An Integrated Single-Item Lot-Sizing Problem in a Two-Stage Industrial Symbiosis Supply Chain with Stochastic Demands

Cheshmeh Chamani, El-Houssaine Aghezzaf, Abdelhakim Khatab, Birger Raa, Yogang Singh, Johannes Cottyn

An Integrated Single-Item Lot-Sizing Problem in a Two-Stage Industrial Symbiosis Supply Chain with Stochastic Demands

Cheshmeh Chamani[1,2][0000−0003−2687−6484], El-Houssaine Aghezzaf[1,2][0000−0003−3849−2218], Abdelhakim Khatab[3][0000−0003−1166−3806], Birger Raa[1][0000−0003−2127−4513], Yogang Singh[1,2][0000−0002−7352−8936], and Johannes Cottyn[1,2][0000−0002−1668−3331]

[1] Dept. of Industrial Systems Engineering and Product Design, Ghent University, Ghent, Belgium

[2] Industrial Systems Engineering (ISyE), Flanders Make, www.FlandersMake.be
{Cheshmeh.Chamani; ElHoussaine.Aghezzaf; Birger.Raa;Yogang.Singh;Johannes.Cottyn}@ugent.be

[3] Laboratory of Industrial Engineering, Production and Maintenance, Lorraine University, Metz, France
abdelhakim.khatab@univ-lorraine.fr

Abstract. We consider a two-stage supply chain in which two production plants are collaborating in an industrial symbiosis to satisfy their respective stochastic demands. We formulate the production planning problems of these two plants as an integrated capacitated lot-sizing problem, in which the second production plant uses as an alternative raw material a by-product obtained as a residue from the production of the first plant. The goal is to minimize the overall total cost in the supply chain, including production and inventory of the final product and by-product transfer costs, while meeting the stochastic demands. First, a natural formulation of the problem is proposed, and is solved using the Sample Average Approximation (SAA) method. The analysis of the gaps exhibits however quite large optimality gaps. To improve these optimality gaps, a plant location like reformulation for this integrated lot-sizing problem is developed. The analysis has been carried out again to evaluate both formulations' performances in terms of the optimality gaps and computational times, both when items demands follow Gamma and Normal distributions.The analysis indicates that despite having a computational time of on average 1.7 times higher than the main formulation, the plant location reformulation provides better optimality gaps on average 22% improved and better ranges for upper and lower bounds under stochastic demands.

Keywords: Lot-sizing · By-product · Industrial Symbiosis · Sample Average Approximation.

1 Introduction and a brief literature review

As the world's population increases, so does the demand for products and other consumption goods. Consequently, the demand for raw materials keeps growing. However, the supply of essential raw materials is now reaching its limit. This awareness forces today's societies and supply chains to consider more creative approaches to use and reuse available resources in a more sustainable manner. Circular economy is an emerging concept that facilitates moving away from the traditional linear economic model based on the take-make-consume-throw-out pattern. Circular economy is a production and consumption model, which involves sharing, leasing, reusing, repairing, refurbishing, and recycling existing materials and products as long as this is possible to extend products life-cycles. Moving towards a more circular economy will bring about significant economic, environmental, and social benefits. The transition from a linear to a circular economy requires, however, a fundamental change in production and consumption systems beyond waste recycling and resources use efficiency.

Production processes have a high impact on product life, supply, resource use, and waste generation. When manufacturing a product usually a collateral flow that is generally considered as waste is generated. A practice that can potentially benefit the transition to a circular economy is to generate and capture value by converting these waste streams (through further processing) into a useful by-products. This practice of turning produced residues, considered as waste, from an industrial process into by-products for another process is generally known as *by-product synergy* [3]. Optimizing jointly the production in both processes while explicitly considering the operational synergy between the original product and the by-product generates value for the involved parties in the supply chain. This paper focuses on a basic building block of the underlying production planning problem to provide evidence for this fact.

