Analysis of the Effect of Number of Players on the Excitement of the Game with Respect to Fairness

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Abstract. Games can be played alone or with multiple people. We often assume that the game is more fun if played with someone, but we do not know much about how the number of players in a game affects the enjoyment of a fair game. From this research, we outlined that more players in a game increase the unpredictability of the game and its excitement overall. By applying the game refinement theory, we can see how the number of players can affect the excitement of a game. We also look at the pressure of a game on the players by utilizing the force-in-mind theory to further understand how games can motivate players without causing them too much stress.

Keywords: Fairness, Number of players, Pressure in games, Game refinement theory, Force-in-mind theory

1 Introduction

The act of playing games initially was something done during free time, to either have fun or relax after a busy day. However, as time went on, people started being more competitive over games. They started playing the game, practicing for long hours to hone their skills and master the game itself. Having more skills often meant that they will most certainly win the game. However, this would probably be not too interesting for viewers if these games are too one-sided. Ben-Naim et al. [5] have done a research to find the most competitive game. From their research, they found that a game with a higher upset probability is much more competitive game. In this research, they categorized the team into two sides, the team with a better performance record is considered the favourite, while the team with a worse performance is the underdog. The upset probability is the chances for the underdog team to win the game and hence, the closer the upset probability to 0.5, the more competitive the game is. Hence, games often introduce luck or chance elements to help balance the game [12]. By implementing luck and chance in a game design, this gives the underdogs a chance to win, and in turn makes it interesting for both the players and viewers, as the outcome of the game will be more uncertain. However, incorporating too much luck would make the game into a game-of-chance like children or gambling games [12]. A good balance between skill and luck in game design must be configured to make the game interesting, competitive and fun for everyone.

Games are further divided into two categories, single-player games or multiplayer games. As the name suggests, single player games usually consist of only one player while multiplayer games consist of two or more players. With the existence of the Internet, multiplayer games have become the norm of many online games as players from all over the world can connect and play with each other. Examples of popular online multiplayer games are Dota 2, Diablo III, Counter Strike, FIFA Online and StarCraft II. These online games can usually consist of a massive number of players, ranging from two up to the hundreds. More players in a game would suggest that an individual will interact with other players more frequent, where the interaction can be cooperation or a stand-off to achieve a goal. These increasing interactions help make the game more interesting to players. With these many players in a game, the main concern that comes to mind is the fairness of the game. How is the game balanced to ensure that each player has the same resources to work with in the game without having huge advantage over the others?

In this paper, we look at how the number of players can influence a game with respect to fairness. In Section 2, we briefly look into the game refinement theory and force-in-mind theory. Section 3 shows how games incorporate skill and chance to create a perceived fairness. Section 4 then outlines the effect of more players in a game on the excitement of the game. Section 5 shows how pressure in mind can be estimated. Concluding remarks are then made to summarize this paper.

2 Game refinement theory

Game Refinement Theory is a theory which was proposed by Iida et al. [9]. The Game Refinement Theory focuses on the sophistication and the uncertainty of the outcome of a game which in turn increases the attractiveness of a game. This theory is not used to calculate winning strategies but is used to see the quality of the game and the amount of entertainment it can provide. The game progress is twofold, where one is the game speed or scoring rate while the other is the game information progress. Game information progress presents the degree of certainty of the games results in time or steps. If one knows the game information progress, for example after the game, the game progress $x(t)$ will be given as a linear function of time t with $0 \leq t \leq t_k$ and $0 \leq x(t) \leq x(t_k)$ as shown in Eq.(1).

$$x\left(t\right) = \frac{x(t_k)}{t_k}t \tag{1}$$

However, the game information progress given by Eq.(1) is usually unknown during the in-game period. Hence, the game information progress is reasonably assumed to be exponential. This is because the game outcome is uncertain until the very end of many games. Hence, a realistic model of game information progress is given by Eq.(2).

$$x(t) = x(t_k) \left(\frac{t}{t_k} \right)^n \tag{2}$$

Here, n stands for a constant parameter which is given based on the perspective of the observer. If one knows the outcome of the game, then we have $n = 1$, where $x(t)$ is a linear function of time t. Iida [8] conjectured that the parameter n has a possible correspondence with the expected feeling of the player in the game as tabulated in Table 1. The parameter $n = 2$ is ideal as it is the most exciting setting.

