Fractional Programming for Robust TX BF Design in Multi-User/Single-Carrier PD-NOMA

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***Abstract*—We present a new Beamforming-based (BB) Multiple-Input Single-Output (MISO)-Non-orthogonal Multiple Access (NOMA) scheme for Power Domain NOMA (PD-NOMA), in which the total transmit power consumption is minimized subjected to prescribed signal-to-interference-plus-noise ratio (SINR) requirements for each user, and under the assumption that only imperfect channel state information (CSI) is available at the transmitter. To this end, the fractional programming (FP)-based quadratic transform is employed to reformulate the non-convex SINR constraint of the original problem into a tractable quadratic form, which contains an estimate of the CSI error vector as a parameter. Taking advantage of the fact that the zero duality gap holds for the non-convex quadratic problems, a closed-form expression for an estimate of the CSI error vector is derived, completing the formulation. Finally, a novel iterative algorithm based on both the herein derived CSI error vector and the semidefinite relaxation (SDR) technique is contributed, which is shown to capable of efficiently solving the constrained min-power problem. Simulation results are given which illustrate the effectiveness of the proposed algorithm, which is found to sacrifice only small quantities of transmit power in return for substantial increase in robustness against CSI imperfection.**

I. Introduction

Non-orthogonality is a key concept in fifth generation (5G) and beyond networks, which aims to improve the efficiency of resource utilization via the carefully designed overlapping of wireless signals. In particular, the advantages of code and power domain Non-orthogonal Multiple Access (NOMA) over classic Orthogonal Multiple Access (OMA) approaches such as Time Division Multiple Access (TDMA) [1], Code Division Multiple Access (CDMA) [2] and Orthogonal Frequency Division Multiple Access (OFDMA) [3], have been demonstrated in [4] and references thereby. The NOMA approach has since become the multiplexing method of consensus to ensure massive connectivity in future wireless networks, while resulting also in significant enhancement of spectrum efficiency [4]–[6], being incorporated in the ongoing standardization of 5G radio, Release 14 and beyond, which determined that non-orthogonal transmission should be considered at least in the uplink of 5G massive Machine Type Communications (mMTC) systems.

In turn, the combination of Multiple-Input Multiple-Output (MIMO) techniques and NOMA has attracted significant attention from both Academia and Industry, as it enables further improvement thanks to the expansion of Degree of Freedoms (DoFs) in spatial domain.

Two distinct strategies for MIMO-NOMA system designs can be found in the literature, namely, a) the Cluster-based (CB) MIMO-NOMA [7]–[9] and b) the Beamforming-based (BB) MIMO-NOMA [10]–[12] designs. In the CB MIMO-NOMA approach, users in a cell are partitioned into groups (clusters), with a conventional Successive Interference Cancellation (SIC)-based PD-NOMA scheme applied within each group, while transmit beamforming vectors are designed for each cluster. The advantage of this scheme is that since the additional spatial DoF is devoted to ensuring that the appropriately designed beam corresponding to a specific cluster holds orthogonality to users assigned to other clusters, the inter-cluster interference can be sufficiently mitigated. On the other hand, the drawback of the CB MIMO-NOMA approach is the combinatorial optimization problem which needs to be solved in order to form the clusters, which becomes prohibitive (NP-hard) for large number of users.

In contrast, in BB MIMO-NOMA, different transmit beamforming vectors are assigned to different users individually, which then perform SIC in an order defined by the relative channel gains associated with each user. In this approach, the computational cost at the base station (BS) is significantly reduced due to the fact that user grouping is not required, such that BB MIMO-NOMA is more scalable than its counterpart. However, due to the larger number of users whose interference need be mitigated, it is easy to foresee that BB MIMO-NOMA is more sensitive to the accuracy of instantaneous channel state information (CSI) knowledge than CB MIMO-NOMA.

Interestingly, despite the great amount of effort dedicated thus far to develop NOMA technologies, not enough attention has been payed to the aforementioned problem as evidence by the fact that perfect CSI knowledge at the transmitter is the standard assumption in related literature. While usually a reasonable simplification in wireless communications systems, the assumption of perfect CSI is particularly unrealistic in NOMA due to inherent overloading, which is known to impact the accuracy of channel estimation [13].

In this paper, we therefore turn our attention to the design of a multi-user/single-carrier BB MISO-NOMA system subjected to norm-bounded channel imperfection, proposing a novel fractional programming (FP)-based transmit beamforming design aiming at the minimization of the total power consumption.

The remainder of the article is as follows. Section II describes the system model and the problem formulation aiming at minimizing the total transmit power subjected to signal-to-interference-plus-noise ratio (SINR) requirements for each user. The Quality of Service (QoS) based min-min problem is reformulated and solved in Section III. The mathematical expression of the CSI error vector estimate is derived therein, and the proposed algorithm are also offered thereby. Simulation results illustrating the effectiveness of the proposed algorithm compared with the non-robust scheme – that is, subject to the same channels, but optimized under the assumption of perfect CSI – are shown in Section IV. Conclusions and discussions on possible future works are given in Section V.

