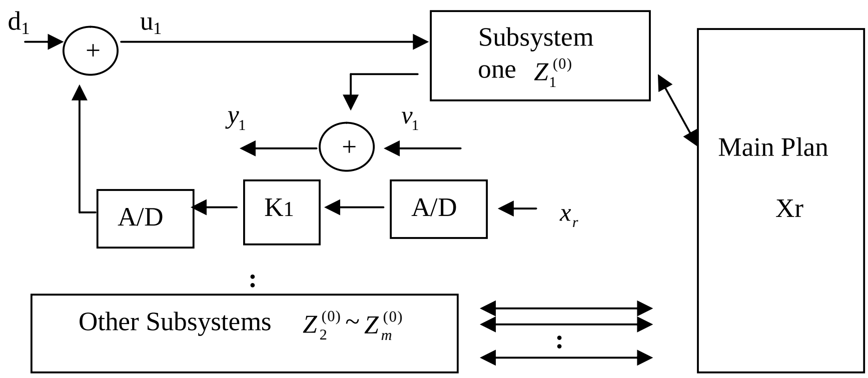

Fig. 1. The decentralized stochastic singularly-perturbed system. All the subsystems are computer processing units and assumed to be zero-order.

Now, if the main station is controlled from the subsystem one, we can assume the $d_1 \sim d_m$ and G_{r1} and G_{r2} are all disturbing noise to the controller one. They only affect the amplitude of system responses. The pole locations are unchanged.

Therefore; we revise (6) as

$$x_r(k+1) = [\Phi_N + \Gamma_1 K_1(k)]x_r(k) + JN(k) + G_r(kh) \tag{14a}$$

where $\Phi_N = (\Phi + \Gamma_2 K_2 + \Gamma_3 K_3 + ...\Gamma_m K_m)$ and $K_2 \sim K_m$ are existing feedback gains. $J = \begin{bmatrix}\Gamma_1 & \Gamma_2 & .... & \Gamma_m\end{bmatrix} = \Gamma$; $N(k) = \begin{bmatrix} d_1(k) \\ d_2(k) \\ : \\ d_m(k) \end{bmatrix}$

$$y_i(k) = C_{ri}x_r(k) + D_{ri}u_i(k) + v_i(k) \tag{14b}$$

Eq. (14) is a closed-loop state model. According to stochastic control theory [11] and singular perturbation methodology [12], the LQ performance index of each subsystem in the full order system:

$$J_i = \frac{1}{2}\sum_{k=0}^{N-1}(w^T(k)Q_i w(k) + u_i^T(k)R_i u_i(k)) \tag{15}$$

where Q_i is the weighting matrix with p. s. d. for each sub-system and $Q_i = \begin{bmatrix} Q_{11}^i & 0 \\ 0 & Q_{22}^i \end{bmatrix}$. R_i is the weighting matrix with p. d. for each sub-system. $w = \begin{bmatrix} x_r \\ z_i \end{bmatrix}$; x_r is a slow state vector and $z_i = \begin{bmatrix} z_1 \\ z_2 \\ : \\ z_m \end{bmatrix}$ is a fast state vector.

It can be presented as Eq. (16) that has the reduced order state vector, x_r.

$$J_i = \frac{1}{2}\sum_{k=0}^{N-1}(x_r^T(k)Q_{ri}x_r(k) + u_i^T(k)R_i u_i(k)) \tag{16}$$

With this constrain (16), if we have control from the subsystem one, we can have the optimal control

$$u_1^{optimal}(k) = -K_{r1}x(k) \tag{17}$$

where
$$K_{r1} = (B_{r1}^T P B_{r1} + R_1)^{-1} B_{r1}^T P \Phi_N \tag{18}$$

and, the P is the solution of the Riccati equation

$$P = \Phi_N^T\{P - PB_{r1}[B_{r1}^T PB_{r1} + R_1]^{-1}B_{r1}^T P\}\Phi_N + Q_{r1} \tag{19}$$

where P is a constant matrix. The $u_1^{optimal}$ not only minimizes the energy use but also stabilizes the system. This stabilizing feedback gain stabilizes the slow state variables of the system. There will be no control to the fast state variables; therefore, stability of the fast state variables is required. Furthermore, in steady state, the optimal control cost from the subsystem one can also be obtained as

$$J_{\infty}^{optimal} = \frac{1}{2} x_r^T(0) P x_r(0) \tag{20}$$

The robust sub-optimal poles are

$$P_d = eig[\Phi_N + B_{r1}K_{r1}] \tag{21}$$

where P_d is the desired sub-optimal pole locations. The optimal feedback control and the optimal costs of the subsystem two to the subsystem m can be found by the same procedure used in the subsystem one. For a successful state feedback design, stabilizability is a necessary condition, and controllability is a sufficient condition.

