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Abstract. In time of financial crisis, bank loans are often extremely difficult to obtain for many companies. However, companies always need free cash flow to efficiently react against to any uncertainty. This work demonstrates the impact of financial consequences on operational decisions in the single-product, single-level, infinite capacity EOQ model. We propose an operation-related working capital requirement (WCR) model in a tactical planning context. The classic EOQ model is extended by integrating the WCR financing cost with a cost minimization objective and deriving its analytical solution. Compared with the optimal policy of the classic EOQ model, our approach leads to a new policy with a smaller production lot size due to the new cost trade-offs. Furthermore, an analytical analysis with a classic EOQ-based formula that considers the cost of capital demonstrates the sensitivity of approximating financial costs compared to our exact approach. Finally, sensitivity analysis and numerical examples are also provides.

Keywords: working capital requirements.· payment delays.· EOQ model.· cost of capital.

1 Introduction

In the context of the recent financial crisis, many companies suffer from a lack of credit and working capital. Furthermore, they must accept longer payment terms from their customers which exacerbates their working capital level [4]. Moreover, tight or unavailable bank credit impacts the company's performance by halting their operations and starving the supply chain[1]. For this reason,

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in recent years working capital management (WCM), especially the working capital requirement(WCR) has drawn more attention for mainly two reasons. The first is the direct link to the cash flow level. The second reason is the potential of generating additional cash flow. As reported in the Working capital management report 2012 of Ernst, $1.1 trillion in cash have been unnecessarily tied up in working capital in the largest American and European companies. For its important role in practice, the WCR is chosen as the financial aspect to be integrated.

To do so, the WCR is remodeled for adapting the context of tactical planning following its financial signification based on cash conversion cycle[2]. However, its non-linear formulation brings additional difficulty in resolution. More essentially, due to this non-linearity, all existing theorems and properties of classic lot-sizing model must be revalidated. New ones may be necessarily proposed. For example, the Zero-Inventory Property has been proven valid in Uncapacitated Lot-Sizing (ULS) case. As mentioned in [3], to tackle more complex and realistic cases, some heuristic algorithms are established based on the Economic Order Quantity(EOQ) and the trade-off between the setup cost and inventory holding cost that minimizes the total cost. (When the EOQ model is adopted for lot-sizing problem, the EOQ represents the optimal production quantity and the fixed order cost is the fixed setup cost for one setup.) Therefore, we do not use the classic Economic Production Quantity (EPQ) model, but adopt the classical EOQ model and consider all operation costs including purchasing, setup, production and inventory holding (noted as holding for the rest of paper) costs. Furthermore, the WCR financing cost is integrated into the cost minimization objective in order to obtain a new form of the EOQ of this new problem. Such a new EOQ can thus be used for building heuristic in the future and to better understand the financial consequence of operation decision on the production program. The main contributions are summarized as follows:

- First, we propose a model of WCR adapted to the EOQ context. With this model, we are able to measure the evolution of WCR over the continuous planning horizon;
- Second, we give the closed-form, analytical formula for the optimal production quantity. Then, we prove that our result extends Wilson's formula. Furthermore, we identify the trade-offs between different costs that minimize the total cost. Sensitivity analysis is also provided in order to show the influence of parameter variations, particularly those which are finance related.

2 EOQ based cost minimization model

2.1 WCR modeling and differences compared with the trade credit concept

In the field of accounting and finance, the WCR is measured adopting the cash conversion cycle (CCC) methodology. It is calculated as the sum of the account payable and the inventory value minus the account receivable using the balance sheet and the profit& loss statement. However, it only represents an aggregated financial situation of the company at the moment of elaboration. Therefore, we propose a new generic WCR model based on the CCC. It is adapted for introducing the financial costs linked to operational decisions (i.e., the financing cost of WCR) in the EOQ context. Both the delay in payment to supplier and from client (i.e., downstream and upstream) are taken into account in this formulation. Moreover, only production-related WCRs are considered in this WCR model. They thus depend on the amount of investment for the related operations (e.g., purchasing, setup, production and inventory holding) and the associated financing duration before recovering by corresponding revenue. More precisely, the financing cost of the WCR is the interest (i.e., cost of capital) multiplied by the amount of WCR.