A. Notation

Throughout the article, matrices and vectors are expressed respectively by bold capital and small letters, such as in $\boldsymbol{X}$ and $\boldsymbol{x}$. The conjugate, Hermitian (transpose conjugate) and inverse operators are be respectively denoted by $(\cdot)^*$, $(\cdot)^{\mathrm{H}}$ and $(\cdot)^{-1}$, while the ℓ_2-norm operators are be depicted by $\|\cdot\|_2$. A complex matrix with a columns and b rows is denoted by $\boldsymbol{X} \in \mathbb{C}^{a\times b}$, and a circularly symmetric complex random scalar variable following the complex Gaussian distribution with mean μ and variance σ^2 is expressed as $x \sim \mathcal{CN}\left(\mu, \sigma^2\right)$. For given $\mathbf{A}, \mathbf{B} \in \mathcal{S}^n$, $\mathbf{A} \succeq \mathbf{B}$ indicate that $\mathbf{A}-\mathbf{B}$ is positive semidefinite, and $\mathrm{Re}(\cdot)$ and $\mathrm{R}(\mathbf{A})$ express the real part of a complex number and the range space of the matrix $\mathbf{A}$, respectively.

II. System Model

Consider a power domain single-carrier NOMA system where one BS, equipped with N_t transmit antennas and capable of performing digital precoding towards all users, attempts to simultaneously transmit U data-stream to a pool of single-antenna downlink users $u \in \{1, 2, \ldots, U\}$.

Without loss of generality, let all channel vectors $\mathbf{h}_u$ between the BS and the u-th user be sorted in ascending order, such that $\|\mathbf{h}_1\|_2 \leq \|\mathbf{h}_2\|_2 \leq \ldots \leq \|\mathbf{h}_U\|_2$. It is assumed then that each u-th user in such a system can decode its own intended signal s_u after successively detecting and removing the first $u-1$ users's messages while regarding the higher order signals as interference. It follows from this assumption that, for a given reference user u, all ℓ_u users with $\ell_u \in \{u, u+1, \ldots, U\}$ ought to detect the u-th user's signal in order to successfully decode their own intended signals.

Letting $\mathbf{w}_u \in \mathbb{C}^{N_t\times 1}$ denote the transmit beamforming vector corresponding to the symbol intended for the u-th user, the received signal at the ℓ_u-th expressed so as to highlight the u-th user's signal can be written as

$$y_{u,\ell_u} = \mathbf{h}_{\ell_u}^{\mathrm{H}}\mathbf{w}_u s_u + \sum_{m=1}^{u-1}\mathbf{e}_{\ell_u}^{\mathrm{H}}\mathbf{w}_m s_m + \sum_{k=u+1}^{U}\mathbf{h}_{\ell_u}^{\mathrm{H}}\mathbf{w}_k s_k + n_{\ell_u}, \tag{1}$$

where $n_\ell \sim \mathcal{CN}(0, \sigma^2)$ is the circularly symmetric additive white Gaussian noise (AWGN) at user ℓ_u and

$$\mathbf{h}_{\ell_u} = \hat{\mathbf{h}}_{\ell_u} + \mathbf{e}_{\ell_u}, \forall \ell_u \tag{2}$$

with $\hat{\mathbf{h}}_{\ell_u} \in \mathbb{C}^{M\times 1}$ and $\mathbf{e}_{\ell_u} \in \mathbb{C}^{M\times 1}$ denoting the estimate of the physical channel $\mathbf{h}_{\ell_u}$ between the BS and the user ℓ_u and the corresponding channel estimation error bounded such that $\|\mathbf{e}_{\ell_u}\|_2 \leq \varepsilon$, respectively.

Let SINR_{u,ℓ_u} denote the SINR of the signal s_u at the user ℓ_u. Then, taking into account the fact that the achievable throughput of the u-th user is determined by the lowest SINR_{u,ℓ_u} among all ℓ_u users, the maximal data rate of the u-th user in order for all users to be able to successfully obtain their own intended signals is given by

$$R_u = \log_2\left(1 + \min_{\ell_u}\mathrm{SINR}_{u,\ell_u}\right), \tag{3}$$

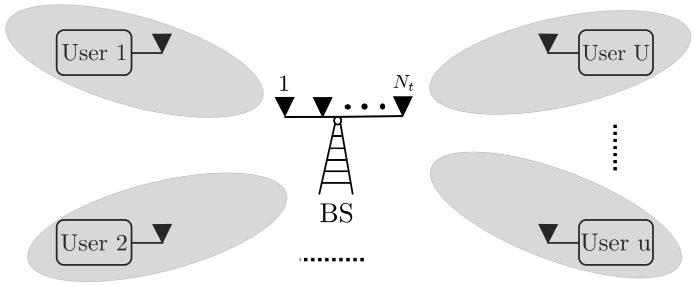


Fig. 1. System Model of BB MISO NOMA with N_t transmit antennas and U single-antenna users.

with

$$\mathrm{SINR}_{u,\ell_u} = \frac{\mathbf{h}_{\ell_u}^{\mathrm{H}}\mathbf{w}_u\mathbf{w}_u^{\mathrm{H}}\mathbf{h}_{\ell_u}}{\sum\limits_{m=1}^{u-1}\mathbf{e}_{\ell_u}^{\mathrm{H}}\mathbf{w}_m\mathbf{w}_m^{\mathrm{H}}\mathbf{e}_{\ell_u} + \sum\limits_{k=u+1}^{U}\mathbf{h}_{\ell_u}^{\mathrm{H}}\mathbf{w}_k\mathbf{w}_k^{\mathrm{H}}\mathbf{h}_{\ell_u} + \sigma^2}. \tag{4}$$

