Managing Perishable Multi-product Inventory with Supplier Fill-Rate, Price Reduction and Substitution

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Abstract. Order-sizing in replenishment planning and control for perishable products is studied in grocery retail context. There is a need for age-based policies that consider multiple products, the impact from price reduction (due to close-to-expiration), and product substitution in order to reduce waste, increase availability and improve freshness. This study develops a theoretical extension to known EWA-models considering positive and/or negative interdependence in substitution between products, impact from price reduction and expired products, as well as the inventory impact from other products safety stocks.

Keywords: inventory control · shelf-life · perishable · substitution

1 Introduction

The grocery market faces ever-growing requirements to product availability and freshness [1]. Majority of consumers often feel disappointed with fresh food products' (FFP) availability and freshness when grocery shopping [2]. The FFPs have down to few days shelf-life with high waste-levels when comparing with other product types [3]. Increasing remaining shelf-life one day causes improved freshness, availability and waste [4].

Grocery demand is stochastic and non-stationary over the week with high sales in weekends [5]. This, as well as the increased focus on food waste and use of automated replenishment systems across product assortments [6], put high requirements on the FFP replenishment planning and control at wholesaler and retail store. Different heuristics have been suggested to manage perishables in automated replenishment systems when considering the product's remaining shelf-life [5, 7–9]. However, they do not reflect certain real-life situations. Grocery wholesaler/retailer faces different product characteristics that influence the order-size decision-making of FFPs:

1. Price-reduction: if "FFP A" is close to expiration, its price is reduced (in rounds) to minimize waste. The demand for the price-reduced "FFP A" depends on the reduction i.e. price elasticity, which influences the available inventory in different degrees.
2. Order fill-rate: FFPs to be delivered in the future, not yet in transit, may be influenced by (suddenly) reduced fill-rate due to factors such as, e.g. sudden raw-material unavailability. This influences the safety stock, hence the ability to withstand variation in demand level, thus order-sizing of FFPs.

3. Substitution demand: if "FFP A" is out-of-stock it may be substituted with "FFP B", causing extraordinary substitution demand on "FFP B" – and vice versa, depending on the products' positive and/or negative interdependence [10].
4. Substitution inventory: FFPs have asymmetrical financial losses[1] with increased food waste focus. Therefore, instead of buying too many "FFP B" (due to e.g. minimum order quantities) which causes excess inventory, hence increased risk of waste from expiration, the available inventory from substituting "FFP A" may satisfy "FFP B"'s demand, thereby mitigate risk.

By investigating current heuristics for perishable (automated) replenishment planning and control, it is possible to see how substitution, price reduction and reduced fill-rate in future orders may be included in the decision-making. The following presents the background, the developed multi-product EWA_{3SL}, and ends with the conclusion.

1.1 Inventory Control for Perishable Products

Numerous inventory control systems have been introduced for perishable products with fixed or random shelf-life and fixed or continuous review period, modelling deterministic or stochastic demand [11–15]. Fixed shelf-life is a known and deterministic time period where a product deteriorates (e.g. fresh meat, dairy and chilled food products), while random shelf-life is a probabilistic time period where a product deteriorates (e.g. fruits and vegetables). Recent studies primarily concern single items assuming deterministic demand, mainly focusing on pricing and lot-sizing or multi-echelon – and shortages are considered through back-ordering [14]. For products with particular short shelf-life, i.e. one day, the newsboy problem is considered appropriate [15]. Extended versions covering two periods with stochastic demand are suggested by e.g. [16].

For products with up to few weeks shelf-life such as fresh meat and dairy products the OIR policy [8], age-and-stock-based (CASB) policy [9] and the EWA policy [5] are considered. The old inventory ratio (OIR) policy is a two-step policy minimizing the expected number of outdated products given a predetermined allowance for out-of-stock. The inventory position is raised to order-up-to level, and then, if the ratio between old (i.e. outdated) and total inventory position on hand is larger than a specified threshold, an additional order quantity corresponding to the number of outdated products is ordered. Simulation results for blood products show significant reduction in outdated products (19,6% to 1,04%) while keeping sufficiently high fill-rate [8]. A variation of the OIR is the CASB policy with a continuous review [9]. An order quantity is suggested either when total inventory position drops to a specified number of products (reorder point) or when the oldest batch has aged *t* units of time; whichever comes first [9]. Since the review is continuous, the required safety stock is lower [15].