Comparing our WCR model with the concept of the trade credit in the inventory model literature, the essential difference is found on the consideration of sales revenue. In the literature, the trade credit is modeled with the assumption that the unpaid revenue from the customer is charged as a whole by the retailer for earning interest. From a financial point of view, it assumes, by default, that the cost part and the profit part of the revenue are reinvested in the same operating cycle. In practice, the profit can be allocated to any of a number of objectives for the firm including debt reduction, internal or external investment, or dividend payments. Consequently, the above-mentioned assumption does not reflect the financial management in real world situation. Since the profit reinvestment problem should be more carefully investigated. Therefore, we separate the investment problem and the WCR management and focus on the latter.

To do so, we assume that the WCR is only financed by the cost part of the revenue, not by the profit part. It is because the profit of the company has never been required, it does thus not correspond to the WCR definition and should not be considered as the WCR. In consequence, we adopt the scheme that the WCR generated by producing a product is only effectively recovered when that product is sold. Thus, we progressively and uniformly receive all production related cost from the sales revenue of products over time. Nonetheless, since we do not designate the allocation of profit (to WCR financing or others objectives), our model remains a partial model for companies.

2.2 Assumptions

In this section, we present an EOQ-based model with a cost minimization objective considering the financing cost of WCR. The WCR formulation and the corresponding proposed model are built under the following assumptions:

- Production:
  - Demand is constant and uniform during the planning horizon;
  - For each production lot, all purchasing, setup, production and inventory holding costs must be financed;
  - Production capacity is infinite;

- • No backlogging is allowed;
  - • Only inventory of final product is considered;
  - • Initial and final stocks are defined as zero;
  - • No delivery delay of material and production is immediate.
- Financial:
  - • Production, inventory and setup costs should be paid instantaneously (but may be financed);
  - • WCR is only financed by the cost part of revenue;

As previously discussed, we only consider the cost portion in revenue (not the profit margin) for financing the WCR because the margin may be reinvested in another operation cycle. With this assumption, we do not need to consider the opportunity cost of using the margin to finance the WCR. Moreover, we consider that the WCR is generated due to the timing mismatch between revenue and costs to finance. Therefore, we set the last assumption in the list in order to precisely calculate the setup cost in revenue of selling each product. More specifically, all products in the same production lot share the fixed setup cost equally and it will be refunded progressively by selling the products.

2.3 Parameters et decision variables

Notation is defined for the single-site, single-level, single-product with infinite capacity case in Table 1. These parameters are all assumed to be nonnegative. The decision variable is the production lot size, denoted as Q. Accordingly, Q^* represents the optimal production lot size.

Table 1. Parameters for WCR modeling

Parameter	Definition
T	Horizon length
D	Total demand in units
d	Demand rate in units per time unit $\left(d = \dfrac{D}{T}\right)$
h	Unit inventory cost in dollars per unit during a time unit
p	Unit production cost in dollars per unit
s	Unit setup cost per time in dollars per lot
a	Unit raw material cost in dollars per unit
r_c	Delay in payment from client in time units
r_f	Delay in payment to supplier in time units
α	Interest rate for financing WCR per dollar per time unit

2.4 WCR formulation

Following the concept of the cash conversion cycle methodology, we propose a generic WCR formulation for purchasing, setup, production and holding inventory by taking into account the interest accrued by financing the WCR during the planning horizon. In the EOQ model, a uniform time division is established in which, for each cycle, a quantity Q is instantly supplied at the beginning of the cycle and uniformly consumed over the cycle. Thus, for a horizon with total duration, T and a global demand, D, each period will last $\dfrac{QT}{D}$ time units. Since the WCR represents the financing need for these operations between when we pay for them and when we collect the payment from customer, these two timings are the key elements of the WCR measurement. Explicitly, we product a lot of Q products at the beginning of the cycle (i.e., instant 0) with the following cost and revenue timing assumptions:

- The setup and production costs are paid immediately, while the purchasing cost is paid at a delay to the supplier at instant $0 + r_f$.
- A product sold (from inventory) at instant $t \in \left[0, \dfrac{QT}{D}\right]$ will be paid at a delay by the customer at the instant $t + r_c$.

