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Age of Sensed Information in a Cognitive Radio Network

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***Abstract*—Age of information is often studied as a primary objective to be optimized, but for problems where age is not the primary objective, it can still have a major role that can be utilized. This work studies a two-user, single-channel cognitive radio network, where the primary user's transmit/idle dynamics are modeled as a binary Markov chain, and the secondary user decides to either sense or transmit. Under this setup, the age of the information sensed by the secondary user has a direct impact on its performance. The secondary user aims to maximize its throughput subject to a constraint on the probability of collision experienced by the primary. Using the Markov chain model of the primary user, the secondary user decides on its transmission and sensing strategy based on the estimated evolution of the primary user transmission state. For a stationary randomized transmission policy that depends on the sensed state, we derive the secondary throughput and the collision probability. Due to the complexity of the resulting expressions, we develop an alternative formulation of the problem by recognizing that the throughput and collision probability are functions of the age of each type of sensed information. Therefore, we transform the problem by converting the randomized policy to its induced age distribution function. As a result, the age distribution-based formulation results in a linear program, which can be solved efficiently. We include numerical results and simulations, and discuss the role of the age distribution and other related qualities of the information.**

I. Introduction

The field of age of information (AoI) has been a breakthrough in studying timeliness in communication networks [1], [2]. Specifically, it has formalized the concept of the "freshness" of information coming from a source as observed at a remote location over time. While most studies focus on analyzing [3]–[5] and optimizing [6], [7] the age of information itself, the age can play a significant role even when it is not the ultimate objective. In this work, we study a two-user cognitive radio network, in which the primary user transmit/idle state is modeled as a Markov chain, and the secondary decides whether to sense or transmit based on information that is aging.

There have been some works that extend beyond age of information as an objective. A minor extension is the idea of an age penalty function [8], which characterizes the level of dissatisfaction with aged information. AoI has also been extended to study real-time remote estimation [9], [10], where the objective is not simply to have the updates be fresh, but to minimize the error in the real-time estimate of the source status. Some works consider competing objectives [11], [12] or mixtures of objectives [13]. In the context of caching [14], the age was combined with popularity to model request rates for cached content. In [15], the authors consider the impact of age on the performance of supervised learning-based forecasting, and they show that the training loss is a function of the age and that adding the age as a feature is beneficial. A closely related work to ours is [16], which also considers a binary Markov model of the channel state (but not in a cognitive radio setting), and they analyze the utility as a function of the age of the channel information.

In this work, we study the impact of the age of sensed information on throughput in a two-user cognitive radio network. A primary user (PU) has priority to utilize the spectrum, and a secondary user (SU) senses the channel to access the spectrum opportunistically while limiting its interference with the primary user [17]. The PU's transmit/idle dynamics is modeled as a binary Markov chain, and the SU is not capable of sensing and transmitting simultaneously, such that its sensed information about the PU is aging. In this work, the goal is for the SU to decide whether to sense or transmit in each slot to maximize its throughput without exceeding a collision rate experienced by the primary.

There have been a number of works on cognitive radio networks with the PU modeled as a Markov chain. Some study hidden Markov models and try to predict the PU dynamics [18], [19], while others consider channel selection using a constrained Markov Decision Process [20] or determining a sensing duration [21]. There have also been a few works on age and spectrum sensing, but with the primary objective of minimizing AoI [22], [23].

We first analyze the binary Markov model to determine the throughput and collision probability as a function of the SU's stationary randomized policy, which results in a complex expression. By recognizing that the distribution of the age of each type of sensed information determines the throughput and collision probabilities, we reformulate the problem using the age distribution, which allows us to express the problem as a linear program, which can be solved efficiently. Our numerical results show the achievable throughput for different collision thresholds, and in general, the optimal policy is shown to be random between two consecutive values of transmission slots and zero elsewhere.

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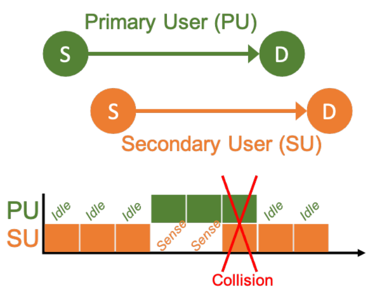

Fig. 1. Two user cognitive radio network.

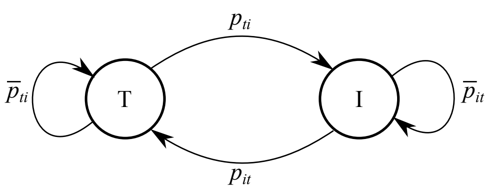

Fig. 2. Primary user transmission model.

II. System Model

We study a single channel cognitive radio network with a primary user (PU) transmitter/receiver pair and a secondary user (SU) pair (Fig. 1). The system is time-slotted, and the timescale of the system is normalized to the sensing duration, such that PU transmissions, SU transmissions, and SU sensing durations are measured in time slots of equal length. Between sensing durations, the age of the SU sensed information is increasing, which reduces the accuracy of the estimate of the PU state. The objective is to maximize the SU throughput while limiting collisions to some maximum percentage of PU transmissions.

A. Primary User Model

The primary user (PU) transmission dynamics follow a binary Markov model. Define the state of the PU in slot t as $S(t) \in \{T, I\}$. The PU transmits or remains idle for a time slot length, where at each slot boundary, the PU may switch states according to the Markov chain shown in Fig. 2. The probability of transitioning from a transmit (T) state to an idle (I) state is p_{ti}, and the probability of staying in a transmit state is $\overline{p}_{ti} = 1 - p_{ti}$. The probability of transitioning from I to T is p_{it}, and the probability of staying in an idle state is $\overline{p}_{it}$.