Various versions of this planning problem have been investigated in the literature. Suzanne et al., [4] carried out a comprehensive review on mid-term production planning in the context of circular economy and reverse logistics. The review presents an overview of the mathematical formulations, and their related solution methods, for production planning problems that arise under disassembly for recycling, in product to raw material recycling, and in by-products and co-production settings. The single-item lot-sizing problem arising in a single production unit, generating a by-product during the production process of the main product to satisfy a deterministic demand, has been investigated by [5]. The authors proved that this problem is NP-Hard when the by-product inventory capacity is time dependent, and proposed a pseudo-polynomial time dynamic programming algorithm to solve it. They also showed that when the inventory capacity is time independent the problem can be solved in polynomial time also via dynamic programming. A capacitated lot-sizing problem involving two collaborating production plants under different collaboration settings in an industrial symbiosis, and under deterministic demand is discussed in [2]. The objective of the model is to minimize total costs associated with supply, disposal, production, inventory, and symbiosis. Our current research investigates the integrated capacitated lot-sizing problem for a two-plants supply chain, collaborating in an industrial symbiosis, in which the second plant uses as raw material a by-product obtained as a residue generated by the production in the first plant to satisfy their respective stochastic demands. In the next section, we present a mathematical model of this planning problem, which we extended further to deal with demand uncertainty.

2 Problem statement and model formulation

We consider a centralized supply chain containing two production plants cooperating in an industrial symbiosis to minimize the supply chain's total costs. We investigate the underlying capacitated lot-sizing problem, called IS-SCLSP for short in the rest of the paper, where a primary plant generates a by-product that is used by the secondary plant to satisfy their respective stochastic demands. Over a planning horizon of T periods, the IS-SCLSP determines when and how much each plant should produce while satisfying the demand d. The amount of by-product generated in a rate α is moved forward to the second plant from its inventory level J_t hold in the first plant and its demand from the second plant. The second plant consumes the by-product at a rate β as raw material. The production system involves fixed setup cost f_t^n and unitary production cost either from raw material p_t^n or by-product $\hat{p}_t$. The finished product's surplus quantity can be stored in inventory at a unitary holding cost h_t^n from period t to t+1. The transportation of the by-product is performed at unitary cost q_t. The three main decisions posed by IS-SCLSP are: (1) when to produce the finished product y_t^n, (2) how much to produce of the finished product x_t^n, and (3) when and how much by-product to transfer w_t. Accordingly, all other related decisions, namely inventory levels of the main products I_t^n and the by-product J_t, are implied.

Before proceeding to the problem modelling we list hereafter the assumptions that have been made: (1) Both production plants produce a single type of product; (2) Initial inventory of the main products as well as of the by-product is null at the beginning of the planning horizon; (3) Production in the first plant is made entirely of purchased raw material, and the resulting by-product can be consumed as raw material to satisfy part of the demand in the second plant; (4) There is a limit on the production capacity in each plant at each planning period; (5)There is no limit on by-product inventory level; (6) No backorder is allowed, and all the demands should be satisfied within their desired due date; (7) Processing the finished product in the second plant using the by-product is more cost effective than when only raw material is used.

Summary of the parameters:

- T: Number of periods in the planning horizon.
- N: Number of production plants in the supply chain.
- t: Index for discrete period of the planning horizon.
- n: Index for production plants in the supply chain.
- p_t^n: Unit production cost of finished product using only raw material in plant n in period t.

- $\hat{p}_t$: Unit production cost of finished product in plant 2, using by-product as raw material,
- f_t^n: Fixed set-up cost for plant n in period t.
- d_t^n: Demand of product for plant n in period t.
- h_t^n: Unit inventory cost of the finished product in plant n in period t.
- q_t: Unit transportation cost of by-product.
- α: Generation rate of the by-product from the production in the plant 1.
- β: Production rate of the finished product for plant 2 per unit of by-product used.
- cap_t^n: Production capacity level of plant n in planning period t.