Table 1. Correspondence of n with Expected Feeling

n	*Expected Feeling*
0	Boring
1	A little challenging
2	Exciting
3	Very challenging
4	Too high challenge

Eq. (2) is then derived twice to obtain the acceleration of the game information. Solving at $t = t_k$, we obtain Eq. (3).

$$x''(t_k) = \frac{x(t_k)}{(t_k)^n} t^{n-2} \, n(n-1) \, |_{t=t_k} = \frac{x(t_k)}{(t_k)^2} n(n-1) \tag{3}$$

It is assumed in the current model that game information progress in any type of game is encoded and transported in our brains. We do not know yet about the physics of information in the brain, but it is likely that the acceleration of information progress is subject to the forces and laws of physics. Therefore, we expect that a larger value of $\frac{x(t_k)}{(t_k)^2}$, the more exciting the game becomes due to the uncertainty of the game outcome. Thus, we use its root square $\frac{\sqrt{x(t_k)}}{t_k}$, as a game refinement ($GR$) measure for other games under consideration. Table 2 shows some of the GR values for variants of boardgames and scoring sports games [16, 13]. We can see that the GR values lie between a zone value of 0.07 to 0.08.

According to Newton's second law of motion, the vector sum of force F is equal to the mass m multiplied by acceleration a of the object as described in Eq. (4).

$$F = ma \tag{4}$$

We can assume that the Newton's second law can be applied to find the Force-in-mind, F_n. Force-in-mind can be defined as the force of the information that is moving in our minds. A higher F_n signifies that the information is more

Table 2. GR value of various games

Game	$x(t_k)$	t_k	*GR*
Chess	35	80	0.074
Go	250	208	0.076
Basketball	36.38	82.01	0.073
Soccer	2.64	22	0.073
Badminton	46.34	79.34	0.086
Table Tennis	54.86	96.47	0.077

interesting and has a higher momentum in our minds. To calculate the F_n, we need to find the acceleration-in-mind as well as the mass-in-mind. As Eq. (3) is the second derivative of Eq. (1), the expression can be said to be the acceleration-in-mind. The expression can be further simplified as shown in Eq. (5).

$$a = (GR)^2 \; n(n-1) \tag{5}$$

GR value shows the balance between the skill and chance aspects of a game. As the variable n stands for the difficulty of the game, we can say that the $n(n-1)$ in Eq. (5) is the variable for game difficulty. Mass-in-mind can be defined as the inverse of branching factor in board-games. In score based games, the mass-in-mind can be defined as the total score of a game Σg over the score of a player g [10]. Hence, the force-in-mind can be expressed in Eq. (6)

$$F_n = \frac{\Sigma g}{g} \; (GR)^2 \; n(n-1) \tag{6}$$

3 Fairness and excitement

Fairness can be defined as the act of being impartial by giving equal treatment to everyone without any bias or favouritism. In a game context, this can be translated into giving equal rules to all players without advantages for any side, ensuring an equal winning probability for each side. If each player has an equal probability of winning, we can assume that the game will always lead to a draw. V.d.Herik et al. [17] defined that a game is fair if the outcome of the game is a draw and both sides of the players have roughly an equal probability of making a mistake which can cause them in the game. Iida [7] also conjectured that the outcome of a sophisticated game is always a draw if both players are of sufficiently strong level. This implies that the perfect play by both sides of the game will always lead the game to a draw. To make the outcome of the game more unpredictable, certain games nowadays do not maintain total fairness by incorporating elements of luck and chance.

Lennart Nacke [12] explained on the significance and importance of the balance of skill and chance in game design. A fair game usually operates on balance of skill or chance. Games that run mainly on chance are usually children games like scissors-paper-stone or gambling. Games with a focus on luck and chance help create fairness for people of various skill level, making it possible for lower skilled players to still be able to win. This will make the game very unpredictable as anyone can win the game, regardless of their skill level. Games that operate mainly on skill are more deterministic and players can master the game if they know the most effective strategies. Games like this allow veteran players to have an advantage over a newer player. This can get very one-sided as the result of the game can be easily predicted before the game even ends. Therefore, games now incorporate both skill and chance elements to their game design. The element of chance helps create unexpectedness into the game, making it more exciting and attractive to players and viewers.