In this work, we do not consider the case where $p_{it} + p_{ti} > 1$, where the state is more likely to alternate every slot. In such a case, the policy would need to account for this alternating. In practice, the PU dynamics are much slower than the sensing duration, and since in this model the sensing and transmission slots are taken to be the same length, this means the values for p_{it} and p_{ti} will typically be small.

B. Secondary User Model

In each slot, the secondary user (SU) can either sense or transmit. In this model, the only cost of transmitting is the potential collision with the PU, and the only cost of sensing is that it uses up an opportunity to transmit. When the SU senses, it senses for the whole slot, and it detects the PU state without error (non-ideal sensing will be considered in future work). When the SU transmits, it transmits a packet containing one unit of data. If the PU is transmitting simultaneously, there is a collision and no data gets through for either user. Otherwise, the unit of data is received without error. We assume no feedback, and the only new information the secondary has is the sensed PU state.

C. Secondary User Sensing/Transmission Policy

We consider a stationary randomized policy $(p_{S_I}(s_I), p_{S_T}(s_T))$, where if the SU senses the PU is idle, the SU randomly transmits for $S_I - 1$ slots according to the probability distribution $p_{S_I}(s_I)$, and if the SU senses the PU is transmitting, the SU transmits for $S_T - 1$ slots according to the probability distribution $p_{S_T}(s_T)$. The transmission period is followed by a sensing slot. Since the policy is stationary, the SU does not act on any real-time information about the interference to the PU, and it must satisfy a collision constraint probabilistically.

We now argue that the stationary randomized policy is equivalent to an age-dependent policy. Consider an alternative class of policies $\mathcal{P}_2$ in which the probability of transmitting in a slot depends on what information was last sensed as well as the age of the sensed information, or the time elapsed since the last sensing slot. If the SU last sensed the PU was idle, the policy is to transmit with probability $\tilde{p}_I(\Delta)$, where Δ is the age since the last sensed slot, otherwise the SU senses (which occurs with probability $1 - \tilde{p}_I(\Delta)$). If the SU senses the PU is transmitting, the policy is to transmit with probability $\tilde{p}_T(\Delta)$ (or sense with probability $1 - \tilde{p}_T(\Delta)$.

Because there is no new information during transmission slots (only during sensing slots), there is an equivalent policy we call $\mathcal{P}_1$, in which the SU first senses and if the PU is idle, the SU transmits for a random $S_I - 1$ slots, and if the PU is transmitting, the SU transmits for a random $S_T - 1$ slots. In $\mathcal{P}_2$, the probability of transmitting for $S_I - 1$ slots after sensing the PU is idle is given by $(1 - \tilde{p}_I(S_I)) \prod_{\Delta=1}^{S_I-1} \tilde{p}_I(\Delta)$. The probability of transmitting for $S_T - 1$ slots after sensing the PU is transmitting is $(1 - \tilde{p}_I(S_I)) \prod_{\Delta=1}^{S_I-1} \tilde{p}_I(\Delta)$. Then in $\mathcal{P}_1$, we can define the probability distributions $p_{S_I}(s_I)$ and $p_{S_T}(s_T)$, such that the two policies are statistically equivalent. Therefore we can simply study the stationary randomized policy, and the age-dependent policies are also accounted for.

III. Throughput and Collision Probability

The problem studied in this work is to determine an SU sensing/transmission policy $p_{S_I}(s_I), p_{S_T}(s_T)$ that maximizes the SU throughput subject to a constraint on the probability of collision experienced by the PU:

$$\begin{aligned} \max_{p_{S_I}(s_I), p_{S_T}(s_T)} \quad & \mu_{SU}(p_{S_I}(s_I), p_{S_T}(s_T)) \\ \text{s.t.} \quad & C(p_{S_I}(s_I), p_{S_T}(s_T)) \leq \eta \end{aligned} \tag{1}$$

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We derive the expressions for $\mu_{SU}(p_{S_I}(s_I), p_{S_T}(s_T))$ and $C(p_{S_I}(s_I), p_{S_T}(s_T))$, by conditioning on whether the primary user is in an idle or transmit state when the SU senses.

A. Primary User Transmission State Probabilities

First, we compute the probability of the PU being in an idle or transmit state in a slot following a sensor slot. Since the PU follows a Markov chain and the SU senses its state without error, we let time be relative to the sensing slot, which we call slot 0. Define $P(T,\Delta) \triangleq P(S(\Delta) = T|\text{SU sensed in slot 0})$ and $P(I,\Delta) \triangleq P(S(\Delta) = I|\text{SU sensed in slot 0})$. Again, we assume error-free sensing, so the probability that the PU is idle or transmitting in the sensing slot is either 0 or 1. That is, $P(I,0) = \mathbb{1}(S(0) = I)$ and $P(T,0) = \mathbb{1}(S(0) = T)$, where $\mathbb{1}(expr)$ is the indicator function, which equals 1 if $expr$ is true and 0 otherwise. As the sensed information ages, the SU can estimate the probability that the PU is transmitting or idle based on its Markov transmission model. We compute the probability of transmitting or idle as a function of the age of sensed information Δ as follows:

$$\begin{aligned}\begin{bmatrix}P(T,\Delta)\\P(I,\Delta)\end{bmatrix} &= \begin{bmatrix}1-p_{ti} & p_{it}\\ p_{ti} & 1-p_{it}\end{bmatrix}^{\Delta}\begin{bmatrix}P(T,0)\\P(I,0)\end{bmatrix}\\ &= \begin{bmatrix}p_{it} & -1\\ p_{ti} & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & 1-p_{it}-p_{ti}\end{bmatrix}^{\Delta}\\ &\quad\times\begin{bmatrix}\frac{1}{p_{it}+p_{ti}} & \frac{1}{p_{it}+p_{ti}}\\ -\frac{p_{ti}}{p_{it}+p_{ti}} & \frac{p_{it}}{p_{it}+p_{ti}}\end{bmatrix}\begin{bmatrix}\mathbb{1}(S(0)=T)\\ \mathbb{1}(S(0)=I)\end{bmatrix}\\ &= \begin{bmatrix}q_1(\Delta) & q_2(\Delta)\\ r_1(\Delta) & r_2(\Delta)\end{bmatrix}\begin{bmatrix}\mathbb{1}(S(0)=T)\\ \mathbb{1}(S(0)=I)\end{bmatrix}\end{aligned}$$

where

$$q_1(\Delta) \triangleq \frac{p_{it} + p_{ti}(1-p_{it}-p_{ti})^{\Delta}}{p_{it}+p_{ti}} \quad (2)$$

$$q_2(\Delta) \triangleq \frac{p_{it} - p_{it}(1-p_{it}-p_{ti})^{\Delta}}{p_{it}+p_{ti}} \quad (3)$$

$$r_1(\Delta) \triangleq \frac{p_{ti} - p_{ti}(1-p_{it}-p_{ti})^{\Delta}}{p_{it}+p_{ti}} \quad (4)$$

$$r_2(\Delta) \triangleq \frac{p_{ti} + p_{it}(1-p_{it}-p_{ti})^{\Delta}}{p_{it}+p_{ti}}. \quad (5)$$

B. Throughput for State-Dependent Sensing

For a given policy $(p_{S_I}(s_I), p_{S_T}(s_T))$, we model the state evolution of the SU's sensed PU state as a binary semi-Markov process, where the "$\hat{I}$" state represents sensing that the PU is idle, which is followed by a transmission phase of length $S_I - 1$, and the "$\hat{T}$" state represents sensing that the PU is transmitting, followed by a transmission phase of length $S_T - 1$. We denote this state as $S(0)$, since the state is determined by the most recent sensing slot. Using the PU state transition probabilities (2)-(5), we first derive the transition probabilities for $S(0)$, which occur at each sensing slot, which is determined by the policy $(p_{S_I}(s_I), p_{S_T}(s_T))$. These are given as follows:

$$p_{\hat{T},\hat{I}} = \sum_{s_T=1}^{\infty} p_{S_T}(s_T) r_1(s_T) = E_{S_T}[r_1(S_T)]$$

$$p_{\hat{I},\hat{T}} = \sum_{s_I=1}^{\infty} p_{S_I}(s_I) q_2(s_I) = E_{S_I}[q_2(S_I)].$$

Then the stationary probabilities of the Markov chain of successive $S(0)$ states can be found by the global balance equation, and are given by

$$\pi_{\hat{I}} = \frac{E_{S_T}[r_1(S_T)]}{E_{S_T}[r_1(S_T)] + E_{S_I}[q_2(S_I)]}$$

$$\pi_{\hat{T}} = \frac{E_{S_I}[q_2(S_I)]}{E_{S_T}[r_1(S_T)] + E_{S_I}[q_2(S_I)]}.$$

The probability distribution of the most recently sensed state $S(0)$ is given by

$$\begin{aligned}P(S(0)=I) &= \frac{\pi_{\hat{I}} E[S_I]}{\pi_{\hat{I}} E[S_I] + \pi_{\hat{T}} E[S_T]}\\ &= \frac{E_{S_T}[r_1(S_T)]E[S_I]}{E_{S_T}[r_1(S_T)]E[S_I] + E_{S_I}[q_2(S_I)]E[S_T]}\\ P(S(0)=T) &= \frac{\pi_{\hat{T}} E[S_T]}{\pi_{\hat{I}} E[S_I] + \pi_{\hat{T}} E[S_T]}\\ &= \frac{E_{S_I}[q_2(S_I)]E[S_T]}{E_{S_T}[r_1(S_T)]E[S_I] + E_{S_I}[q_2(S_I)]E[S_T]}\end{aligned}$$

where $E[S_I] = \sum_{s_I=1}^{\infty} s_I p_{S_I}(s_I)$. Under the policy $(p_{S_I}(s_I), p_{S_T}(s_T))$, we derive the SU throughput as follows:

$$\begin{aligned}&\mu_{SU}(p_{S_I}(s_I), p_{S_T}(s_T))\\ &= P(S(0)=I)P(\text{SU tx}, \overline{\text{PU tx}}|S(0)=I)\\ &\quad + P(S(0)=T)P(\text{SU tx}, \overline{\text{PU tx}}|S(0)=T)\\ &= P(S(0)=I)\sum_{\Delta=1}^{\infty} P(\Delta|S(0)=I)\\ &\quad \times P(\text{SU tx}, \overline{\text{PU tx}}|S(0)=I,\Delta) + P(S(0)=T)\\ &\quad \times \sum_{\Delta=1}^{\infty} P(\Delta|S(0)=T)P(\text{SU tx}, \overline{\text{PU tx}}|S(0)=T,\Delta)\\ &= P(S(0)=I)\sum_{\Delta=1}^{\infty}\frac{\overline{F}_{S_I}(\Delta+1)}{E[S_I]} r_2(\Delta)\\ &\quad + P(S(0)=T)\sum_{\Delta=1}^{\infty}\frac{\overline{F}_{S_T}(\Delta+1)}{E[S_T]} r_1(\Delta) \quad (6)\end{aligned}$$

where $(\text{SU tx}, \overline{\text{PU tx}}|S(0)=I)$ is the event where the SU transmits and the PU does not transmit in a random slot given that it follows a sensed "idle," and $\overline{F}_{S_I}(s)$ and $\overline{F}_{S_T}(s)$ are the complementary cumulative distribution function for S_I and S_T, respectively.