Summary of the variables - Natural formulation:

- x_t^n: Production amount in plant n in planning period t.
- y_t^n: Set up variable in plant n, assuming value 1 if production takes place in planning period t.
- I_t^n: Inventory level of the finished product in plant n at the end of planning period t.
- J_t: Inventory level of generated by-product in plant 1 at the end of planning period t.
- w_t: Amount of by-product that is transferred from plant 1 to plant 2 during the planning period t.

Summary of the variables - Plant location reformulation:

- $x_{t't}^n$: Fraction of the demand of plant n in period t that is satisfied from the production in plant n during period t' not involving the by product.
- $w_{t't}$: Fraction of the demand of plant 2 in period t that is satisfied form the production in plant 2 during period t' involving the by-product.

2.1 The natural formulation of IS-SCLSP

The integrated capacitated lot-sizing problem for a two-plants supply chain exchanging a by-product can naturally be formulated as a (deterministic) mixed-integer linear program given below:

$$\text{Minimize } Z = \sum_{n=1}^{N}\sum_{t=1}^{T}(f_t^n \cdot y_t^n + p_t^n \cdot x_t^n + h_t^n \cdot I_t^n) + \sum_{t=1}^{T}(q_t \cdot w_t) + \sum_{t=1}^{T}(\hat{p}_t \cdot \beta \cdot w_t) \quad (1)$$

Subject to

$$I_{t-1}^1 + x_t^1 = d_t^1 + I_t^1, \quad \forall t \in T, n = 1 \quad (2)$$

$$I_{t-1}^2 + x_t^2 + \beta \cdot w_t = d_t^2 + I_t^2, \quad \forall t \in T, n = 2 \quad (3)$$

$$J_{t-1} + \alpha \cdot x_t^1 = w_t + J_t, \quad \forall t \in T, n = 1 \quad (4)$$

$$x_t^1 \leq cap_t^1 \cdot y_t^1, \quad \forall t \in T, n = 1 \quad (5)$$

$$(x_t^2 + \beta \cdot w_t) \leq cap_t^2 \cdot y_t^2, \quad \forall t \in T, n = 2 \quad (6)$$

$$I_t^n, x_t^n, J_t, w_t \geq 0 \quad y_t^n \in \{0,1\}, \quad \forall t \in T, n \in N \quad (7)$$

The objective function (1) minimizes the sum of the fixed and variable production costs, inventory holding costs of the finished products, transportation cost of the by-product, and processing of finished product in plant 2 involving the by-product as the raw material. Constraints (2) and (3) represent the inventory flow of the finished product in the two production plants respectively. Constraint (4) expresses the flow conservation of the by-product in the first plant. Inequalities (5) and (6) ensure that the fixed setup costs in the two plants are paid and that production capacities are not exceeded. Non-negativity and binary requirements on the variables are expressed through (7).

2.2 Reformulation of IS-SCLSP

As the result section below will show, the optimality gaps achieved through implementing SAA on the main problem for different sample sizes are relatively high. Adopting the reformulation idea from [1], we provide a reformulation based on a plant location problem for the primary model to provide better LP-relaxation gaps compared to the formulation in the original variables.

$$\begin{aligned}\text{Minimize } Z = &\sum_{n=1}^{N}\sum_{t=1}^{T} f_t^n \cdot y_t^n + \sum_{n=1}^{N}\sum_{t=1}^{T}\sum_{t'=1}^{t}(p_{t'} + \sum_{r=t'}^{t-1} h_r) x_{t't}^n \cdot d_t^n \\ &+ \sum_{t=1}^{T}\sum_{t'=1}^{t}(\hat{p}_{t'} + \sum_{r=t'}^{t-1} h_r) w_{t't} \cdot d_t^2 + \sum_{t'=1}^{t}\sum_{t=1}^{T} q_{t'} \cdot \frac{1}{\beta} \cdot w_{t't} \cdot d_t^2 \end{aligned} \tag{8}$$