The EWA policy considers the estimated number of products to outdate within the review period. Based on [15] the EWA batches store orders according to case sizes with positive lead-times and weekly time-varying demand, as known in the grocery industry [5]. They obtain 17,7% increase in inventory availability and 3,4% waste reduction for

[1] Too few products mean lost sales i.e. profit – too many means lost purchase and handling costs.

products with 4-7 days shelf-life when comparing to stock-based policy. [7] extends the EWA to EWA_{SS} considering the size of safety stock relative to the expected number of products outdated within the review period. They simulate grocery products with short shelf-life and compare with a stock-based policy and obtain improved results on waste reduction compared to [5]: 10,3% increase in inventory availability and 10,7% waste reduction. The latest EWA_{SS} suggested by [7] is in equation (1)-(2):

If ,

$$I_t - \sum_{i=t+1}^{t+R+L-1} \hat{O}_i < \sum_{i=t+1}^{t+R+L} E[D] + SS \tag{1}$$

then,

$$Q_t = \begin{cases} \max\left(\dfrac{\sum_{i=t+1}^{t+R+L} E[D] + \sum_{i=t+1}^{t+R+L-1} \hat{O}_i - I_t}{B}, 0\right) & \text{if, } SS < \sum_{i=t+1}^{t+R+L-1} \hat{O}_i \\ \max\left(\dfrac{\sum_{i=t+1}^{t+R+L} E[D] + SS - I_t}{B}, 0\right) & \text{if, } SS \geq \sum_{i=t+1}^{t+R+L-1} \hat{O}_i \end{cases} \tag{2}$$

$E[D]$ = expected product demand within review time
I_t = inventory position of product at time t
$\hat{O}_i$ = estimated number of product*s* to expire within review time
SS = safety stock for product

Although EWA_{SS} includes the size of safety stock relative to the estimated number of products that will outdate, it is for a single product as with EWA, OIR and CASB. Since including only one product, they do not consider the additional demand created from other products sold out and out-of-stock (i.e. substitutions demand). Further, they do not include the impact of when selling product close to expiration at a reduced price.

1.2 Product Characteristics

Different planning environment characteristics influence FFPs [17]. In this study, the focus is on the impact of price reduction when FFPs are close to expiration, the supplier order fill-rate for future orders and impact from substitution on demand and inventory.

Due to FFPs short shelf-life, any excess inventory will be prone to the risk of expiration and thus subject to a price reduction. Depending on how excessive the inventory level is, a price reduction can be used as a tool to increase the demand in due time [13]. This decreases the inventory level with the desired speed and timing. Price-elasticity can support the order sizing of FFPs by estimating how much the inventory position will decrease each time products are reduced in the price, and is also suggested by [18].

The FFPs are processed down to every day with immediate shipment from the supplier, for fresh meat products see e.g. [17]. The raw materials for FFPs are scarce and can usually not be stored for any longer time, as well as they are often influenced from factors such as, e.g. available only in certain season(s) and nature (storm, rain etc.). Sudden scarcity may, therefore, influence future orders, not yet in transit, within the review period. By including a supplier order fill-rate, this order sizing of FFPs may encounter this and increase order size as needed.

The last two product characteristics concern substitutions and the impact on demand and inventory availability [10, 19]. Focusing on "FFP A", we consider substitution demand for "FFP A" when "FFP B" has too low inventory, and substitution inventory from "FFP B" when "FFP A" has too low inventory. [10] describes how the well-used exogenous substitution factors may be used for creating a substitution probability matrix. We represent the two by available substitution inventory of other FFPs and substitution demand from other FFPs.

2 A Multi-product EWA with Supplier Fill-Rate, Price Reduction & Substitution

To control inventories in a way which reflects the consumer requirements (availability and freshness) and impact from substitution as well as mitigates the risk of causing quality reduction and food waste, it is necessary to use a multi-product approach. To ensure the size of safety stock relative to outdating products, we build on the EWA_{SS}. As with both current EWA policies [5, 7], we use a fixed review period. This fits with the grocery industry and wholesaler/retail stores placing orders at specified time points regardless of demand type (normal or campaign demand). Having a safety stock for perishable items means a chance for reducing the sales price of the product to adjust the inventory position, so waste is avoided. Based on the four FFP characteristics, EWA_{3SL} is suggested. The 3SL in EWA_{3SL} relates to the supplier (S), shelf-life (SL) and substitution (S). It follows the logic as depicted in Figure 1, where one of three different order-sizing decisions applies.