We can consider the special case of geometrically distributed transmission phases with parameters p_I and p_T as the probability of transmitting in each slot following an idle or transmit detection, respectively. More specifically, the probability distributions of S_I and S_T are $p_{S_I}(s) = p_I^{s-1}(1-p_I)$

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and $p_{S_T}(s) = p_T^{s-1}(1-p_T)$. The transition probabilities are given by

$$\begin{aligned}
&E_{S_T}^{geom}[r_1(S_T)] \\
&\quad = \sum_{s_T=1}^{\infty} p_T^{s_T-1}(1-p_T)\frac{p_{ti} - p_{ti}(1-p_{it}-p_{ti})^{s_T}}{p_{it}+p_{ti}} \\
&\quad = \frac{p_{ti}}{p_{it}+p_{ti}}\left(1 - \frac{(1-p_T)(1-p_{it}-p_{ti})}{1-p_T(1-p_{it}-p_{ti})}\right) \\
&\quad = \frac{p_{ti}}{1-p_T(1-p_{it}-p_{ti})}
\end{aligned}$$

and similarly, $E_{S_I}^{geom}[q_2(S_I)] = \frac{p_{it}}{1-p_I(1-p_{it}-p_{ti})}$. The probabilities of sensing idle and transmit are thus given by

$$\begin{aligned}
P^{geom}(S(0)=I) &= \frac{1}{1+\frac{p_{it}(1-p_I)(1-p_T(1-p_{it}-p_{ti}))}{p_{ti}(1-p_T)(1-p_I(1-p_{it}-p_{ti}))}} \\
P^{geom}(S(0)=T) &= \frac{1}{1+\frac{p_{ti}(1-p_T)(1-p_I(1-p_{it}-p_{ti}))}{p_{it}(1-p_I)(1-p_T(1-p_{it}-p_{ti}))}}
\end{aligned}$$

and the conditional throughputs are given by

$$\begin{aligned}
&\sum_{\Delta=1}^{\infty}\frac{\overline{F}_{S_I}(\Delta+1)}{E[S_I]}r_2(\Delta) = p_I\left(1-\frac{p_{ti}}{1-p_I(1-p_{it}-p_{ti})}\right) \\
&\sum_{\Delta=1}^{\infty}\frac{\overline{F}_{S_T}(\Delta+1)}{E[S_T]}r_1(\Delta) = \frac{p_T p_{ti}}{1-p_T(1-p_{it}-p_{ti})}.
\end{aligned}$$

From (6), the throughput is then given by

$$\begin{aligned}
&\mu_{SU}^{geom}(p_I,p_T) \\
&\quad = \Big[p_{ti}(p_I(1-p_T)(1-p_I(1-p_{it}-p_{ti})-p_{it}) \\
&\qquad + p_{it}p_T(1-p_I))\Big]\Big[p_{ti}(1-p_T)(1-p_I(1-p_{it}-p_{ti})) \\
&\qquad + p_{it}(1-p_T)(1-p_T(1-p_{it}-p_{ti})\Big]^{-1}.
\end{aligned} \tag{7}$$

C. Probability of Collision when Primary User Transmits

Here we compute the probability of a collision when the primary user transmits. In the following derivation, we first define the probability of collision in a random slot as the probability that the SU is transmitting given that the PU is transmitting, and then we condition on whether the SU last sensed idle or transmit:

$$\begin{aligned}
&C(p_{S_I}(s_I), p_{S_T}(s_T)) \\
&\quad = P(\text{SU tx}|\text{PU tx}) \\
&\quad = \frac{P(\text{SU tx,PU tx})}{P(\text{PU tx})} \\
&\quad = \frac{1}{P(\text{PU tx})}\Big[P(S(0)=I)P(\text{SU tx,PU tx}|S(0)=I) \\
&\qquad + P(S(0)=T)P(\text{SU tx,PU tx}|S(0)=T)\Big] \\
&\quad = \frac{1}{P(\text{PU tx})}\Big[P(S(0)=I)\sum_{\Delta=0}^{\infty}P(\Delta|S(0)=I) \\
&\qquad \times P(\text{SU tx,PU tx}|S(0)=I,\Delta) + P(S(0)=T) \\
&\qquad \times \sum_{\Delta=0}^{\infty}P(\Delta|S(0)=I)P(\text{SU tx,PU tx}|S(0)=T,\Delta)\Big] \\
&\quad = \frac{1}{P(\text{PU tx})}\Big[P(S(0)=I)\sum_{\Delta=1}^{\infty}\frac{\overline{F}_{S_I}(\Delta+1)}{E[S_I]}q_2(\Delta) \\
&\qquad + P(S(0)=T)\sum_{\Delta=1}^{\infty}\frac{\overline{F}_{S_T}(\Delta+1)}{E[S_T]}q_1(\Delta)\Big].
\end{aligned}$$

We can use this expression to derive the collision probability for the geometric case as follows:

$$\begin{aligned}
&C_{geom}(p_{S_I}(s_I), p_{S_T}(s_T)) \\
&\quad = P(S(0)=I)\sum_{\Delta=1}^{\infty}p_I^{\Delta}(1-p_I)(1-(1-p_{it}-p_{ti})^{\Delta}) \\
&\qquad + P(S(0)=T)\sum_{\Delta=1}^{\infty}p_T^{\Delta}(1-p_T) \\
&\qquad \times \left(1-\frac{p_{ti}}{p_{it}}(1-p_{it}-p_{ti})^{\Delta}\right) \\
&\quad = \Big[(p_{it}+p_{ti})(p_{ti}p_I(1-p_T)+p_T(1-p_I) \\
&\qquad \times (1-p_T(1-p_{it}-p_{ti})-p_{ti})\Big]\Big[p_{ti}(1-p_T) \\
&\qquad \times (1-p_I(1-p_{it}-p_{ti}))+p_{it}(1-p_T) \\
&\qquad \times (1-p_T(1-p_{it}-p_{ti})\Big]^{-1}.
\end{aligned} \tag{8}$$

The above expressions for throughput and collision probability as functions of the randomized policy are rather complex, and the constrained optimization is difficult to solve in this form. Therefore, we seek an alternative formulation.