Game refinement theory has been studied mainly based on the game outcome uncertainty [9]. However, it essentially concerns about game length with respect to justification of game outcome. If the length is too short, the game outcome would be not justified, i.e. unfair. If the length is too long, the outcome would be more than stable, i.e. boring or unexcited. Thus, justification of game outcome postulates the appropriate game length to maintain fairness and excitement. Therefore, the measure of game refinement for sophisticated games is ranged in a zone value like 0.07 to 0.08, which may correspond to the lower limit (engagement) and upper limit (fairness) respectively. A game which has a low GR value shows that it is more skill based, while those with a high GR show that it is more chance based [13]. The ideal game refinement zone is between 0.07 and 0.08 which can be seen by various popular boardgames and sports. Hence, we can say that a game is most interesting at a balance of skill and chance to make the game competitive and exciting at the same time.

4 Influence of the number of players

In this section, we study on how the number of players can affect the excitement of a game in team games and non-team games. The GR value as explained above is used to decribe the properties of the game, whether it is more deterministic or stochastic.

4.1 The parameter k

Figure 1 shows a curve of $y = GR^2x^2$ and two linear lines of formula $y = x$ and $y = \frac{x}{2}$ respectively for values of $x \geq 0$. There are two intersection points in this graph, which are the intersection of the curved line with the line $y = x$ and the curved line with the line $y = \frac{x}{2}$. The value of x and y from the graph can be assumed as the total score (Σg) and the winner's score (g) of a score based game respectively. Hence, the line $y = x$ and $y = \frac{x}{2}$ can also be expressed as $g = \Sigma g$ and $g = \frac{\Sigma g}{2}$ respectively.

The line $y = x$ shows that the total score of the game is equal to the winner's score. This signifies that the game is very one sided. For example, in a game of soccer, if the score is 4:0, both the total score and the winner's score will be equals to four. The game may be too easy for the winning team.

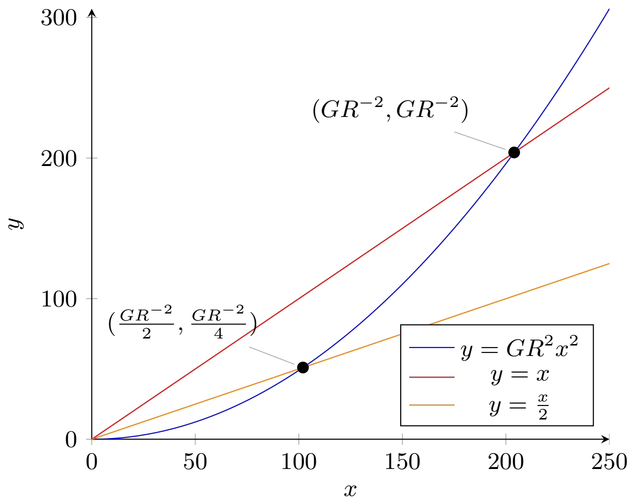

Fig. 1. Graph of $y = GR^2x^2$

The line $y = \frac{x}{2}$ shows that the winner's score is half of the total score. This signifies that the game is equally fair to both players. A similar example would be in a game of score with the score 2:2, where the total score is four while the winner/players' score is two. This shows that there is an equal distribution of score among the players.

If we draw a new straight line where $y = \frac{x}{3}$, this new line would signify that the players' score is one-third of the total score as $g = \frac{\Sigma g}{3}$.

Conjecture 1. The line $y = \frac{x}{k}$ is an even distribution of score for k number of players in an even and fair game scenario, where each player will have an equal win probability of $\frac{1}{k}$.

In Figure 1 a curve $y = GR^2x^2$ shows the progress of the game over time with respect to the score of the game. At the intersection of the curve with line $y = x$ at coordinates (GR^{-2}, GR^{-2}), the total score is equal to the winner's score, as was previously discussed above. The points on the curve after this point shows that the winner's score is higher than the total points. Hence, we can assume that the game ends at the intersection point of the curve and the line $y = x$ at point (GR^{-2}, GR^{-2}).