Although several beamforming (BF) techniques aiming at maximizing the downlink sum data rate in PD-NOMA systems have been proposed in the past few years [10], we argue that the sum rate criterion in an overkill, considering that it is sufficient to maintain a certain SINR in order for a user to enjoy its intended application. We therefore consider instead the optimization of transmit beamformers aimed at minimizing total power consumption minimization, while constrained to satisfying worst-case individual target throughput requirements, *i.e.*

$$\min_{\mathbf{w}_u,\forall u} \quad \sum_{u=1}^{U}\|\mathbf{w}_u\|_2^2 \tag{5a}$$

$$\text{s.t.} \quad \min_{\|\mathbf{e}_{\ell_u}\|_2\leq\varepsilon} R_u \geq \log_2(1+\Gamma_u)\ \forall u, \tag{5b}$$

where Γ_u denotes the target SINR for the u-th user, and $\mathbf{e}_{\ell_u}$ is the worst-case CSI error vector corresponding to user ℓ_u.

From the fact that the logarithmic function is a non-decreasing function, the above optimization problem can be readily rewritten as

$$\min_{\mathbf{w}_u,\forall u} \quad \sum_{u=1}^{U}\|\mathbf{w}_u\|_2^2 \tag{6a}$$

$$\text{s.t.} \quad \min_{\|\mathbf{e}_{\ell_u}\|_2\leq\varepsilon}\ \min_{\ell_u}\ \mathrm{SINR}_{u,\ell_u} \geq \Gamma_u\ \forall u. \tag{6b}$$

It can be recognized from the constraint in inequality (6b), that the problem formulated in equation (6) is not convex and therefore intractable in its original form. In the next section, we therefore tackle the above problem by 1) transforming the $\min\min$ operation in (6b); and 2) recasting the non-convex ratio constraints into a tractable quadratic convex form.

III. Proposed robust beamforming design

A. Transformation of SINR Constraints

Following related literature [14], each constraint on Γ_u constructed with basis on the $\min$ operator over all ℓ_us in equation (6b) can be replaced by a set of $(U-u+1)$ simultaneous constraints.

That is, $\forall u$,

$$\min_{\|\mathbf{e}_{\ell_u}\|_2\leq\varepsilon}\min_{\ell_u}\mathrm{SINR}_{u,\ell_u}\geq\Gamma_u\equiv\begin{cases}\min\limits_{\|\mathbf{e}_{\ell_u}\|_2\leq\varepsilon}\mathrm{SINR}_{u,u}\geq\Gamma_u,\\ \min\limits_{\|\mathbf{e}_{\ell_u}\|_2\leq\varepsilon}\mathrm{SINR}_{u,u+1}\geq\Gamma_u,\\ \quad\vdots\\ \min\limits_{\|\mathbf{e}_{\ell_u}\|_2\leq\varepsilon}\mathrm{SINR}_{u,U}\geq\Gamma_u.\end{cases}\tag{7}$$

Thanks to the recasted constraints above, the original optimization problem given in equation (6) can be rewritten as

$$\min_{\mathbf{w}_u,\forall u}\quad\sum_{u=1}^{U}\|\mathbf{w}_u\|_2^2\tag{8a}$$

$$\text{s.t.}\quad\min_{\|\mathbf{e}_{\ell_u}\|_2\leq\varepsilon}\mathrm{SINR}_{u,\ell_u}\geq\Gamma_u\;\;\forall u,\ell_u.\tag{8b}$$

where, for clarity, we highlight that the constraints in (8b) are for all u and ℓ_u.

B. Quadratic Transform of Ratio Constraints

Unfortunately, the problem formulated in equation (8) is still not convex, due to the SINRs defined in equation (4). While several methods such as the the Taylor series approximation [15] and the semidefinite relaxation (SDR) [14] have been proposed for the transformation of non-convex ratios constraints in the past decade, a novel quadratic transformation technique for such non-convex ratio problems has been recently proposed in [16], which has been shown not to result in any approximation gap at the optimal point.

Indeed, consider a generic maximization problem with a sum of ratios as objective, such as

$$\max_{\boldsymbol{x}}\quad\sum_{m=1}^{M}\boldsymbol{a}_m^{\mathrm{H}}(\boldsymbol{x})\,\mathbf{B}_m^{-1}(\boldsymbol{x})\,\boldsymbol{a}_m(\boldsymbol{x})\tag{9a}$$

$$\text{s.t.}\quad\boldsymbol{x}\in\mathcal{X},\tag{9b}$$

where $\boldsymbol{a}_m(\boldsymbol{x})$ denotes an arbitrary complex function, $\mathbf{B}_m(\boldsymbol{x})$ is an arbitrary symmetric positive definite matrix, and $\boldsymbol{x}$ is a variable to be optimized in a constraint set $\mathcal{X}$.