In the forgoing process, we use the reduced order state model (6) and existing feedback gains $K_{r2} \sim K_{rm}$ to compute the sub-optimal feedback gain of the subsystem one: $K_{r1} = (B_{r1}^T P B_{r1} + R_1)^{-1} B_{r1}^T P \Phi_N$, if the control is performed from the subsystem one. Now, we would like to find the sub-optimal feedback gain, K_1, for the original full order system by using the aggregation matrix, then the input of the subsystem one.

$$u_1 = -K_{r1} x_r \tag{22}$$

where x_r denotes reduced-order state. According to the state transformation technique, Eq. (22) can be shown as

$$u_1 = -K_{r1}\overline{T}x_f \tag{23}$$

$$= -K_1 x_f \tag{24}$$

where x_f denotes full-order state. By comparing with Eq. (23) and Eq. (24), we can find the relationship

$$-K_1 = -K_{r1}\overline{T} \tag{25a}$$

Also,

$$K_1 = K_{r1}\overline{T} \tag{25b}$$

where K_1 is the robust sub-optimal feedback gain implementing in the original full order system. K_{r1} is the robust sub-optimal feedback gain obtained from the reduced order system. $\overline{T}$ is the fast aggregation matrix.

For the subsystem two to the subsystem m can follow the same method as the subsystem one to find the optimal poles by the aggregation matrix.

4 Illustrations

The whole system is a fifth-order system with three first order subsystems and three inputs. The state model is shown as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \varepsilon\dot{z}_1 \\ \varepsilon\dot{z}_2 \\ \varepsilon\dot{z}_3 \end{bmatrix} = \begin{bmatrix} -0.5 & 0 & 0.1 & -0.2 & 0.1 \\ 0 & -1 & 0.1 & 0.3 & -0.2 \\ 0.4 & -0.3 & -0.5 & 0 & 0 \\ -0.4 & 0.4 & 0 & -0.45 & 0 \\ 0.35 & 0.3 & 0 & 0 & -0.4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ z_1 \\ z_2 \\ z_3 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0.4 & 0 & 0 \\ 0 & -0.5 & 0 \\ 0 & 0 & 0.6 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} + G_i\omega_i \quad \text{(26a)}$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} + v_i \quad \text{(26b)}$$

where x_1 and x_2 are slow state vectors that are second-order. z_1, z_2, z_3 are all fast state vectors and first-order individually. w_i and v_i are disturbing noises of inputs and outputs. Therefore, when the system researches quasi-steady state, $\varepsilon = 0$ and the system can be reduced to a second order system such as

$$\dot{x} = \begin{bmatrix} -0.1545 & -0.163 \\ -0.363 & -0.943 \end{bmatrix} x + \begin{bmatrix} 0.08 & -0.222 & 0.15 \\ 0.08 & 0.333 & -0.3 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \quad \text{(27)}$$

Next, we digitize this reduced-order model to discrete-time domain with the sampling period 0.1.

$$x(k+1) = \begin{bmatrix} 0.985 & -0.0154 \\ -0.0344 & 0.9103 \end{bmatrix} x(k) + \begin{bmatrix} 0.0079 & -0.0223 & 0.0151 \\ 0.0075 & 0.0322 & -0.0289 \end{bmatrix} \begin{bmatrix} u_1(k) \\ u_2(k) \\ u_3(k) \end{bmatrix} \quad \text{(28)}$$

In this example, the overall state vector w is concerned with $w = \begin{bmatrix} x \\ z_i \end{bmatrix}$. Now, if we want to have the optimal control in the subsystem one, by assuming the existing $K_2 = [1 \quad 1]$ and $K_3 = [5 \quad 5]$, we can rewrite the model as

$$x(k+1) = \begin{bmatrix} 1.0382 & 0.0378 \\ -0.1467 & 0.7980 \end{bmatrix} x(k) + \begin{bmatrix} 0.0079 \\ 0.0075 \end{bmatrix} u_1(k) \quad \text{(29)}$$