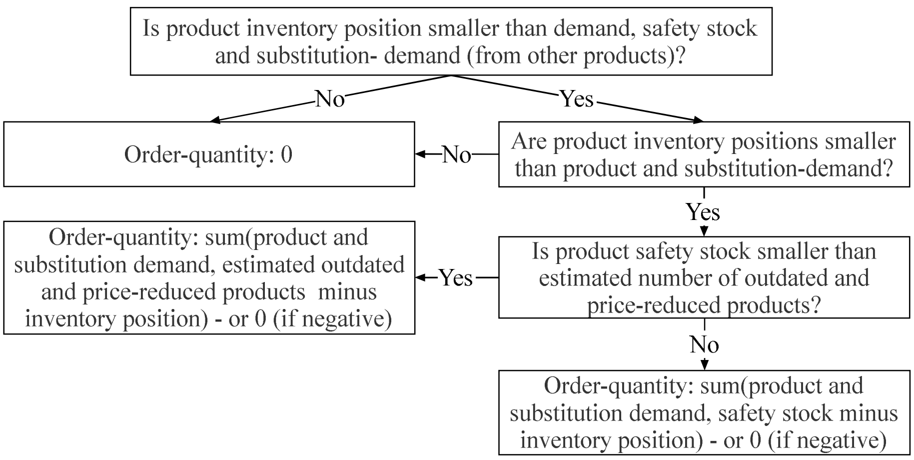

Fig. 1. Decision Diagram for EWA_{3SL}

To ensure simplicity in presentation, we first define the available inventory as in equation (3). For product p_1 at time t we consider current inventory level (on hand and in transit), plus all quantities ordered but not yet received/in transit multiplied by the fill-rate (*β*) for each supplier (*l*), minus already reserved quantities[2], within the review- and lead-time (*i*) [15]. Then, the estimated outdated (i.e. expired) quantities and estimated quantities sold at a reduced price (due to close to expiration) up until the immediate prior time period are subtracted. For quantities sold at a reduced price, please notice that there may be products with different expiration dates, i.e. different price-reduced quantities each day as identified by ε.

$$I_{p_1,t}^{available} = I_{p_1,t} + \sum_{i=t+1}^{R_{p_1}+L_{p_1}} \sum_{l=1}^{S_1 \to S_x} Q_{p_1,i,l}^{ordered} \beta_{p1,i,l} - \sum_{i=t+1}^{R_{p_1}+L_{p_1}} Q_{p_1,i}^{reserved}$$
$$- \sum_{i=t+1}^{R_{p_1}+L_{p_1}-1} \hat{Q}_{p_1,i}^{outdate} - \sum_{i=t+1}^{R_{p_1}+L_{p_1}-1} \sum_{k=1}^{\varepsilon_{p_1}} \hat{Q}_{p_1,i,k}^{reduced} \quad (3)$$

$I_{p_1,t}$ = starting inventory position, after expired products are subtracted
$Q_{p_1,i,l}^{ordered}$ = number of product p_1 already ordered but arriving later, within review time
$\beta_{p1,i,l}$ = fill-rate on ordered quantities of product p_1 from supplier l $(S_1 \to S_x)$
$Q_{p_1,i}^{reserved}$ = number of product p_1 reserved from inventory due to, e.g. campaign or customer
$\hat{Q}_{p_1,i}^{outdate}$ = estimated number of product p_1 to expire within review time
$\hat{Q}_{p_1,i,l}^{reduced}$ = estimated number of product p_1 sold a reduced price within review time

In addition to the classical demand plus safety stock as order-up-to point, the EWA_{3SL} considers the substitution effect, when evaluating relative to available inventory. Also, that the substitution for "FFP A" and "FFP B" may not necessarily be one-to-one, i.e. equal interdependence. As an example, while a substitute for ground beef 8-12% may be ground beef 4-7%, the substitute for 4-7% may be a completely different product, i.e. thus not necessarily symmetrical demand-effect.