IV. Age Distribution Formulation

While prior AoI works focus on average age in general, the average age is insufficient for determining the throughput/collision in this problem, in part because they depend on whether the SU sensed the PU as idle or transmitting. Therefore, we first separate out the age for each type of information, where the age of sensing idle is given by $\Delta_I(t_I)$, and the age of sensing transmit is given by $\Delta_T(t_T)$, where t_I

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is the index for the t_Ith time slot in which the SU last sensed idle, and likewise t_T for transmit slots.

Under the current model, the age in a given slot ($\Delta(t)$) and the sensed information ($\hat{I}$ or $\hat{T}$) determines the probability of the PU transmitting (or idling), from the perspective of the SU. Therefore, for a given sensing/transmission policy π, the throughput and collision probability after $\mathcal{T}$ slots can be derived as a function of the age and sensed information as follows:

$$\mu_\pi(\mathcal{T}) = \frac{1}{N_{\hat{I},\pi}(\mathcal{T}) + N_{\hat{T},\pi}(\mathcal{T})}\left(\sum_{t_{\hat{I}}=1}^{N_{\hat{I},\pi}(\mathcal{T})} P(I(t_{\hat{I}}), \Delta_\pi(t_{\hat{I}})|\hat{I}) + \sum_{t_{\hat{T}}=1}^{N_{\hat{T},\pi}(\mathcal{T})} P(I(t_{\hat{T}}), \Delta(t_{\hat{T}})|\hat{T})\right)$$

$$C_\pi(\mathcal{T}) = \frac{1}{P(\text{PU tx})(N_{\hat{I},\pi}(\mathcal{T}) + N_{\hat{T},\pi}(\mathcal{T}))} \times\left(\sum_{t_{\hat{I}}=1}^{N_{\hat{I},\pi}(\mathcal{T})} P(T(t_{\hat{I}}), \Delta_\pi(t_{\hat{I}})|\hat{I}) + \sum_{t_{\hat{T}}=1}^{N_{\hat{T},\pi}(\mathcal{T})} P(T(t_{\hat{T}}), \Delta(t_{\hat{T}})|\hat{T})\right)$$

where $N_{\hat{I},\pi}(\mathcal{T})$ and $N_{\hat{T},\pi}(\mathcal{T})$ are the number of slots where the SU last estimated the PU as idle or transmit, respectively, under the policy π after $\mathcal{T}$ slots. We then take the limit of the above expressions as $\mathcal{T} \to \infty$, assuming the ages under the policy π are ergodic. Since the PU state probabilities can be viewed as functions of age and the sensed information, the expected throughput and collision probabilities are given as follows:

$$E[\mu_\pi] = \sum_{\delta_{\hat{I}}=1}^{\infty} P(I|\hat{I}, \delta_{\hat{I}}) P(\Delta_{\pi,\hat{I}} = \delta_{\hat{I}}) + \sum_{\delta_{\hat{T}}=1}^{\infty} P(I|\hat{T}, \delta_{\hat{T}}) P(\Delta_{\pi,\hat{T}} = \delta_{\hat{T}})$$

$$E[C_\pi] = \frac{1}{P(\text{PU tx})} \sum_{\delta_{\hat{I}}=1}^{\infty} P(T|\hat{I}, \delta_{\hat{I}}) P(\Delta_{\pi,\hat{I}} = \delta_{\hat{I}}) + \sum_{\delta_{\hat{T}}=1}^{\infty} P(T|\hat{T}, \delta_{\hat{T}}) P(\Delta_{\pi,\hat{T}} = \delta_{\hat{T}}).$$

The above expression comes from the fact that an SU sensing corresponds to the age being equal to 0 ($\delta = 0$), and the SU is transmitting for all other values of the age ($\delta \geq 1$). Given these expressions, we observe that what matters is not the average age, but rather the distribution of age (Δ) for each type of information or state (S), We denote this age distribution as $f_{\Delta,S}(\delta, s)$. The expected throughput and collision probabilities as a function of the age distribution can be stated as

$$E[\mu(f_{\Delta,S}(\delta, s))] = \sum_{s=0}^{1}\sum_{\delta=1}^{\infty} P(I|\delta, s) f_{\Delta,S}(\delta, s)$$

$$E[C(f_{\Delta,S}(\delta, s))] = \frac{1}{P(\text{PU tx})}\sum_{s=0}^{1}\sum_{\delta=1}^{\infty} P(T|\delta, s) f_{\Delta,S}(\delta, s) = \frac{1}{P(\text{PU tx})}\sum_{s=0}^{1}\sum_{\delta=1}^{\infty} (1 - P(I|\delta, s)) f_{\Delta,S}(\delta, s)$$

where we let $s = 0$ be the state where SU last sensed PU is idle, and $s = 1$ be the state where the SU last sensed the PU is transmitting.