The three diagrams in Figure 1 illustrate the financing needs for purchase, production and setup operations over time. Since the WCR is the financial need to cover between the timing of paying for expenses and collecting the revenue, the surfaces in these figures represent the WCR of these operations in each production cycle. We can then easily establish the formulation of the WCR for each

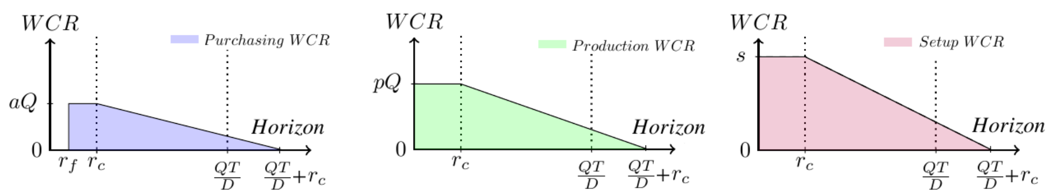

Fig. 1. Illustration of WCR of purchasing, setup and production costs in a cycle

lot, generated by purchasing, setup and production, as follows :

$$
\begin{aligned}
WCR_{purchasing}(Q) &= a\left[Q(r_c - r_f) + \frac{Q^2T}{2D}\right]\\
WCR_{production}(Q) &= p\left(Qr_c + \frac{Q^2T}{2D}\right)\\
WCR_{setup}(Q) &= s\left(r_c + \frac{QT}{2D}\right)
\end{aligned}
$$

Contrary to purchasing, set-up and production, which are all one-time payments, a unitary inventory holding cost must be paid regularly for each item for the entire duration of its storage. Therefore, there is a cumulative effect in financing the inventory holding costs. More precisely, for one product sold at $t \in \left[0, \frac{QT}{D}\right]$, we must pay its inventory holding costs generated in all instants over the entire duration 0 to t. Thus, all these costs must be financed from the instant when they occur (instant 0) until the arrival of client's payment at $t+r_c$, as presented in Figure 2.

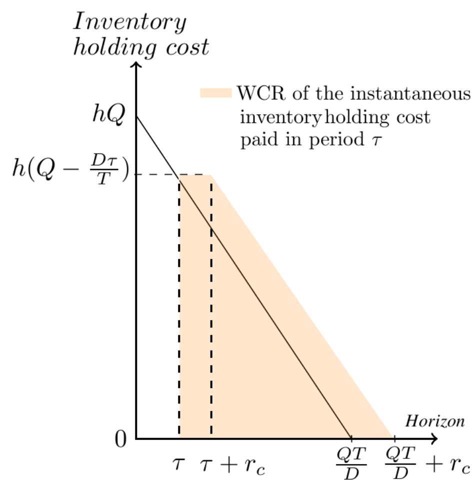

Fig. 2. Illustration of the WCR calculation for inventory holding cost

Consider an item that is sold at insant t, where $t \in \left[0, \frac{QT}{D}\right])$ and one of its inventory payment which occurs at $\tau \in [0, t]$. This cost must be financed until the customer pays the product at $t + r_c$. Therefore, the financing duration will be $t + r_c - \tau$. Consequently, the WCR for inventory holding cost in a production cycle is formulated as follows:

$$\begin{aligned} WCR_{inventory}(Q) &= \int_0^{\frac{QT}{D}} \int_0^t h\frac{QT}{D}\,(t + r_c - \tau)\ d\tau dt \\ &= h\left(\frac{Q^3T^2}{6D^2} + \frac{r_cQ^2T}{2D}\right) \end{aligned} \tag{1}$$

Following the above formulations of WCR, the total WCR for a production cycle can be expressed as follows:

$$\begin{aligned} WCR(Q) &= WCR_{inventory} + WCR_{purchasing} + WCR_{production} + WCR_{setup} \\ &= \frac{hT^2}{6D^2}Q^3 + \frac{T}{2D}(hr_c + a + p)Q^2 + \left[a(r_c - r_f) + pr_c + \frac{sT}{2D}\right] Q + sr_c \end{aligned}$$

Accordingly, the total WCR is formulated as follows :

$$WCR_{total} = \frac{hT^2}{6D}Q^2 + \frac{T}{2}(hr_c + a + p)Q + D\left[a(r_c - r_f) + pr_c + \frac{sT}{2D}\right] + sr_cD\frac{1}{Q}$$

2.5 Objective function

The aim of our contribution is to minimize the total cost (TC) including the production-related logistic cost and associated financial cost for satisfying constant demands. In our case, the interest is the cost of financing the WCR. The formulations of these components are

- The logistic cost (denoted as CL) is the sum of purchasing, setup, production and inventory holding costs: $CL = aD + s\frac{D}{Q} + pD + hT\frac{Q}{2}$;
- The interest to pay for financing the WCR (denoted as CF) based on an interest rate α: $CF = \alpha WCR_{total}$.