Then, the equivalent problem after applying the *quadratic transformation* [16] can be written as

$$\max_{\boldsymbol{x}}\quad\sum_{m=1}^{M}2\mathrm{Re}\left\{\mathbf{t}_m^{\mathrm{H}}\boldsymbol{a}_m(\boldsymbol{x})\right\}-\mathbf{t}_m^{\mathrm{H}}\mathbf{B}_m(\boldsymbol{x})\,\mathbf{t}_m\tag{10a}$$

$$\text{s.t.}\quad\boldsymbol{x}\in\mathcal{X},\;\;\mathbf{t}_m\in\mathbb{C},\tag{10b}$$

where $\mathbf{t}_m$ is a scaling quantity designed so as to ensure that, for a given $\boldsymbol{x}$, the original function in equation (9a) is equivalent to the transformed function as outlined in equation (10a) and given by

$$\mathbf{t}_m=\mathbf{B}_m^{-1}(\boldsymbol{x})\,\boldsymbol{a}_m(\boldsymbol{x})\,.\tag{11}$$

From the above, equation (4) can be rewritten in the form

$$\mathrm{SINR}_{u,\ell_u}=2\mathrm{Re}\{t_{u,\ell_u}^{*}\mathbf{h}_{\ell_u}^{\mathrm{H}}\mathbf{w}_u\}-|t_{u,\ell_u}|^2\left[\sum_{m=1}^{u-1}\mathbf{e}_{\ell_u}^{\mathrm{H}}\mathbf{w}_m\mathbf{w}_m^{\mathrm{H}}\mathbf{e}_{\ell_u}+\sum_{k=u+1}^{U}\mathbf{h}_{\ell_u}^{\mathrm{H}}\mathbf{w}_k\mathbf{w}_k^{\mathrm{H}}\mathbf{h}_{\ell_u}+\sigma^2\right],\tag{12}$$

with

$$t_{u,\ell_u}=\left[\sum_{m=1}^{u-1}\mathbf{e}_{\ell_u}^{\mathrm{H}}\mathbf{w}_m\mathbf{w}_m^{\mathrm{H}}\mathbf{e}_{\ell_u}+\sum_{k=u+1}^{U}\mathbf{h}_{\ell_u}^{\mathrm{H}}\mathbf{w}_k\mathbf{w}_k^{\mathrm{H}}\mathbf{h}_{\ell_u}+\sigma^2\right]^{-1}\mathbf{h}_{\ell}^{\mathrm{H}}\mathbf{w}_u.\tag{13}$$

C. Closed-form of Worst Sum-SINR CSI Error Vector

With possession of equation (12), the optimization problem in equation (8) can be recast as a quadratically constrained quadratic convex problem with respect to $\mathbf{w}_u$. To this end, however, all SINR_{u,ℓ_u}'s in the inequalities (12) must be optimized with respect to the CSI error vector $\mathbf{e}_{\ell_u}$, which can lead to overwhelming complexity due to the large number of distinct pairs u,ℓ_u. In order to keep overall complexity under control, we introduce a new variable $\ell\in\{1,2,\ldots,U\}$ and circumvent this problem by relaxing these requirement into the sum-SINR minimization problems (for each ℓ)

$$\min_{\mathbf{e}_\ell}\quad\sum_{j=1}^{\ell}\mathrm{SINR}_{j,\ell}\tag{14a}$$

$$\text{s.t.}\quad\|\mathbf{e}_\ell\|_2\leq\varepsilon,\tag{14b}$$

which under the quadratic transform yields

$$\min_{\mathbf{e}_\ell}\quad-\mathbf{e}_\ell^{\mathrm{H}}\mathbf{A}_\ell\mathbf{e}_\ell+2\mathrm{Re}\{\mathbf{e}_\ell^{\mathrm{H}}\mathbf{b}_\ell\}+c_\ell\tag{15a}$$

$$\text{s.t.}\quad\|\mathbf{e}_\ell\|_2\leq\varepsilon,\tag{15b}$$

where

$$\mathbf{A}_\ell=\sum_{j=1}^{\ell}|t_{j,\ell}|^2\sum_{i\neq j}\mathbf{w}_i\mathbf{w}_i^{\mathrm{H}},\tag{16a}$$

$$\mathbf{b}_\ell=\sum_{j=1}^{\ell}t_{j,\ell}^{*}\mathbf{w}_j-|t_{j,\ell}|^2\sum_{k=j+1}^{U}\mathbf{w}_k\mathbf{w}_k^{\mathrm{H}}\hat{\mathbf{h}}_\ell,\tag{16b}$$

$$c_\ell=\sum_{j=1}^{\ell}2\mathrm{Re}\{t_{j,\ell}^{*}\hat{\mathbf{h}}_\ell^{\mathrm{H}}\mathbf{w}_j\}-\sum_{j=1}^{\ell}|t_{j,\ell}|^2\left[\sum_{k=j+1}^{U}\hat{\mathbf{h}}_\ell^{\mathrm{H}}\mathbf{w}_k\mathbf{w}_k^{\mathrm{H}}\hat{\mathbf{h}}_\ell+\sigma^2\right].\tag{16c}$$

Notice that due to the relaxation of equation (7) into equation (14), the CSI error vector $\mathbf{e}_\ell$ obtained is a worst-case vector in the sum-SINR sense. Furthermore, since $\mathbf{A}_\ell\succeq 0$, the objective function in equation (15) is a concave function and therefore can not be minimized via numerical convex optimization tools. Fortunately, the strong duality holds for non-convex quadratic problems [17], [18], so that equation (15) can be solved via techniques such as SDR and the Lagrange multiplier method.

The Lagrangian function of equation (15) is given by

$$\mathcal{L}(\mathbf{e}_\ell,\lambda_\ell)=\mathbf{e}_\ell^{\mathrm{H}}\big(\lambda_\ell\mathbf{I}-\mathbf{A}_\ell\big)\mathbf{e}_\ell+2\mathrm{Re}\{\mathbf{e}_\ell^{\mathrm{H}}\mathbf{b}_\ell\}+c_\ell-\lambda_\ell\varepsilon^2\tag{17}$$

where $\lambda_\ell\geq 0$ denotes the dual variable.