Subject to

$$\sum_{t'=1}^{t} x_{t't}^1 = 1, \quad \forall t \in T, n = 1 \tag{9}$$

$$\sum_{t'=1}^{t} (x_{t't}^2 + w_{t't}) = 1, \quad \forall t, t' \in T, t' \leq t \tag{10}$$

$$\sum_{t=t'}^{T} d_t^1 \cdot x_{t't}^1 \leq cap_{t'}^1 \cdot y_{t'}^1, \quad \forall t' \in T, n = 1 \tag{11}$$

$$\sum_{t=t'}^{T} d_t^2 \cdot (x_{t't}^2 + w_{t't}) \leq cap_{t'}^2 \cdot y_{t'}^2, \quad \forall t' \in T, n = 2 \tag{12}$$

$$\sum_{t'=1}^{t}\sum_{r=t'}^{T} \alpha \cdot x_{t'r}^1 \cdot d_r^1 - \sum_{t'=1}^{t}\sum_{r=t'}^{T} \frac{1}{\beta} \cdot w_{t'r} \cdot d_r^2 \geq 0, \quad \forall t \in T \tag{13}$$

$$0 \leq x_{t't}^n, w_{t't} \leq 1, \quad y_t^n \in \{0, 1\}, \quad \forall t, n \tag{14}$$

In this reformulation, the variables $x_{t't}^n$ and $w_{t't}$ represent the combined fraction of the demand d_t^2 of the second plant in period t that is satisfied by production in period t' in the second plant not involving the by-product, ($x_{t't}^n$), and that involving the by-product, $w_{t't}$, respectively. Constraints (9) and (10) ensure that demands in both plants should be fully satisfied. Constraints (11) and (12) assure that fixed set-up costs are paid and the capacity limit is respected in case of production. Constraint (13) indicates that the amount of demand satisfied through consuming by-products in the second plant must not exceed the available by-product generated in plant one. Non negativity and binary requirements are expressed by (14).

3 The Sample Average Approximation (SAA) procedure

The SAA procedure is based on solving M samples of the stochastic problem, under a limited number of S scenarios taken from the original distributions. Assuming each replication is optimally solved, the M problem's average objective value provides a statistical lower bound to the original problem. The M problems' optimal solutions are then reevaluated under a more extensive set of scenarios ($S' \geq S$) to estimate their actual objective function values. The solution which achieves the lowest estimated cost is assumed to be the best upper bound to the original problem. The methodology presented in the following are derived from [7].

Summary of SAA methodology

1. Generate M samples each of size S,(ξ_m^i) i=1,...,S, m=1,...,M.
2. For each sample m solve the problem $Z_S^m = min_{x \in X}\{c^T x + \frac{1}{S}\sum_{i=1}^{S} Q(x, \xi_m^i)\}$.
3. set $\hat{x}^{*m}$ as the candidate solution.
4. Equation $\overline{z}_S = \frac{1}{M}\sum_{m=1}^{M} z_S^m$ provides a statistical estimate for the lower bound.
5. For $S' \geq S$, $\hat{z}_{S'}(\hat{x}^m) = min\{c^T\hat{x}^m + \frac{1}{S'}\sum_{i=1}^{S'} Q(\hat{x}^m, \xi^i)\}$.
6. Select $\hat{x}^{*m} = \{\hat{x}^m : \hat{z}_{S'}(\hat{x}^m) = min_{1\leq i \leq m}\hat{z}_{S'}(\hat{x}^i)\}$.
7. The optimal gap would be $\frac{\hat{z}_{S'} - \overline{z}_S}{\hat{z}_{S'}}$.

Our problem is a two-stage stochastic capacitated lot sizing-by-product in which the first stage variables are the setup variable before demand for both plants are revealed. After the demand is revealed, the second stage variables determine the amount of production, the inventory level of the main product, and the amount of by-product that will be transferred. Below, we provide details on our implementation of the SAA procedure for our problem. The objective function aims to minimize the first stage costs and the mean of second stage variables costs under a defined set of scenarios.The stochastic formulation considering a limited set of M samples each of S scenarios (IS-SCLSP-SAA) is defined as follows, subject to Constraints (2)-(7):

$$\text{IS-SCLSP-SAA:} \quad \min \sum_{n=1}^{N}\sum_{t=1}^{T} f_t^n \cdot y_t^n + \frac{1}{S} \cdot [\sum_{i=1}^{S} \varphi(y, \xi_m^i) |\ (2) - (7)] \tag{15}$$

$$\varphi(y, \xi_m^i) = \min \sum_{n=1}^{N}\sum_{t=1}^{T}(p_t^n \cdot x_t^n + h_t^n \cdot I_t^n) + \sum_{t=1}^{T}(q_t \cdot w_t + \hat{p}_t \cdot \beta \cdot w_t) \tag{16}$$