If the performance index of the slow state vector from the subsystem one is

$$J_1 = \frac{1}{2} \sum_{k=0}^{N-1} (x^T(k) Q_1 x(k) + u_1{}^T(k) R_1 u_1(k)) \quad \text{(30)}$$

where $Q_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$; $R_1 = 1$. The optimal control of the subsystem one:

$$u_1^{optimal} = [-3.2814 \quad -0.5737]\, x(k) \quad \text{(31)}$$

with $P = \begin{bmatrix} 364.8580 & 61.0517 \\ 61.0517 & 13.3957 \end{bmatrix}$. The pole locations of this optimal control are 0.9820 and 0.8240; therefore, the system is stabilized by the controller, too. If the initial condition $x_r(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, the optimal cost can be calculated as

$$J_1^{optimal} = \frac{1}{2} x_r^T(0) P x_r(0) \approx 500 \tag{32}$$

For the robust control test, this optimal reduced-order control will be placed back to the original full-order model. If the desired performance still exist, we have a robust control system. If $\varepsilon = 0.001$, we can have the same discrete-time model as:

$$w(k+1) = \begin{bmatrix} 0.9842 & -0.0151 & 0.0002 & -0.0004 & 0.0003 \\ -0.0334 & 0.9102 & 0.0002 & 0.0004 & -0.0005 \\ 0.8072 & -0.559 & 0 & -0.0007 & 0.0005 \\ -0.9042 & 0.8239 & 0 & 0.0001 & -0.0006 \\ 0.837 & 0.6713 & 0.0003 & 0.0001 & -0.0001 \end{bmatrix} w(k) + \begin{bmatrix} 0.0077 & 0.0.0218 & 0.0147 \\ 0.0074 & -0.0315 & -0.0282 \\ 0.8017 & 0.0356 & 0.0281 \\ -0.0003 & -1.1574 & -0.0373 \\ 0.012 & -0.0045 & 1.4919 \end{bmatrix} \begin{bmatrix} u_1(k) \\ u_2(k) \\ u_3(k) \end{bmatrix}$$

Now, we use the optimal feedback gain, $K_1^{optimal} = [-3.2814 \quad -0.5737]$, in the full-order system with $K_2 = [1 \quad 1]$ and $K_3 = [5 \quad 5]$. The pole locations of the system are 0.9853 and 0.8006. We can see the locations are very close to the desired pole locations; therefore, we have a robust control system.

Also, by assuming $y_1 = [0 \quad 0 \quad 1 \quad 1 \quad 1] \begin{bmatrix} x \\ z_1 \\ z_2 \\ z_3 \end{bmatrix}$, we can have the system responses based on the subsystem one with h=0.1 and ε=0.001 as follows:

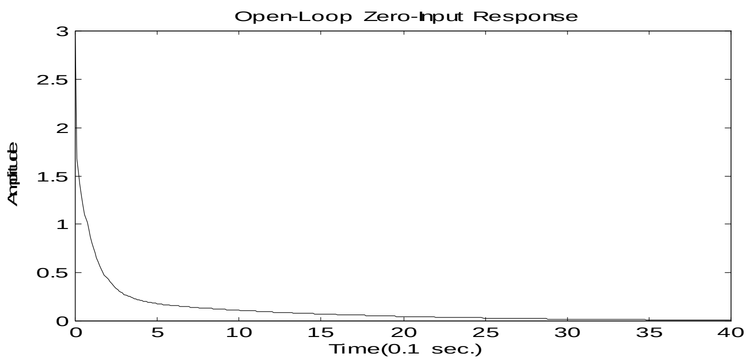

Fig. 2. The open-loop zero-input response of the full-order system with the slow state poles at 0.9915 and 0.9038.

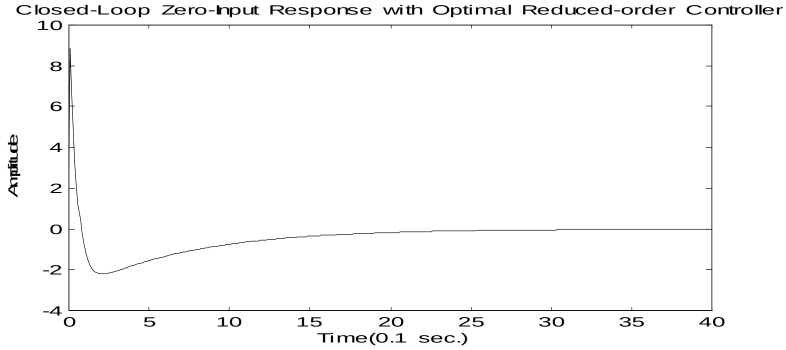

Fig. 3. The closed-loop zero-input response with the optimal reduced-order controller shifting poles to 0.9820 and 0.8240.

The robustness bound of the robust control system can be found by changing the value of ε. Table 1 shows how the poles shift when the value of ε changes.

Table 1. The robust control test.