For a fixed λ_ℓ, the optimal CSI error vector $\mathbf{e}_\ell$ and the corresponding dual function $g(\lambda_\ell)$ are, respectively, given by

$$\mathbf{e}_\ell=-\big(\lambda_\ell\mathbf{I}-\mathbf{A}_\ell\big)^{-1}\mathbf{b}_\ell,\tag{18}$$

and

$$g(\lambda_\ell)=\begin{cases}-\mathbf{b}_\ell^{\mathrm{H}}\big(\lambda_\ell\mathbf{I}-\mathbf{A}_\ell\big)^{-1}\mathbf{b}_\ell+c_\ell-\lambda_\ell\varepsilon^2 & \text{if }\mathbf{b}_\ell\in\mathrm{R}(\lambda_\ell\mathbf{I}-\mathbf{A}_\ell)\text{ and }\lambda_\ell\mathbf{I}-\mathbf{A}_\ell\succeq 0\\ -\infty & \text{(otherwise).}\end{cases}\tag{19}$$

By virtue of the Schur complement, the dual variable λ_ℓ is optimized by solving an epigraph form of the maximization of the dual function in equation (19), such that

$$\max_{\lambda_\ell,\beta_\ell}\quad\beta_\ell\tag{20a}$$

$$\text{s.t.}\quad\begin{bmatrix}\lambda_\ell\mathbf{I}-\mathbf{A}_\ell & \mathbf{b}_\ell\\ \mathbf{b}_\ell^{\mathrm{H}} & c_\ell-\lambda_\ell\varepsilon^2-\beta_\ell\end{bmatrix}\succeq 0\tag{20b}$$

$$\lambda_\ell\geq 0.\tag{20c}$$

Algorithm 1: FP-based Robust TX BF for PD-NOMA.

Input: Channel Estimate: $\hat{\mathbf{h}}_\ell \ \forall \ell$
Target SINR: $\mathbf{\Gamma}_u \ \forall u$
Maximum number of iteration: $i_{\max}$
Initial TX beamforming vector: $\mathbf{w}_u^{(0)} \ \forall u$
CSI error bounding parameter: ε

Generate random CSI error $\mathbf{e}_\ell^{(0)} \ \forall \ell$ such that $\|\mathbf{e}_\ell\|_2 \leq \varepsilon$.
Set $i = 0$.
repeat
  $i \leftarrow i + 1$
  $t_{u,\ell_u} \ \forall u, \ell_u \leftarrow$ Equation (13) for given $\mathbf{e}_\ell^{(i-1)}, \mathbf{w}_u^{(i-1)}$.
  $\lambda_\ell \ \forall \ell \leftarrow$ Solve SDP in equation (20).
  $\mathbf{e}_\ell^{(i)} \leftarrow$ Compute from equation (18).
  $\mathbf{w}_u^{(i)} \leftarrow$ Solve SDP in equation (21).
  Check convergence
  $\delta = \sum_{u=1}^{U} \left\| \mathbf{w}_u^{(i-1)} - \mathbf{w}_u^{(i)} \right\|_2 / (N_t \cdot U)$
until $\delta < 10^{-4}$ *or reach the maximum iteration i_{max}*;

Proof. See Appendix A. □

One can readily notice that equation (20) is a standard semidefinite programming (SDP) problem with a linear objective function, which can be efficiently solved in polynomial time via interior point methods [19].

D. Robust TX BF Algorithm

With estimates of the CSI error vectors $\mathbf{e}_\ell$ obtained as per equation (18) in hand, the original optimization problem given in equation (8) becomes a standard non-convex power minimization problem, which can be relaxed and solved in a number of different manners. One possibility is, for instance, to solve the problems via SDR [20]. In this case, defining $\mathbf{W}_u = \mathbf{w}_u \mathbf{w}_u^{\mathrm{H}} \succeq 0, \mathbf{H}_{\ell_u} = \mathbf{h}_{\ell_u} \mathbf{h}_{\ell_u}^{\mathrm{H}} \succeq 0$ and $\mathbf{E}_{\ell_u} = \mathbf{e}_{\ell_u} \mathbf{e}_{\ell_u}^{\mathrm{H}} \succeq 0$, equation (8) can be transformed into

$$\min_{\mathbf{W}_u, \forall u} \quad \sum_{u=1}^{U} \mathrm{Tr}\left(\mathbf{W}_u\right) \tag{21a}$$

$$\text{s.t.} \quad \mathrm{Tr}\left(\mathbf{H}_{\ell_u} \mathbf{W}_u\right) - \Gamma_u \left[\sum_{m=1}^{u-1} \mathrm{Tr}\left(\mathbf{E}_{\ell_u} \mathbf{W}_m\right) \right. \tag{21b}$$

$$\left. + \sum_{k=u+1}^{U} \mathrm{Tr}\left(\mathbf{H}_{\ell_u} \mathbf{W}_k\right) \right] \geq \Gamma_u \sigma^2 \ \forall u, \ell_u$$

$$\mathbf{W}_u \succeq 0 \ \forall u, \tag{21c}$$

where we omitted the rank-one constraint.