In step 1 (equation 4, below) in the EWA_{3SL}, if the available inventory of product p_1 at time t is less than the sum of expected demand within the review- and lead-time, the safety stock and the expected substitution-demand from other products (not having sufficient inventory) (product *2* to x, $p_2 \to p_x$), then continue to step 2. $E\left[D_{j,i}^{sub}\right]$ is expected substitution demand for all products p_j, when product p_1 has excess inventory and p_j has too low inventory to satisfy demand and thus substitute with product p_1. This is influenced by the substitution probability factor $\mu_{p_1|j}$ for all j products [10]. Similarly, when the substituting products p_j have excess inventory, allowing substituting demand from product p_1. In the formula we account for an FFP may have several other substituting FFPs as the case of, e.g. multiple brands (brand#1, brand#2 and private label). For expected demand, this may be particularly relevant when a certain product may not be available from the supplier for a (longer) period. This is depicted in equation (4).

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2 Customer orders placed long time in advance, e.g. pre-orders for campaigns.

In step 2 (equation 5), the substituting inventory available from the product $p_2 \rightarrow p_x$ is included when evaluating against product p_1 demand and product $p_2 \rightarrow p_x$ substitution demand. If the total available inventory is less than the total expected demand, proceed to step 2a. Here the evaluation of safety stock and outdated/price-reduced products determines the order-size as described by [7]. In the EWA$_{3SL}$, we additionally add the number of products price-reduced due to close to expiration as well as the substituting demand from other products if safety stock is smaller than the two. This is depicted in equations (5-9).

In step 3, if the available inventory is larger or equal to the expected product and substitution demand, no order should be placed. This may be of particular relevance if experiencing too high inventory levels of substituting products that need to be reduced. Depending on the substitutability, different products inventories may be included in the calculation. Thus, EWA$_{3SL}$ includes risk mitigation by evaluating with substitution inventory that could otherwise end up as potential waste if inventory levels are high. This is depicted in equation (10).

1) If:

$$I_{p_1,t}^{available} < \sum_{i=t+1}^{R_{p_1}+L_{p_1}} E\left[D_{p_1,i}\right] + SS_{p_1} + \sum_{j=1}^{p_2 \rightarrow p_x} \sum_{i=t+1}^{R_{p_1}+L_{p_1}} E\left[D_{j,i}^{sub}\right] \mu_{p_1|j} \quad (4)$$

where:

$D_{j,i}^{sub} = 0 \quad if \quad I_{j,i}^{available} \geq D_{j,i}$ and $D_{j,i}^{sub} > 0 \quad if \quad I_{j,i}^{available} < D_{j,i}$

$$\mu_{p_x|j} = \begin{pmatrix} 0 & \mu_{p_1 2} & \cdots & \mu_{p_1 j} & \cdots \\ \mu_{p_2 1} & 0 & \cdots & \mu_{p_2 j} & \cdots \\ \vdots & \vdots & \ddots & \vdots & \cdots \\ \mu_{p_x 1} & \mu_{p_x 2} & \cdots & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

then,

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for all $\boldsymbol{I_{p_x,t}^{available} < E\left[D_{p_x,i}\right]}$,

2) if,

$$I_{p_1,t}^{available} + \sum_{j=1}^{p_2 \rightarrow p_x} \sum_{i=t+1}^{R_{p_1}+L_{p_1}} I_{j,i}^{sub.avail.} < \sum_{i=t+1}^{R_{p_1}+L_{p_1}} E\left[D_{p_1}\right] + \sum_{j=1}^{p_2 \rightarrow p_x} \sum_{i=t+1}^{R_{p_1}+L_{p_1}} E\left[D_{j,i}^{sub}\right] \mu_{p_1|j} \quad (5)$$

then,

2a) if,

$$SS_{p_1} < \sum_{i=t+1}^{R_{p_1}+L_{p_1}-1} \hat{Q}_{p_1,i}^{outdate} + \sum_{i=t+1}^{R_{p_1}+L_{p_1}-1} \sum_{l=1}^{\varepsilon_{p_1}} \hat{Q}_{p_1,i,l}^{reduced} \quad (6)$$