4 Computational experiments

We conduct our computational experiments on the data set described in [6] for CLSP with deterministic demand. As some of our required information was not defined in this data set, we randomly generated the data associated with production and transportation cost to solve our defined problem.The research aims to optimize total costs associated with setup, production, inventory, and by-product transportation cost for two collaborating production plants during a six-week planning horizon. It has been assumed that the generation rate of by-product from production in plant 1 (α) is equal to 0.3, and the production rate of the finished product in plant 2 through consumption per unit of by-product (β) is equal to 0.5. The available production capacity at each planning period is calculated as follows $cap_t^n = \sum_t^T d_t^n * \gamma$, in which a predefined percentage (γ=0.85) of cumulative demand will be satisfied from the same period to the end of the planning horizon. We set the scale of gamma distribution for demand to $\lambda = 1$. Each model described in this paper was implemented using the Julia 1.4.2 programming language and solved using Gurobi 9.0.2. We performed all experiments on an Intel(R) Core(TM) i7-7700HQ CPU @ 2.80GHz, with 32GB of RAM.

4.1 The sample average approximation procedure

We implemented the SAA approach on our primary formulation. For one replication, we run the experiment for different sample sizes varying from 10 to 100, assuming that demand follows gamma distribution with scale 1. Table 1 shows the results. The first and second columns represent the lower and upper bounds of the problem, respectively. The gap in the third column refers to the relative optimality gap computed as $(\frac{UB-LB}{UB}) * 100$. The fourth column represents the computational time. As it can be seen, the optimality gap for different sample sizes is relatively high. To obtain a better optimality gap, we conduct the same experiments on a reformulated version of the main problem discussed in section 2.2. We carried out an analysis of the effect of reformulation on improving the optimality gap. As we can observe from Table 1, there is a reduction in the gap but a slight increase in the computational time for the reformulated version. As shown in the reformulated section in Table 1 scenario size 100, has the slightest optimality gap among other sample sizes. Therefore, 100 will be the selected number of scenarios for testing the SAA procedure under different sample sizes.

Table 2 and 3 present the results of the SAA procedure for 100 scenarios under different sample sizes. The sample size s' to obtain the upper bound is considered to be 500. Due to statistical computation of the bounds, it is probable that the computed lower bound exceeds the upper bound. In this regard, to still have a good indication of how good the solution is, we consider the lower bound to be the minimum of all the lower bounds and the upper bound to be the average of all the upper bounds achieved under different sample sizes in the fourth and fifth columns, respectively, and this for both formulations. For ten replications, we obtain the best optimality gap. Therefore we set the number of scenarios and sample size to 100 and 10 respectively to compare over two probability distribution outcomes on obtained optimality gap and variance of lower bound. Figure 1 illustrates the difference in achieved optimality gaps for both reformulations applied for sample sizes 10,20,30,40,50 and 60, respectively, each of 100 scenarios. As we can see, sample size 10 in plant location formulation has the best performance in reducing the optimality gap. On average, the plant location formulation provides almost 22 % reduction in optimality gap compared to the natural formulation. The comparison in computational time for both formulations with the mentioned sample sizes is depicted in Figure 2. On average, the plant location formulation takes 1.7 times longer time. Table 4 explains the comparison of the SAA performance in terms of the optimality gap and the variance in the obtained objective function lower bounds. The experiments were conducted with demand data following Gamma and Normal distributions with scale and standard deviation set to 0.5, 1, 1.5 and 2. As it can be seen, there is a significant difference in the optimality gap as well as in the obtained lower bound variance, which indicates the importance of better estimating the probability distribution of the demand parameters.