We remark that although the relaxed problem in equation (21) can be efficiently solved by interior point methods, it is not guaranteed that such relaxed solutions are rank-one. It is known, however, that if the largest eigenvalue of $\mathbf{W}_u$ is sufficiently larger than the second, the approximation gap between the global optimum and the solution to equation (21) is tight [21], [22], while randomization procedures can be used to generate an approximate solution otherwise.

IV. Simulation results

In this section, we evaluate via computer simulations the effectiveness of the proposed algorithm in a downlink BB MIMO-NOMA system with $N_t = 3$ transmit antennas at the BS serving $U = 3$ single-antenna users, comparing its performance against that of a "Non-Robust" scheme which does not take into account channel uncertainties, both under perfect and imperfect CSI knowledge conditions.

In all simulations, it is assumed that $\hat{\mathbf{h}}_\ell$ are independent and identically distributed (i.i.d) Rayleigh fading channel estimates, with underlying vectors following a complex Gaussian distribution with zero-mean and variance adjusted by the number of transmit antennas N_t for all ℓ, namely, $\hat{\mathbf{h}}_\ell \sim \mathcal{CN}\big(\mathbf{0}, \frac{1}{N_t}\mathbf{I}_{Nt}\big)$, while the CSI error $\mathbf{e}_\ell$ is uniformly distributed within a disk of radius 0.01 ($i.e.$, $\varepsilon = 0.01$). In addition, we use the following simulation setup unless otherwise mentioned. The noise variance σ^2 is set to be 0.01 [mW] and the number of iterations for the proposed algorithm is upper-bounded by 10, $i.e.$, $i_{\max} = 10$, whereas, for the sake of simplicity but without loss of generality, the target SINR Γ_u is assumed to be identical, namely, $\Gamma = \Gamma_u \ \forall u$, and scaled from 0 [dB] to 10 [dB]. All the results are averaged over 500 channel realizations with 100 CSI error realizations for each channel realization.

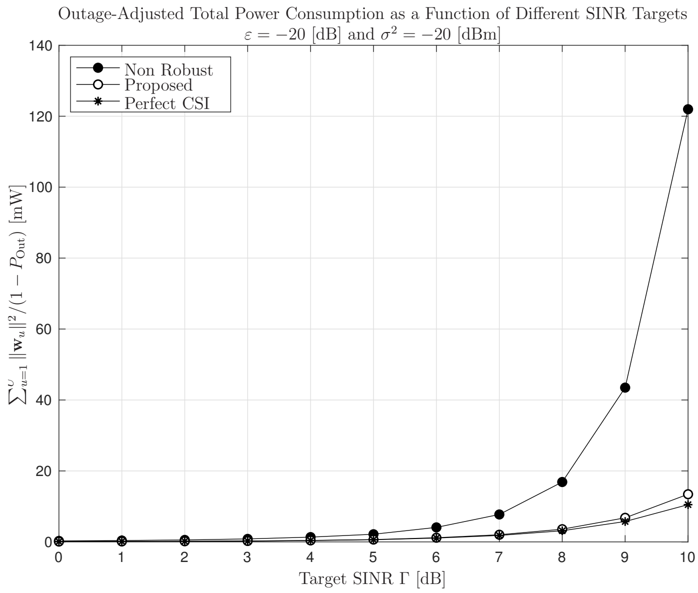


Fig. 2. Outage-adjusted total transmit power consumption at different SINR targets for imperfect CSI error bounded according to $\varepsilon = -20$ [dB]

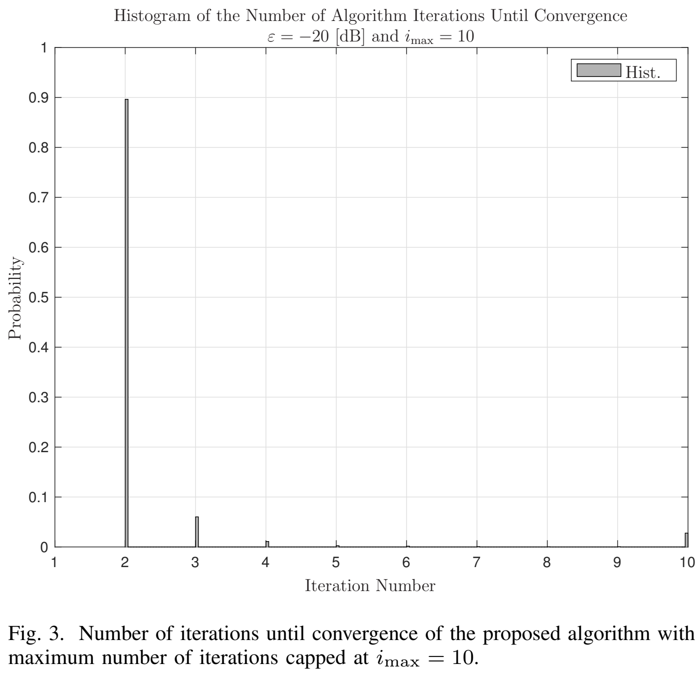
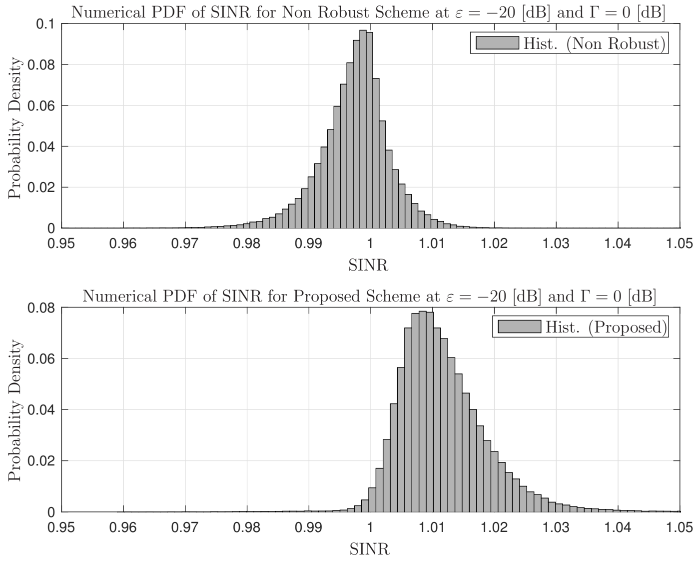


