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Abstract. In the digital economy, information systems have a significant impact on supply chain management. However, there is a need for further development of theoretical knowledge and mathematical models, including methods for managing risk in complex supply networks to best serve customer orders. In the supply chain operations reference (SCOR) model, reliability is assessed by calculating perfect order parameters. The component/process reliability is calculated as the product of the weighted averages of the perfect order parameters, and possible combinations of failure features are not taken into account. This paper presents an approach to probabilistic estimation of perfect order parameters based on the general theorem on the repetition of experiments, and proposes to use a binomial distribution to approximate the values obtained. The obtained results make it possible to assess the efficiency of possible measures (increasing the insurance stock, replacing the carrier, etc.) to improve the reliability of perfect order fulfilment.

Keywords: Supply Chain Management, Perfect Order Fulfilment, Combinations of Failure Features.

1 Introduction

Considerable attention is now being paid to the study of technologies and concepts such as artificial intelligence (AI), blockchain, Industry 4.0, big data, and several others [1-4]. However, there is a need to further develop theoretical knowledge and mathematical models to assess the impact of information systems on supply chain management [5-7]. This is necessary to achieve the primary goal of the supply chain – customer satisfaction [8-11]. Recent literature suggests some indicators to evaluate the impacts of uncertainty on supply chain (SC) execution. Most popular are reliability, robustness, stability and resilience [12]. In the supply chain operations reference (SCOR) model, reliability is assessed by calculating perfect order parameters [13-15]. Vishnu et al. [14] note that the reliability index of each supply chain component/process is calculated as a weighted average of the reliabilities of supplying the right quantity, the right quality, and at the right time. A similar approach to assessing the perfect order is found in studies of Bowersox, Closs, and Cooper [16]; Christopher [17]; Ballou [18]; Cousins et al. [19]; Grant et al. [20]; and Heizer and Render [21].

Bowersox, Closs, and Cooper [16] claim that as many as 20 different logistic service elements may impact a perfect order. Ho et al. [22] emphasise that there is no research measuring the correlations between risk factors and the corresponding risk types, or the probability of occurrence of particular risk types associated with their factors. On the other hand, Walker [23] says that business has embraced the key performance indicator (KPI) for real-time process decision making. Unfortunately, KPIs multiply like rabbits, and unless they are properly managed, tend to be defined within the narrow context of functional silos.

Thus, the literature does not take into account possible combinations of failure features, and there is no probabilistic assessment of perfect order parameters. Consequently, to improve the reliability of perfect order parameter estimation, it is necessary to develop a new methodology that takes into account the specifics of supply chain operation.

This paper presents an approach to the probabilistic estimation of perfect order parameters based on the general theorem on the repetition of experiments, and proposes to use binomial distribution to approximate the obtained values. The obtained results make it possible to assess the efficiency of possible measures (increasing the insurance stock, replacing the carrier, etc.) to improve the reliability of perfect order fulfilment.

The paper is organised as follows. The methods for the probabilistic estimation of perfect order parameters are presented in Section 2. Section 3 lists the calculations used to test the suggested methods in different scenarios. Section 4 provides directions for future research.

2 Developing Methods for the Probabilistic Estimation of Perfect Order Parameters

To describe analytically the performance of a perfect order, a probabilistic model is used [16-21], which allows estimating the failure-free performance of the system in the form of the dependence

$$p_0 = \prod_{i=1}^{n} p_i \,, \tag{1}$$

where p_0 is the probability of a failure-free order, n is the number of parameters considered in the perfect order, and p_i is the probability of error-free execution of the i-th operation (the p_i probabilities are independent).

In most papers, the calculated dependence (1) includes three or four parameters: p_1 is the probability of delivery in-full (quantity); p_2 is the probability of delivery in perfect condition (quality); p_3 is the probability of ‘just in time’ (JIT) order performance; p_4 is the probability of error-free documentation.

However, experience with formula (1) has shown that it can lead to significant errors for the following reasons.

1. Formula (1) corresponds to one of the extreme cases of a combination of failures in a single order; i.e., it does not reflect the variety of possible options.

2. The p_i parameters are derived from statistical information, and the relationships and interactions between them are not represented or quantified.

3. Other statistical dependencies, such as the general theorem on the repetition of experiments [24-25], must be used to estimate the probability p_0. The general theorem on the repetition of experiments describes the case when experiments are carried out under different conditions, and the probability of an event varies from experiment to experiment.