Figure 2 shows the curve of $y = GR^2x^2$ with varying GR values from 0.06 to 0.09 and its intersections with the line $y = x$. As the GR increases, we can see that the intersection point of the curve and the line $y = x$ approaches the origin. A high GR value indicates that the game is more sophisticated and in turn, more exciting to the players and the observers. A higher GR value also indicates that the game has a more stochastic nature. A lower GR on the other hand shows that the game is more deterministic and requires more skills from the players.

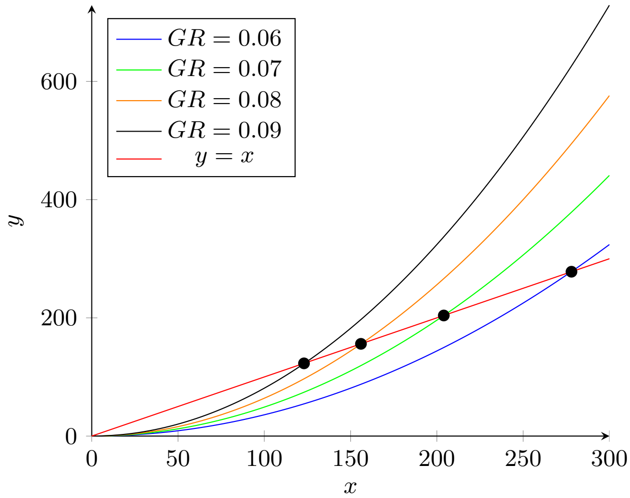

Fig. 2. Graph of $y = GR^2x^2$ with varying GR values

Conjecture 2. The game is more exciting, sophisticated and stochastic in nature as the intersection points get closer to the origin. Conversely, if the points are further away, the game is much more deterministic and less sophisticated.

4.2 The parameter k in non-team games

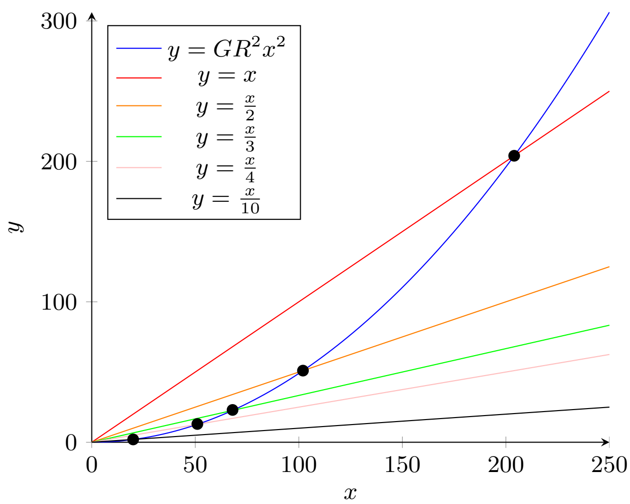

Fig. 3. Graph of $y = GR^2x^2$ with lines $y = \frac{x}{k}$

Figure 3 shows the same curve of $y = GR^2x^2$ and five straight lines of line $y = \frac{x}{k}$ where k is equal to 2,3,4 and 10 respectively. The parameter k shows the number of players in the game. From the graph in Figure 3, we can see that the intersection point between each of the straight line gets closer to the origin as the parameter k increases. It was conjectured previously in Conjecture 2 that as the intersection point approaches the origin, the game gets more stochastic and exciting to the players.

A previous research on the effect of number of players in an UNO game has been done by Alfian et al.[15]. UNO is a card game that can be played with multiple players for up to ten players, with the recommended number of players being four to six people. The finding from the research showed that the GR measure for the UNO game varied for different number of players. The GR measure increased as the number of players increased to four players, but showed continual decrease when the number of players gets more than four people. Hence, having four-players may be the optimum way to play UNO.

A research by Hainey et al.[6] showed that players who preferred multiplayer games over single player games were motivated by the amount of competition available in the game. Multiplayer games provide them with more competition, recognition, curiosity and cooperation which makes the game more interesting to the players.