We directly derive the distribution of the age under the policy $(p_{S_I}(s_I), p_{S_T}(s_T))$ as the probability of being in state s and age equal to δ. We truncate the age distribution to a maximum age of δ_{max}. For $s = 0$, we have

$$f_{\Delta,S}(\delta, 0) = P(S(0) = I)\frac{P(s_I \geq \delta + 1)}{E[S_I]}$$

from which we can derive

$$f_{\Delta,S}(0, 0) = \frac{E_{S_T}[r_1(s_T)]}{E_{S_T}[r_1(s_T)]E[S_I] + E_{S_I}[q_2(s_I)]E[S_T]} \tag{9}$$

$$f_{\Delta,S}(\delta, 0) = f_{\Delta,S}(0, 0)\sum_{s_I=\delta+1}^{\delta_{max}} p_{S_I}(s_I), 1 \leq \delta \leq \delta_{max} - 1, \tag{10}$$

and likewise for $s = 1$,

$$f_{\Delta,S}(0, 1) = \frac{E_{S_I}[q_2(s_I)]}{E_{S_T}[r_1(s_T)]E[S_I] + E_{S_I}[q_2(s_I)]E[S_T]}$$

$$f_{\Delta,S}(\delta, 1) = f_{\Delta,S}(0, 1)\sum_{s_T=\delta+1}^{\delta_{max}} p_{S_T}(s_T), 1 \leq \delta \leq \delta_{max} - 1.$$

The inverse mapping from age distribution to sense/transmit policy can be derived from (10). For s_I and s_T $= 1, \ldots, \delta_{max} - 1$,

$$p_{S_I}(s_I) = \frac{f_{\Delta,S}(s_I - 1, 0) - f_{\Delta,S}(s_I, 0)}{f_{\Delta,S}(0, 0)} \tag{11}$$

$$p_{S_T}(s_T) = \frac{f_{\Delta,S}(s_T - 1, 0) - f_{\Delta,S}(s_T, 0)}{f_{\Delta,S}(0, 0)} \tag{12}$$

and for s_I and $s_T = \delta_{max}$,

$$p_{S_I}(\delta_{max}) = \frac{f_{\Delta,S}(\delta_{max} - 1, 0)}{f_{\Delta,S}(0, 0)} \tag{13}$$

$$p_{S_T}(\delta_{max}) = \frac{f_{\Delta,S}(\delta_{max} - 1, 0)}{f_{\Delta,S}(0, 0)} \tag{14}$$

In addition to being a probability mass function, the policy imposes an additional constraint on the possible age distributions. We derive this constraint by substituting (11) and (12)

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into (9). We first derive the following terms from (9) using $f_{\delta,s} \triangleq f_{\Delta,S}(\delta,s)$:

$$\begin{aligned} E[S_I] &= \sum_{s_I=1}^{\delta_{max}} s_I p_{S_I}(s_I) \\ &= \sum_{s_I=1}^{\delta_{max}-1} s_I \frac{f_{s_I-1,0} - f_{s_I,0}}{f_{0,0}} + \delta_{max} \frac{f_{\delta_{max}-1,0}}{f_{0,0}} \\ &= \frac{\sum_{\delta=0}^{\delta_{max}} f_{\delta,0}}{f_{0,0}}. \end{aligned}$$

$$\begin{aligned} E_{S_I}[q_2(S_I)] &= \sum_{s_I=1}^{\delta_{max}} \left(\frac{p_{ti}}{p_{ti}+p_{it}}(1-(1-p_{it}-p_{ti})^{s_I}) p_{S_I}(s_I)\right) \\ &= \frac{p_{ti}}{f_{0,0}} \left(\sum_{\delta=0}^{\delta_{max}-2} \overline{\phi}^{\delta} f_{\delta,0} + \frac{\overline{\phi}^{\delta_{max}-1}}{\phi} f_{\delta_{max}-1,0} \right. \\ &\quad \left. - \frac{\overline{\phi}^{\delta_{max}-1}}{\phi} f_{\delta_{max},0} \right) \end{aligned}$$

where $\phi \triangleq p_{it} + p_{ti}$. Similar expressions arise for $E[S_T]$ and $E_{S_I}[q_2(S_I)]$. Substituting into (9), we have

$$\begin{aligned} f_{0,0} E_{S_I}[q_2(S_I)] E[S_T] &= E_{S_T}[r_1(S_T)](1 - f_{0,0} E[S_I]) \\ f_{0,0} E_{S_I}[q_2(S_I)] &= E_{S_T}[r_1(S_T)] \end{aligned}$$

which after substituting the terms $E_{S_I}[q_2(S_I)]$ and $E_{S_T}[r_1(S_T)]$ gives us the constraint in the following formulation. Defining $\beta_{\delta,s} \triangleq P(I|\delta,s)$, we have the following optimization problem (**LP1**):

$$\max_{f_{\delta,s}} \quad \sum_{s=0}^{1} \sum_{\delta=1}^{\delta_{max}} \beta_{\delta,s} f_{\delta,s} \tag{15}$$

$$\text{s.t.} \quad \sum_{s=0}^{1} \sum_{\delta=1}^{\delta_{max}} \frac{1}{P(\text{PU tx})} (1-\beta_{\delta,s}) f_{\delta,s} \leq \eta \tag{16}$$

$$f_{\delta,s} \leq f_{\delta-1,s} \qquad s \in \{0,1\}, \delta \geq 1 \tag{17}$$

$$\begin{aligned} & p_{it} \left(\sum_{\delta=0}^{\delta_{max}-2} \overline{\phi}^{\delta} f_{\delta,0} + \frac{\overline{\phi}^{\delta_{max}-1}}{\phi} f_{\delta_{max}-1,0} \right. \\ & \left. - \frac{\overline{\phi}^{\delta_{max}}}{\phi} f_{\delta_{max},0} \right) - p_{ti} \left(\sum_{\delta=0}^{\delta_{max}-2} \overline{\phi}^{\delta} f_{\delta,1} \right. \\ & \left. + \frac{\overline{\phi}^{\delta_{max}-1}}{\phi} f_{\delta_{max}-1,1} - \frac{\overline{\phi}^{\delta_{max}}}{\phi} f_{\delta_{max},1} \right) = 0 \end{aligned} \tag{18}$$