Fig. 3. Number of iterations until convergence of the proposed algorithm with maximum number of iterations capped at $i_{\max} = 10$.

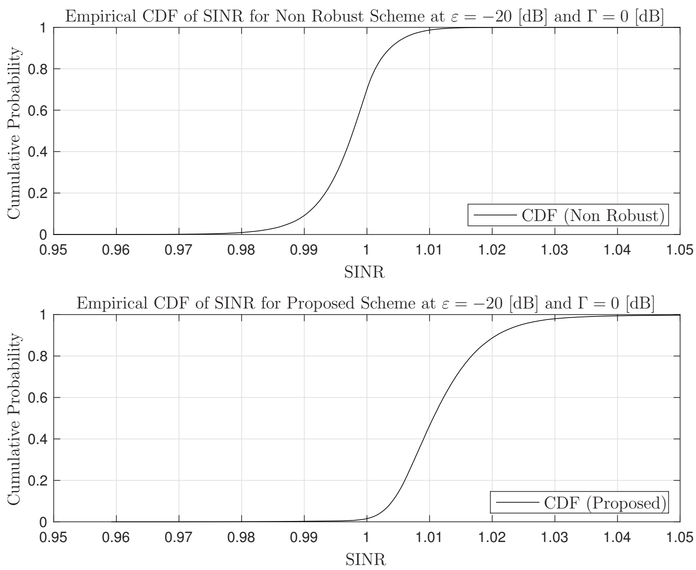
Throughout the simulations, the MATLAB-based convex optimization toolbox CVX with its default solver SDPT3 is utilized to solve convex optimization problems such as SDPs in equation (20) and equation (21).

First, in Figure 2, plots of the average transmit power consumption scaled by taking into account the outage probability defined by $P_{\text{Out}} = \Pr(\text{SINR} < \Gamma)$ are shown, as a function of different target SINRs. Figure 2 clearly illustrates the consequence of ignoring CSI imperfection, demonstrating that the proposed method is capable of significant power savings compared to the non-robust alternative, in fact coming very close to the performance of a system operating under perfect CSI.

The latter results are even more motivating when considered next to the results offered in Figure 3, which shows the empirical probability mass function of the number of iterations till convergence required by for the proposed algorithm. The figure indicates that in approximately 90% of the times convergence is achieved after only 2 iterations, and the maximum allowed number of iterations ($i_{\max} = 10$) is required less than 5% of the times.

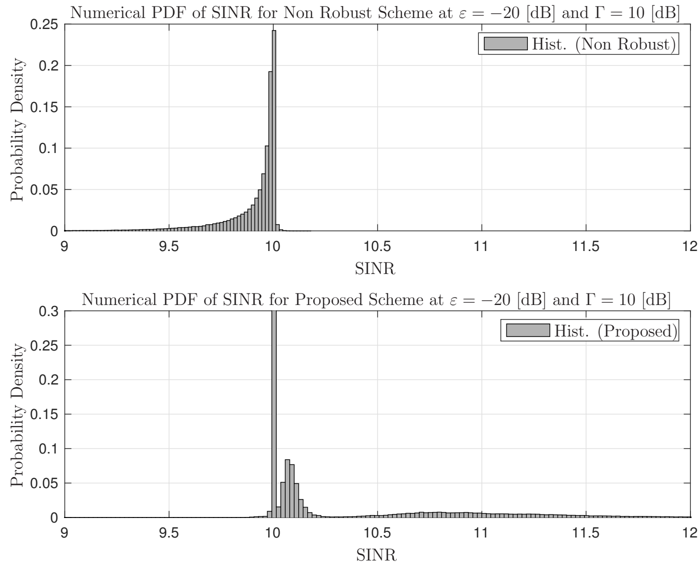


(a) Empirical PDF.

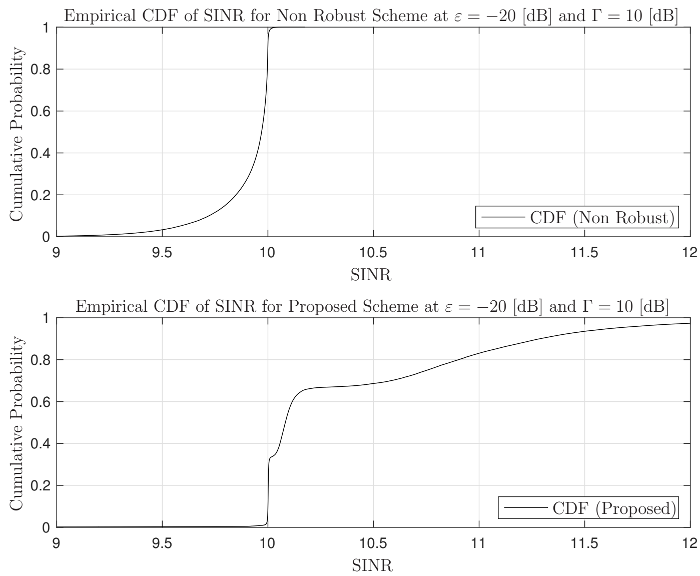


(b) Empirical CDF.