ε	Poles
5.0000e-004	0.9853, 0.8008
0.0060	0.9853, 0.7981
0.0115	0.9853, 0.7951
0.0170	0.9854, 0.7919
0.0225	0.9854, 0.7883
0.0280	0.9854, 0.7842
0.0335	0.9855, 0.7795
0.0390	0.9855, 0.7739
0.0445	0.9855, 0.7673

In this case, if we assume the system performance allows 0.03 shift at each pole location, when $\varepsilon < 0.0115$, we can have a robust control system. The sub-optimal, reduced-order control that performs inside this bound is call robust, decentralized, sub-optimal reduced-order control. In this case, the approximated optimal poles, 0.9820 and 0.8240, are used to compare with the shifting poles caused by system uncertainties.

The robust sub-optimal control, the sub-optimal costs, and the robust control tests of the subsystem two and the subsystem three can just follow the same procedure used in the subsystem one.

After the reduced-order feedbacks are affirmed to be robust, for the full order feedback gains can be found by Eq. (8) to Eq. (10). In Eq. (9)

$$\begin{bmatrix} 0.08 & -0.222 & 0.15 \\ 0.08 & 0.333 & -0.3 \end{bmatrix} .= \begin{bmatrix} t_1 & t_2 & t_3 & t_4 & t_5 \\ t_6 & t_7 & t_8 & t_9 & t_{10} \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0.4 & 0 & 0 \\ 0 & -0.5 & 0 \\ 0 & 0 & 0.6 \end{bmatrix} \tag{33}$$

$$\overline{T} = \begin{bmatrix} t_1 & t_2 & 2 & 0.444 & 0.25 \\ t_6 & t_7 & 2 & 0.667 & -0.5 \end{bmatrix}$$

In Eq. (11),

$$[\Delta \quad C_i]^L = ([\Delta \quad C_i]^T [\Delta \quad C_i])^{-1} [\Delta \quad C_i]^T \cong \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Therefore;

$$\begin{bmatrix} -0.1545 & -0.163 \\ -0.363 & -0.943 \end{bmatrix} = \begin{bmatrix} t_1 & t_2 & 2 & 0.444 & 0.25 \\ t_6 & t_7 & 2 & 0.667 & -0.5 \end{bmatrix}$$
$$\begin{bmatrix} -0.5 & 0 & 0.1 & -0.2 & 0.1 \\ 0 & -1 & 0.1 & 0.3 & -0.2 \\ 0.4/\varepsilon & -0.3/\varepsilon & -0.5 & 0 & 0 \\ -0.4/\varepsilon & 0.4/\varepsilon & 0 & -0.45 & 0 \\ 0.35/\varepsilon & 0.3/\varepsilon & 0 & 0 & -0.4 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.8 & -0.6 \\ -0.889 & 0.889 \\ 0.875 & 0.75 \end{bmatrix} \tag{34}$$

The aggregation matrix is solved as $\overline{T} \cong \begin{bmatrix} 0.001 & 0.07 & 2 & 0.444 & 0.25 \\ 2 & 3.4 & 2 & 0.667 & -0.5 \end{bmatrix}$ and, the full-order feedback $K_1 = K_{r1}\overline{T} = [1.1507 \quad 2.1803 \quad 7.7102 \quad 1.8396 \quad 0.5335]$ where $K_{r1} = [-3.2814 \quad -0.5737]$ found in Eq. (31). The full order feedback gains of the rest of the subsystems can follow the same procedure as above.

5 Conclusions

The full order sub-optimal feedback of the decentralized computer control of stochastic singularly-perturbed system can be found easily through the aggregation matrix and couple steps; moreover, the found robust sub-optimal reduced order feedback gain can also achieve the desired performance with decreasing cost.

By using the reduced-order state model obtained from performing the singularly methodology, the robust reduced order feedback gain can be calculated based on the slow LQ perform index. Next, the full-order feedback gain can be found by multiplying the fast aggregation matrix as Eq. (25b). The effect by applying the full order feedback and reduced order feedback will have similar performance. These two types of feedback gains provide the demand of system to adjust the control status and performance. The completion of this algorithm helps us to analysis the decentralized computer control of stochastic singularly-perturbed system and fast to find the sub-optimal feedback gain for the full-order control and reduced-order control.

Three steps of finding the robust sub-optimal poles of the system are presented as below:

1. Find the fast aggregation matrix $\bar{T}$ from Eq. (8)-(11).
2. Find the reduced-order sub-optimal robust feedback gain of the system from Eq. (17).
3. Find the full-order sub-optimal feedback gain of the original system from Eq. (25b).

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