$$\sum_{s=0}^{1} \sum_{\delta=0}^{\delta_{max}} f_{\delta,s} = 1 \tag{19}$$

$$0 \leq f_{\delta,s} \leq 1 \tag{20}$$

where (16) is the collision probability constraint and (17)-(20) are the constraints for the age distribution, where (18) comes from the constraint based on (9). This is a linear program in the variables $f_{\delta,s}$, which can be solved efficiently. From Sec. III-A, we have $\beta_{\delta,0} = r_2(\delta)$ and $\beta_{\delta,1} = r_1(\delta)$.

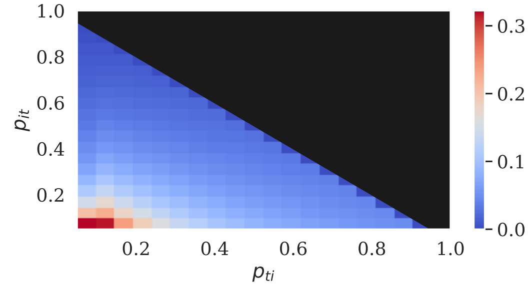

Fig. 3. **LP1** SU Throughput, $\eta = 0.05$.

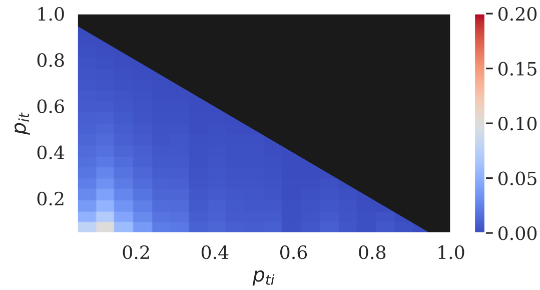

Fig. 4. SU Throughput Gap, $\eta = 0.05$.

V. Numerical Results

We solved the linear program **LP1** computationally for various parameter values for η, p_{it}, and p_{ti}, for $p_{it} + p_{ti} < 1$ (in all plots, the region $p_{it} + p_{ti} \geq 1$ is blacked out). The policy is truncated to $\delta_{max} = 40$. In Fig. 3, we plot the optimal SU throughput for collision constraint $\eta = 0.05$. We observe that the throughput is small except when p_{it} is small, which is the case where the PU is more likely to be idle. Throughput is also small when p_{ti} is small, because the PU is transmitting for very short bursts, so it is difficult to meet the collision constraint (as a percentage of PU transmissions) if transmitting too aggressively. For comparison, we used numerical optimization on the expressions in (7) and (8) to determine the maximum throughput for the random geometric policy. The gap between the optimal throughput for **LP1** and the geometric policy is shown in Fig. 4, and **LP1** outperforms the geometric policy by as much as 0.101 or 46.7%. We also plotted the throughput for $\eta = 0.25$ in Fig. 5, and the SU throughput experiences an increase as the collision constraint η is loosened (increases). The gap between the two policies is shown in Fig. 6, and **LP1** outperforms the geometric policy by as much as 0.098 or 26.4%.

From the solution for the optimal age distribution $f_{\delta,s}$, we use (11)–(14) to convert it to the optimal policy $(p_{S_I}(s_I), p_{S_T}(s_T))$. We plot the optimal policy for $\eta = 0.05$ and $p_{it} = 0.05$ in Figs. 7–8. In general, $p_{S_I}(s_I)$ is only nonzero for at most 2 consecutive values of s_I. I.e., when the SU senses the PU is idle, it transmits for some $\tilde{S}_I$ and $\tilde{S}_I+1$ with some probability where $p_{S_I}(\tilde{S}_I)+p_{S_I}(\tilde{S}_I+1) = 1$.

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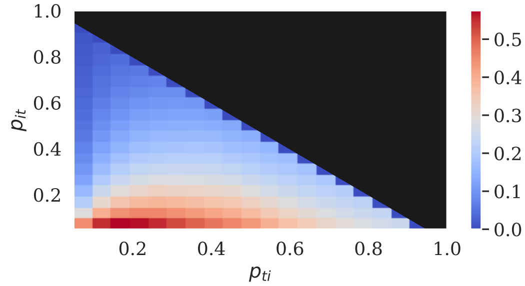

Fig. 5. **LP1** SU Throughput, $\eta = 0.25$.

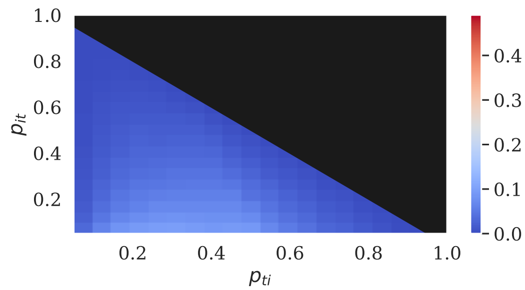

Fig. 6. SU Throughput Gap, $\eta = 0.25$.

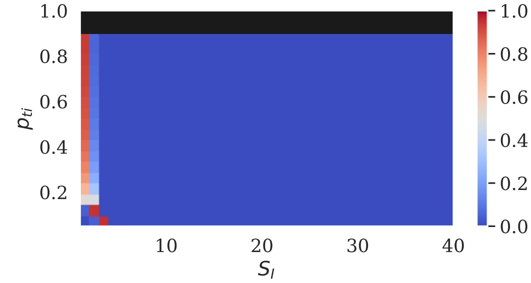

Fig. 7. **LP1** SU Idle Policy, $\eta = 0.05$, $p_{it} = 0.05$.