Let us consider versions of expert analysis of the occurrence of failures in supply chains. The essence of the problem of accounting for the number of parameters in a single order is presented in Table 1 for two extreme cases. The first case reflects the independence and uncorrelatedness of failure features in each order (of the eight orders, only orders 7 and 8 are perfect, and the probability of failure-free order formation is 0.250); the second case reflects the maximum possible combination of failure features in a single order (only orders 4, 5, 6, 7, and 8 are perfect, and the probability of failure-free order formation is 0.625). On the other hand, according to Bowersox, Closs, and Cooper [16], Christopher [17], and Ballou [18], the values of perfect order parameters are on-time p_1 = 7/8 = 0.875, in-full p_2 = 6/8 = 0.75, and error-free p_3 = 5/8 = 0.625. Then formula (1) yields the probability of failure-free order formation $p_0 = 0.875{\cdot}0.75{\cdot}0.625 = 0.41$ for both extreme cases.

Table 1. Possible combinations of failure features in a single order.

Case	Parameter	Order number								Number of failures	Probability of failure-free order
		1	2	3	4	5	6	7	8		
1	On-time	X	0	0	0	0	0	0	0	1	0.250
	In-full	0	X	X	0	0	0	0	0	2	
	Error-free	0	0	0	X	X	X	0	0	3	
2	On-time	X	0	0	0	0	0	0	0	1	0.625
	In-full	X	X	0	0	0	0	0	0	2	
	Error-free	X	X	X	0	0	0	0	0	3	

X: failure; 0: failure-free performance

The dependencies of all possible variations of a perfect order estimate can be calculated based on the general theorem on the repetition of experiments, according to which the probabilities of failure-free order fulfilment are determined by the formula

$$\prod_{i=1}^{n}\left(p_i + q_i z\right) = \sum_{m=0}^{n} P_{m,n} z^m , \tag{2}$$

where p_i is the probability of no failure in the i-th experiment, $q_i = 1 - p_i$; $P_{m,n}$ is the probability of no failure in n experiments exactly m times; and z is an arbitrary parameter.

After expanding equation (2), we find the sum of similar terms for each z^m, which represent probabilities $P_{0,n}$, $P_{1,n}$, ..., $P_{m,n}$ and are the components of a perfect order with m simultaneously observed failures for different values of m. For example, equation (2) for calculating the probabilities of a perfect order when $n = 3$ and the number of failure features m is between 0 and 3 is written as

$$\prod_{i=1}^{n=3}(p_i + q_i z) = p_1 p_2 p_3 + (p_2 p_3 q_1 + p_1 p_3 q_2 + p_1 p_2 q_3)z + \\ + (p_3 q_1 q_2 + p_2 q_1 q_3 + p_1 q_2 q_3)z^2 + q_1 q_2 q_3 z^2 \quad (3)$$

If $n = 3$ and $m = 0$, we obtain $P_{0,3} = p_1 \cdot p_2 \cdot p_3$, i.e., formula (1); if $m = 2$, then $P_{2,3} = p_3 \cdot q_1 \cdot q_2 + p_2 \cdot q_1 \cdot q_3 + p_1 \cdot q_2 \cdot q_3$.

Our research has shown that the probabilities $P_{m,n}$ can be approximated by discrete probability distributions, in particular the binomial distribution

$$P_{m,n} = \frac{n!}{m!(n-m)!} q^m p^{n-m}, \quad (4)$$

where p is the average value of the probability of no failures; $q = 1 – p$.

The value p is calculated according to the formula

$$p = \frac{\sum_{i=1}^{n} p_i}{n} \quad (5)$$

Obtaining the values of probabilities $P_{m,n}$ makes it possible to assess comprehensively the impact of changes in probabilities p_i, as well as the efficiency of possible measures for improving reliability (increasing the insurance stock, replacing the carrier, etc.). According to Inman and Blumenfeld [26], countermeasures may be available to mitigate the impact – such as substituting a different part, installing a feature at a later date, or deleting a feature altogether – but countermeasures carry at least some additional cost or negative side-effect. While no firm wants to carry inventory, it may be a cost-effective strategy for reducing supply chain disruption risk.

Clearly, further research needs to consider the use of discrete distribution laws other than the binomial to approximate the probability (e.g., hypergeometric, Poisson, etc.).

3 Testing

We collected and processed statistical data on supplies for company X, which allowed us to determine the following values of perfect order parameters: on-time $p_1 = 0.90$, in-full $p_2 = 0.80$, error-free $p_3 = 0.85$; number of failure features $n = 3$; simultaneous occurrence of failures m from 0 to 3.

Table 2 shows the results of calculations of probabilities $P_{m,3}$ using formula (3), e.g., $P_{0,3} = 0.9 \cdot 0.8 \cdot 0.85 = 0.612$. The probabilities of occurrence of one or two failures in an order are, respectively, $P_{1,3} = 0.329$ and $P_{2,3} = 0.056$.