Table 1: Results of the SAA for $M = 1$, with a gamma distribution of scale=1

	Main Formulation				Plant-Location Formulation			
size	LB	UB	Gap(%)	CT(s)	LB	UB	Gap(%)	CT(s)
10	15 510	15 565	0.35	6.10	15 368	15 431	0.40	9.40
20	15 540	15 585	0.28	5.70	15 337	15 412	0.48	9.50
30	15 479	15 583	0.66	6.0	15 439	15 457	0.11	9.60
40	15 597	15 630	0.21	5.30	15 375	15 388	0.08	9.20
50	15 553	15 589	0.23	6.10	15 353	15 423	0.45	10.0
60	15 434	15 560	0.80	4.90	15 387	15 396	0.05	9.50
70	15 487	15 574	0.55	5.10	15 333	15 349	0.10	10.0
80	15 564	15 597	0.21	5.01	15 390	15 392	0.01	9.60
90	15 576	15 623	0.30	5.40	15 428	15 453	0.16	9.50
100	**15582**	**15606**	**0.10**	4.90	**15385**	**15385**	≈ 0.0	9.60

Table 2: Results for SAA implemented for main formulation with 100 scenarios and a gamma distribution of scale=1

rep	LB	UB	Min_{LB}	$\overline{UB}$	Gap(%)	σ^2_{LB}	CT (s)
10	**15579**	**15566**	**15520**	**15585**	**0.41**	**101.0**	**34.8**
20	15590	15546	15487	15584	0.62	95.8	68.5
30	15589	15556	15503	15589	0.55	60.5	102.5
40	15593	15552	15495	15588	0.59	71.2	161.0
50	15576	15541	15495	15588	0.70	44.2	170.0
60	15575	15543	15492	15584	0.59	24.0	201.0

5 Conclusion and perspectives

This paper investigates a new version of the capacitated lot-sizing problem, under demand uncertainty, arising in a supply chain transitioning to circular econ-

Table 3: Results for SAA implemented for plant location formulation with 100 scenarios with gamma distribution of scale=1

rep	LB	UB	Min_{LB}	$\overline{UB}$	Gap(%)	σ^2_{LB}	CT (s)
10	**15411**	**15391**	**15381**	**15412**	**0.20**	**54.3**	**92**
20	15413	15393	15336	15409	0.47	77.5	188
30	15409	15336	15333	15408	0.48	50.5	270
40	15413	15356	15324	15404	0.51	49.3	377
50	15409	15356	15317	15407	0.58	34.9	434
60	15407	15359	15329	15407	0.50	41.9	529