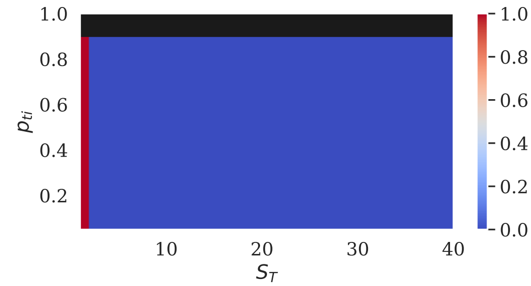

Fig. 8. **LP1** SU Transmit Policy, $\eta = 0.05$, $p_{it} = 0.05$.

This makes intuitive sense, since the probability of the primary being idle decreases with age, so for increasing throughput and reducing collisions, it is better to maximize the number of transmissions in lower age slots. For example, if the policy has nonzero probabilities of sensing after 1, 2, and 3 slots, it would be better to take the probability of sensing after 1 slot and put it into 2 and 3 (with some rebalancing), since it benefits both throughput and collisions. This two-value policy is akin to a threshold policy, but randomized between the two values that yield a collision probability closest to the collision threshold from above and below, thus utilizing the full collision probability budget. In all the cases studied here, the optimal transmit policy is to sense in the next slot ($p_{S_T}(1) = 1$, Fig. 8).

The idle policy for $p_{it} = 0.25$ and $\eta = 0.05$ is shown in Fig. 9. The policy is very similar to that of the $p_{it} = 0.05$, $\eta = 0.05$ case (Fig. 7), with the SU sensing slightly more frequently since the PU is more likely to transition from "idle" to "transmit." For $p_{it} = 0.05$ and $\eta = 0.25$, the idle policy (Fig. 10) senses less often than the $\eta = 0.05$ case since there is higher tolerance for collisions. For $p_{it} = 0.25$ and $\eta = 0.25$, the idle policy (Fig. 11) is similar to that of the previous case of $p_{it} = 0.05$, but just as in the $\eta = 0.05$ case, the higher p_{it} causes the SU to sense slightly more frequently. However, it is clear that the collision threshold η has a much greater impact on the policy than the PU parameters p_{it} and p_{ti}.

VI. AoI as an Information Quality Metric

While AoI is not the ultimate objective in this work, its role is significant, and focusing on it was critical for enabling an efficient solution. We identify AoI as an information quality of interest, i.e., some measure of the information being monitored or used. Another important information quality is the prediction *accuracy* at different ages, since we know that lower age yields better accuracy, though it also depends on the PU Markov model. If we were considering sensing errors, that would also impact the accuracy, and therefore affect how to optimize the throughput and the throughput performance itself. *Completeness*, or how much sensor data acquired, is another relevant quality of the information and correlated with AoI and accuracy. In this system, throughput is directly impacted by completeness in that more sensor data results in less resources (time slots) for transmission. In future work, we plan to investigate the achievable space for all relevant information qualities over all policies, as an intermediate step

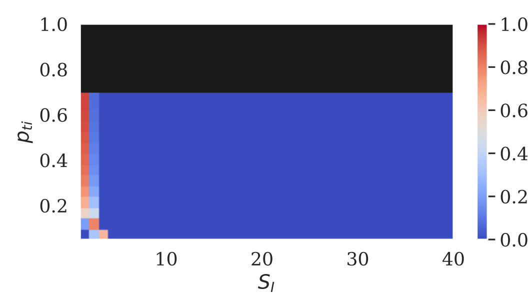

Fig. 9. **LP1** SU Idle Policy, $\eta = 0.05$, $p_{it} = 0.25$.

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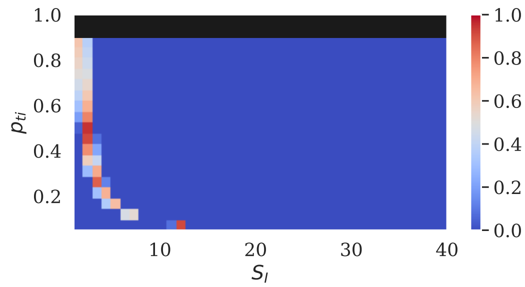

Fig. 10. **LP1** SU Idle Policy, $\eta = 0.25$, $p_{it} = 0.05$.

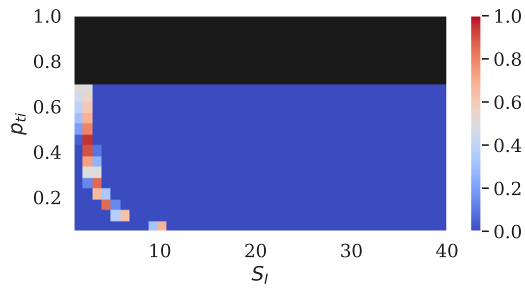

Fig. 11. **LP1** SU Idle Policy, $\eta = 0.25$, $p_{it} = 0.25$.

towards optimizing any number of higher order objectives.

VII. Summary and Future Work

In this work, we have studied a cognitive radio network in which a secondary user seeks to maximize its throughput subject to a collision probability constraint on the primary user. Our analysis yielded complex expressions for throughput and collision probability, which led us to reformulate the problem from the perspective of the distribution of the age of sensed information. This reformulation yielded a linear program, which can be solved efficiently, and the numerical results show its superior performance to the optimal random geometric policy. We also discuss AoI, specifically the age distribution, as an enabling tool, and how it and other qualities of the information can be explored to form a foundation for optimizing other higher order metrics, which we will continue to investigate in future work.

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