To calculate $P_{m,3}$ under approximation by binomial distribution, we define the average value of the probability of no failures as $p = 0.85$. Then, using formula (4), we find the probability of the occurrence of, for example, one failure in the order ($m = 1$):

$$P_{1,3} = \frac{3!}{1! \cdot 2!} 0.15 \cdot 0.85^2 = 0.325$$

The results of the remaining $P_{m,3}$ values are shown in Table 2. An analysis of Table 2 shows that there is good consistency between the $P_{m,3}$ values calculated by different methods.

After considering these results, the management of company X decided that it was necessary to improve delivery efficiency by selecting an alternative route ($p_{1+} = 0.95$), creating an insurance stock ($p_{2+} = 0.85$), and implementing e-documentation ($p_{3+} = 0.90$). The results of the calculations are shown in Table 2, which indicates that the probability of forming a perfect order has increased to $P_{0,3} = 0.729$, i.e., by 15.5%.

Table 2. The results of calculating the probabilities of a perfect order $P_{m,3}$.

Simultaneous occurrence of failures, m	Probabilities $P_{m,3}$		
	General theorem on the repetition of experiments	Binomial distribution	
		Before changes in the supply organisation*	After changes in supply organisation**
0	0.612	0.614	0.729
1	0.329	0.325	0.243
2	0.056	0.058	0.027
3	0.003	0.003	0.001

*on-time $p_1 = 0.90$, in-full $p_2 = 0.80$, error-free $p_3 = 0.85$; **on-time $p_{1+} = 0.95$, in-full $p_{2+} = 0.85$, error-free $p_{3+} = 0.90$.

A closer inspection revealed that in addition to the parameters mentioned above ($p_1 = 0.90$, $p_2 = 0.80$, $p_3 = 0.85$), three additional perfect order parameters were present ($p_4 = 0.99$, $p_5 = 0.97$, $p_6 = 0.95$). In order to assess the impact of these additional parameters on the formation of a perfect order, calculations were carried out using formulas (2) and (4), the results of which are shown in Table 3. An analysis of Table 3 shows that there is good consistency between the $P_{m,n}$ values calculated by different methods.

The results of calculations of $P_{m,n}$ show that an increase in the number of parameters of failure, taken into account in formulas (2) and (4), leads the probability of formation of the perfect order to decrease from 0.61 (at $n = 3$) to 0.56 (at $n = 6$), i.e., by approximately 5%. Thus, even a slight decrease (by 1–2%) in the reliability of operations in supply chains (delay in transportation, errors in order picking, cargo damage, etc.) can have a significant impact on the formation of a perfect order.

It follows that the decrease in the probability of perfect order formation is much greater when the p_i values are below 0.8. This issue requires special attention and further research, because according to Bowersox, Closs, and Cooper [16], even the best logistics organisations report only 60 to 70% perfect order performance.

Table 3. The results of calculating the probabilities of a perfect order $P_{m,n}$ ($P_{m,4}$, $P_{m,5}$, and $P_{m,6}$).

Simultaneous occurrence of failures, m	Number of failure features and average value of the probability of no failures					
	$n = 4$; $p = 0.885$		$n = 5$; $p = 0.902$		$n = 6$; $p = 0.910$	
	1*	2**	1*	2**	1*	2**
0	0.60588	0.61344	0.58770	0.59708	0.55831	0.56786
1	0.33183	0.31881	0.34005	0.32149	0.35243	0.33698
2	0.05850	0.06214	0.06500	0.06487	0.07875	0.0833
3	0.00353	0.00538	0.00510	0.00766	0.00809	0.0110
4	0.00003	0.000175	0.00013	0.00045	0.00038	0.00081
5			$0.9 \cdot 10^{-6}$	$0.9 \cdot 10^{-5}$	$0.75 \cdot 10^{-5}$	0.00003
6					$0.45 \cdot 10^{-7}$	$0.5 \cdot 10^{-6}$

*General theorem on the repetition of experiments; **Binomial distribution

4 Conclusion

In the digital economy, information systems have a significant impact on supply chain management. However, there is a pressing need for further development of theoretical knowledge and mathematical models, including methods that enable risk management for complex supply chains to best serve customer orders. This paper presents an approach to the probabilistic estimation of perfect order parameters based on the general theorem on the repetition of experiments, and proposes to use binomial distribution to approximate the obtained values. In future research, three considerations will be important. The first is to investigate the interdependencies and interactions between the parameters of the perfect order p_i, which must be taken into account in formulas (2) and (4). The second is to pay attention to different types of reservations for reliable fulfilment of a perfect order, taking into account the quality values of the supply chain performance. The third is to carry out a cost analysis of the proposed approaches by taking into account transport, current and insurance stock costs, costs due to shortages, and various penalties in the case of an imperfect order.

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