Conjecture 3. The game gets more exciting and stochastic as the number of players increases in a game. Conversely, the game will be more deterministic if it has less players.

4.3 The parameter *k* in team games

The parameter k is different for team-based or cooperative games. For example, in the previous example of UNO game, playing with four players would mean that the parameter $k = 4$ and the probability of each player to win under an equal fairness condition is $\frac{1}{k} = 0.25$.

In the case of a soccer game, there are two teams, where each team has 11 players. The total number of players in a soccer game is 22 people. However, the parameter $k \neq 22$ as each player does not have a win probability of $\frac{1}{22}$. Since there are two teams, the parameter k should be equal to 2, as each team has an equal probability to win of $\frac{1}{2}$. The players of each team have the same winning probability as their team, which is 0.5 under fair conditions.

Hence, the parameter k is not affected by the number of players in a team-based game but is only affected by the number of teams in the game.

5 Game pressure on players

In this section, we investigate how pressure in mind affects the excitement of a game and how it can be estimated.

5.1 Pressure

Pressure can be defined as force that is acted perpendicularly upon a surface area. Eq. (7) shows the formula to calculate the pressure that is exerted on a surface. The unit for pressure is Newton per square meter (Nm^{-2}) or Pascal.

$$Pressure = \frac{Force}{Area} \tag{7}$$

Pressure is directly proportional to the amount of force applied and is inversely proportional to the surface area. Pressure acting upon a matter can change its properties or appearances. For example, if enough pressure is applied to gases, it can become a liquid. This can be seen in aerosol cans as the gas are compressed with high pressure to from liquid. Once it is sprayed and released to the atmosphere, the decrease in the pressure causes the liquid to revert back to gas.

Pressure is also known as stress. We often hear people complaining that their daily life is very stressful. This stress can come from various sources such as working long hours, heavy workload, being unhappy and much more. We do not know the physics in our brain, but we can say that our minds feel pressure from our daily activities.

5.2 Pressure in games

Previously, we discussed that pressure existed in our minds, where we feel pressure from working on heavy workloads or working for long hours. Hence, playing a game should also exert pressure in our minds. Pressure in games can be said as the stress to earn score or win in a game. A difficult game might put more pressure on the players as it is significantly harder to earn points or score in those games. Conversely, an easier game would put less pressure on the players as scoring is much easier. The difficulty of the games can be seen in the parameter n of the force-in-mind formula shown in Eq. (6). Table 1 also shows the expected feelings of a player based on the parameter n. As the force-in-mind is directly proportional to the value of n, an increase in n would signify that more force will be applied by the game to our minds.

Catarina et al. [3] showed the influence of the number of players on heart rate responses and physical demands in a small-sided soccer game. A small-sided soccer games are training drills used by coaches to ensure that their players are in top form. In their research, they analyzed the effects of the game type and number of players on the physical performances of each player. They found that the player's heart rate response and exertion of the players are much higher when it is a 3vs3 game compared to a 4vs4 game.

Owen et al. [14] explained that this is maybe due to the presence of less players made it so the players will have to move more with the ball and have a higher possession of the ball. They also pointed out that a high number of player in team will result in less technical actions performed by each player, but generally increases the technical actions performed in the whole game. The amount of technical actions performed by players can be assumed to be the player contribution to the game.

A player's contribution to the game can be generally evaluated as in Eq. (8).

$$Player\ Contribution = \frac{Team\ Score}{No.\ of\ Players} = \frac{g}{k_t} \tag{8}$$

Where k_t stands for the number of players in a team. Having more players in a game would signify that each player would contribute lesser to the team, and hence cause them to not feel much exertion.