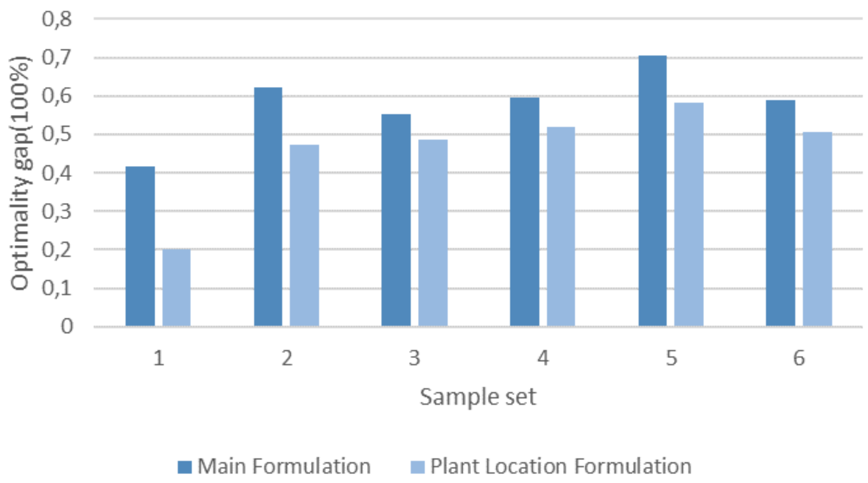

Fig. 1: Comparison between optimality gaps provided by each of the formulations

omy business model. The problem is formulated as an integrated capacitated lot-sizing problem for a supply chain involving two collaborating plants in form of an industrial symbiosis. In this model, the second plant uses as raw material a by-product obtained as a residue generated by the production in the first plant. In this analysis, it is assumed that the product demand of two cooperating plants follow gamma and normal probability distributions. To solve the resulting problem the Sample Average Approximation (SAA) method was implemented. The SAA approach's performance in terms of optimality gaps has been investigated in the original model and its plant-location reformulation version. The results indicate that the reformulated version has better performance in terms of optimality gap and upper and lower bounds but leads to higher computational time in comparison with the original formulation. Some extensions of the supply chain as well as the solution methodology are currently investigated.

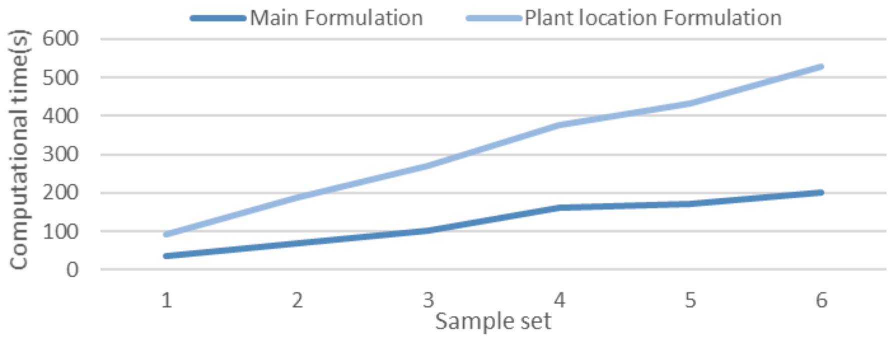

Fig. 2: Comparison between computational times for each of the formulations

Table 4: Results of the SAA procedure for Gamma and Normal distributions with 10 samples and 100 scenarios

Gamma distribution					Normal distribution				
scale	Min_{LB}	$\overline{UB}$	Gap(%)	σ^2_{LB}	σ	Min_{LB}	$\overline{UB}$	Gap(%)	σ^2_{LB}
0.5	15368	15401	0.21	104	0.5	15404	15406	0.01	0.61
1	15520	15585	0.41	101	1	15399	15404	0.03	1.27
1.5	15324	15406	0.41	278	1.5	15396	15405	0.05	3.75
2	15289	15402	0.41	627	2	15397	15407	0.06	4.84

References

1. Barany, I., Van Roy, T., and Wolsey, L. A :Uncapacitated lot-sizing: The convex hull of solutions. In: Mathematical programming at Oberwolfach II (pp. 32-43). Springer, Berlin, Heidelberg (1984)
2. Daquin, C., Allaoui, H., Goncalves, G., and Hsu, T: Collaborative lot-sizing problem for an industrial symbiosis. In: IFAC-PapersOnLine, 52(13), 1325-1330 (2019)
3. Lee, D :Turning waste into by-product. Manufacturing and Service Operations Management,**14**(1), 115-127 (2012)
4. Suzanne, E., Absi, N., and Borodin, V : Towards circular economy in production planning: Challenges and opportunities. European Journal of Operational Research, **287**(1), 168-190 (2020)
5. Suzanne, E., Absi, N., Borodin, V., and van den Heuvel, W : A single-item lot-sizing problem with a by-product and inventory capacities. European Journal of Operational Research,**287**(3), 844-855 (2020)
6. Trigeiro, W. W., Thomas, L. J., and McClain, J. O: Capacitated lot sizing with setup times. Management science, **35**(3), 353-366 (1989)
7. Verweij, B., Ahmed, S., Kleywegt, A. J., Nemhauser, G., and Shapiro, A : The sample average approximation method applied to stochastic routing problems: a computational study. Computational optimization and applications, **24**(2), 289-333 (2003)