then,

$$Q_{p_1,t} = \max\left(\left(\begin{matrix} \sum_{i=t+1}^{R_{p_1}+L_{p_1}} E\left[D_{p_1}\right] + \sum_{i=t+1}^{R_{p_1}+L_{p_1}-1} \hat{Q}_{p_1,i}^{outdate} + \sum_{i=t+1}^{R_{p_1}+L_{p_1}-1} \sum_{l=1}^{\varepsilon_{p_1}} \hat{Q}_{p_1,i,l}^{reduced} \\ + \sum_{j=1}^{p_2 \rightarrow p_x} \sum_{i=t+1}^{R_{p_1}+L_{p_1}} E\left[D_{j,i}^{sub}\right] \mu_{p_1|j} - I_{p_1,t}^{available} \end{matrix}\right), 0\right) \quad (7)$$

2b) if,

$$SS_{p_1} \geq \sum_{i=t+1}^{R_{p_1}+L_{p_1}-1} \hat{Q}_{p_1,i}^{outdate} + \sum_{i=t+1}^{R_{p_1}+L_{p_1}-1} \sum_{l=1}^{\varepsilon_{p_1}} \hat{Q}_{p_1,i,l}^{reduced} \tag{8}$$

then,

$$Q_{p_1,t} = \max\left(\left(\sum_{i=t+1}^{R_{p_1}+L_{p_1}} E[D_{p_1,i}] + \sum_{j=1}^{p_2 \to p_x} \sum_{i=t+1}^{R_{p_1}+L_{p_1}} E[D_{j,i}^{sub}]\mu_{p_1|j} + SS_{p_1} - I_{p_1,t}^{available}\right), 0\right) \tag{9}$$

for all $I_{p_x,t}^{available} \geq E[D_{p_x,i}]$,

3) if,

$$I_{p_1,t}^{available} + \sum_{j=1}^{p_2 \to p_x} \sum_{i=t+1}^{R_{p_1}+L_{p_1}} I_{j,i}^{sub.avail.} \geq \sum_{i=t+1}^{R_{p_1}+L_{p_1}} E[D_{p_1}] + \sum_{j=1}^{p_2 \to p_x} \sum_{i=t+1}^{R_{p_1}+L_{p_1}} E[D_{j,i}^{sub}]\mu_{p_1|j} \tag{10}$$

then,

$$Q_{p_1,t} = 0$$

$I_{p_1,t}^{available}$ = inventory position (on hand plus in transit) at time t for product p_1
$I_{j,i}^{sub.avail.}$ = beginning inventory at time i for substituting product *j* $(p_2 \to p_x)$
$\hat{Q}_{p_1,i}^{outdate}$ = estimated number of product p_1 to expire within review time
$\hat{Q}_{p_1,i,l}^{reduced}$ = estimated number of product p_1 sold a reduced price within review time
$E[D_{j,i}^{sub}]$ = expected substitution demand from product *j* $(p_2 \to p_x)$
$E[D_{p_1,i}]$ = expected demand from product p_1
SS_{p_1} = safety stock for product p_1
$Q_{p_1,t}$ = order quantity for product p_1
$\mu_{p_1|j}$ = substitution matrix for product *j* $(p_2 \to p_x)$ substituting with product p_1 when $I_{j,i}^{available} < D_{j,i}$
ε_{p_1} = price elasticity of product p_1 for price reduction when p_1 gets close to expiration

3 Conclusion

This study extends the inventory control for stochastic demand and fixed review time to a multi-product model, by suggesting a new heuristics considering four product characteristics. The model includes substitution factors across all products as well as includes potential noise in supply-signal through estimated fill-rate during future orders to receive. It is based on previous studies on EWA. By allowing asymmetrical evaluation according to the product characteristics, the EWA_{3SL} reflects the real-life situations, even more, causing effective decision-making when order-sizing. This means that, e.g. the impact from different rounds of price-reduction on the product demand is considered. The EWA_{3SL} is expected to bring even lower waste and improved availability than previous results by supporting the mitigation of risks across products. For practical implications, determining the substitution factor may be challenging and rather subjective given the limited literature on the subject matter and the influence from the geographical area, culture etc. [10, 19]. A solution may be to then apply a binary system: 0 if not substitutable and 1 if substitutable. Further, the model is yet to be tested, and further research governs checking how robust the heuristic is, the impact on inventory levels, fill-rate and waste.

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