Moreover, it is observed that heart beating rate usually increases when we are feeling excited, anxious or angry [1]. Previously, we conjectured that a higher number of players ensured that the game gets more exciting, but the number of players are not influenced in the case of a team game. Therefore, excitement might not be a major cause for the increase in heart rate [3]. The anxiety produced by the game may be the reason for the increase in heart rate. Anxiety is the feeling of fear, worry, or uneasiness that is a reaction from stress. Hence, we can assume that lesser players in a team game causes the stress on the players to increase as the 3v3 small-sided soccer game showed a higher heart rate. Referring to the pressure formula in Eq. (7), we can formulate a new model for calculating the pressure in our minds, assuming that k_t is the number of players in a team.

$$Pressure = \frac{F_n}{k_t} = \frac{\Sigma g\ (GR)^2\ n(n-1)}{g \cdot k_t} \tag{9}$$

Eq. (9) suggests that pressure is directly proportional to the total score (Σg), game refinement value and parameter n but is inversely proportional to the player/team score (g) parameter k_t, which is the number of players in a team. When the player/team is losing, this would mean that more force and pressure is applied on the players. This would further explain why players feel frustrated when losing a game. Astor et al. [4] reported that a losing player's heartbeat rate drops when losing, suggesting that they felt negative emotions or frustration. Their findings also suggested that players experience a more intense emotion when losing than winning. These emotions may be caused by the pressure as explained above.

In the scenario of a non-team game like chess or shogi, the pressure is equivalent to the force-in-mind as $k_t = 1$. Games with high GR value are more sophisticated and exciting. This creates a pressure to the players to work harder to obtain the points or score in a game. However, an extremely high GR or n value may exert too much pressure to the player that may result in anxiety or choking. This can explain why players may feel frustration over an extremely high GR value.

Choking can be defined as performing poorly under pressure due to anxiety [11]. An intense amount of stress can also result in depression among players. An article [2] showed that there is an increasing numbers of athletes who are suffering from depression due to high stress due to the attention, expectations and money involved in the sports. Some of these athletes decided to quit the sports entirely to relieve themselves from the pressure while some even resulted into hurting themselves.

Low pressure in games would mean that there is not much force-in-mind or motivation for the players to score. Hence, it is better for the pressure to be a little bit higher to ensure a fun game experience. On the other hand, an extremely high pressure would cause frustration and anxiety among players, ruining their experience in the game. Therefore, the pressure of a game must be balanced and not be too high to ensure that the player will still have an enjoyable experience throughout the game.

6 Concluding remarks

In this paper, we have looked at how the number of players can influence the enjoyment of the game. We conjectured that the games get more exciting and stochastic with more players. In a team game scenario however, the number of players do not affect the excitement of the game, but the excitement is only influence by the number of teams. We also applied the concept of pressure using the force obtained from the force-in-mind theory dividing the number of players in a team for a team based game. The pressure is directly proportional to the F_n, GR and the n values while inversely proportional to the number of players, k_t.

A high value of F_n, GR and n can cause the pressure to be high, causing negative effects on the player as it can cause them to perform poorly due to the stress induced from the game. Losing a game also creates negative emotions to the players resulting to frustration. This can cause the player to lose interest in the game itself and eventually quit. Hence, a good balance of game difficulty and force-in-mind must be discovered to ensure that the players will be continually motivated to continue playing the game.

In the model formulated to calculate pressure, we assumed that each player in a team will be exerted by an equal amount of pressure as they have the same contribution to the game. It should be noted that each player can have different levels of contribution to the game based on their roles in the game as well as their individual skill. In a game of soccer, players are generally divided into strikers, mid-fielders, defenders and goalkeepers. Each role will have different levels of contribution in a typical game of soccer. A striker will have a higher responsibility in scoring goals, while defenders and goalkeepers will have a higher responsibility in ensuring that the opponent does not score a goal. In a real game scenario, we cannot simply generalize that all players will have the same contribution to the game. Similarly, all players in a team may not always be of the same skill level. Higher skilled players will have a higher contribution to the game and hence have higher pressure exerted on them compared to the other players in the team. Hence, it is quite hard to generalize the pressure to be equivalent to the force divided by the number of players in the team.

More research has to be done on this domain to strengthen the model produced above for pressure. Future works will work on formulating a more accurate model for determining the pressure on players with varying roles and skill levels in a team, as well as look more deeply in to the interaction of the number of players, excitement and pressure of the game to determine the right balance of pressure to ensure that the game is balanced, fun and exciting to the players and viewers alike.

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