Consequently, the objective function is formulated as

$$\begin{aligned} TC &= CL + CF \\ &= D\left\{a + p + \alpha\left[a(r_c - r_f) + pr_c + \frac{sT}{2D}\right]\right\} \\ &\quad + \frac{hT + \alpha(hr_c + a + p)T}{2}Q + \frac{\alpha hT^2}{6D}Q^2 + \frac{Ds(1+\alpha r_c)}{Q} \end{aligned} \tag{2}$$

We observe that the objective function is convex because all negative terms are either stationary, linearly increase or convex which means a unique minimum solution exists.

3 Optimal solution and structural properties

To calculate the optimal production lot size, Q^*, we take the first-order derivative of TC.

$$\frac{\mathbf{d}TC}{\mathbf{d}Q} = -\frac{s(1+\alpha r_c)D}{Q^2} + \frac{\alpha hQT^2}{3D} + \frac{[h + \alpha(hr_c + a + p)]T}{2} \tag{3}$$

The optimal solution is obtained when the equation (3) is set equal to zero. In order to calculate it, we adopt the Cardano method which is devoted to resolve a cubic equation in a special form.

Proposition 1. *We denote that* $\Delta = \frac{3\alpha^2h^2T^4}{s^2(1+\alpha r_c)^2D^4} - \frac{[h+\alpha(hr_c+a+p)]^3T^3}{2s^3(1+\alpha r_c)^3D^3}$

- *if* $\Delta > 0$, $Q^* = \frac{1}{\sqrt[3]{\frac{hT^2\alpha}{6D^2s(1+\alpha r_c)}}\left\{\sqrt[3]{1+\sqrt{1-\frac{D[h(1+\alpha r_c)+\alpha(a+p)]^3}{6h^2s(1+\alpha r_c)\alpha^2T}}} + \sqrt[3]{1-\sqrt{1-\frac{D[h(1+\alpha r_c)+\alpha(a+p)]^3}{6h^2s(1+\alpha r_c)\alpha^2T}}}\right\}}$
- *if* $\Delta = 0$, $Q^* = \frac{D[h+\alpha(hr_c+a+p)]}{2\alpha hT}$
- *if* $\Delta < 0$, $Q^* = \frac{1}{2\sqrt{\frac{[h(1+\alpha r_c)+\alpha(a+p)]T}{6Ds(1+\alpha r_c)}}\cos\left\{\frac{1}{3}\arccos\left[\sqrt{\frac{6h^2s(1+\alpha r_c)\alpha^2T}{[h(1+\alpha r_c)+\alpha(a+p)]^3D}}\right]\right\}}$

We can immediately remark that the optimal solution does not depend on the delay in payment to the supplier. This is not surprising as it remains constant whatever the value of Q in the WCR formulation. We can also deduce from this proposition to the fact that our result extends the Wilson formula.

Proposition 2. *For any value of Δ,*

- Q^* *is an increasing function of s and r_c*
- Q^* *is a decreasing function h, a, p and α*

Proposition 3. *For any value of Δ, let Q^*_{EOQ} be the optimal solution for the classic EOQ model, then $Q^* \leq Q^*_{EOQ} = \sqrt{\dfrac{2Ds}{hT}}$*

From a managerial point of view, this corollary has great significance as the setup and inventory holding costs are often very difficult to estimate in practice. The fact that our proposed model is less sensitive to variations in these costs yields a more robust economic order quantity.

4 Conclusions

We propose a new generic model of operations-related working capital requirements. Such a modeling allows us to measure the evolution of WCR over the entire planning horizon. Furthermore, an EOQ-based cost minimization model is developed considering the financing cost of WCR for the single-site, single-level, single-production and infinite production capacity case. We derive the analytical formula of the optimal EOQ and provide analysis to highlight some managerial insights.

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