Fig. 4. Comparisons of PDF and CDF of obtained SINR for both the Non-Robust and the proposed schemes at $\varepsilon = 0.01$, $\sigma^2 = 0.01$, $i_{\max} = 10$ and $\Gamma = 0$ [dB].



(a) Empirical PDF.



(b) Empirical CDF

Fig. 5. Comparisons of PDF and CDF of obtained SINR for both the Non-Robust and the proposed schemes at $\varepsilon = 0.01$, $\sigma^2 = 0.01$, $i_{\max} = 10$ and $\Gamma = 10$ [dB].

Since this empirical distribution shown in Figure 3 was computed over 500 channel realizations for each different SINR target ranging from 0 [dB] to 10 [dB], it can be concluded that the convergence behavior of the proposed scheme is very stable for different QoS requirements. Together, Figures 2 and 3 suggest that with the proposed robust algorithm, BB MIMO-NOMA is almost immune to the level of CSI imperfection studied.

In Figures 4 and 5, the empirical PDF and corresponding CDF of the instantaneous SINRs attained at each user over different channel and CSI uncertainty levels are shown, for two different SINR targets, namely, $\Gamma = 0$ [dB] (in Figure 4) and $\Gamma = 10$ [dB] (in Figure 5), respectively. The figures further illustrate the robustness of our proposed algorithm. In particular, it is observed that the SINR distributions obtained as a consequence of the proposed method are significantly skewed to the right, regardless of the target QoS requirements, indicating that negligible levels of outages are achieved.

In contrast, the corresponding distributions associated with a system designed under the assumption of perfect CSI, but subjected to CSI uncertainties (non-robust scheme) are found to have significant portions of their probability masses on the left side, with the condition worsening with the QoS requirement.

Taking into account that the latter results are obtained without requiring much more power than would necessary under perfect CSI conditions, and without requiring more than a few iterations of the algorithm, as shown in Figures 2 and 3, respectively, it can be confidently stated that the proposed method is effective, efficient and practical. In addition to all the above, while generating all the data used in the figures above – a total of 550000 Monte-Carlo data points including 500 channel realizations over 11 different SINR targets with 100 CSI uncertainty realizations each – we kept track of the *feasibility ration* r_{Feas} of the proposed algorithm, defined as the fraction of times a rank-one solution of equation (21) was obtained. It was found that the (empirical) feasibility ratio of the method is of $r_{\text{Feas}} = 99.99\%$ at the simulated scenario of $\varepsilon = 0.01$, $\sigma^2 = 0.01$, $N_t = 3$ and $U = 3$.

V. Conclusion

We considered the downlink of a BB MISO PD-NOMA system serving multiple single-antenna users and subjected to a norm-bounded CSI uncertainty. For such a system, we presented a new transmitter (TX) BF scheme, in which the total transmit power consumption is minimized subjected to prescribed SINR requirements for each user. The proposed scheme takes advantage of two contributions: the first is a novel method to obtain closed-form estimates of the CSI error vectors that result in the worst sum of the SINRs of decodable signals at each user; and the second is a FP-based quadratic transformation of the original and intractable non-convex SINR constrained power minimization problem into a problem which is quadratic and therefore solvable using standard interior point methods. Simulation results were shown which confirmed that the proposed algorithm ensures the robustness to imperfect CSI, requiring only small amounts of additional transmit power compared to a system operating under idealistic (perfect CSI) conditions.

The proposed algorithm was also shown to be fast, requiring only a few iterations to converge, and consistent, offering large gains in robust power savings even when subjected to large target SINR requirements.

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Appendix A
Proof of Equation (20)

Although a similar problem has been discussed in [23], we herein provide details for the sake of complementarity. Introducing a new slack variable β_ℓ, the maximization of the dual function $g(\lambda_\ell)$ can be simply written in a form

$$\max_{\lambda_\ell,\beta_\ell} \quad \beta_\ell \tag{22a}$$

$$\text{s.t.} \quad -\mathbf{b}_\ell^{\text{H}}\big(\lambda_\ell\mathbf{I}-\mathbf{A}_\ell\big)^{-1}\mathbf{b}_\ell + c_\ell - \lambda_\ell\varepsilon^2 \geq \beta_\ell \tag{22b}$$

$$\mathbf{b}_\ell \in \mathrm{R}(\lambda_\ell\mathbf{I}-\mathbf{A}_\ell) \tag{22c}$$

$$\lambda_\ell\mathbf{I}-\mathbf{A}_\ell \succeq 0 \tag{22d}$$

$$\lambda_\ell \geq 0. \tag{22e}$$

Recalling the Schur complement, we obtain

$$\mathbf{X} \succeq 0 \quad \Leftrightarrow \quad \begin{cases} \lambda_\ell\mathbf{I}-\mathbf{A}_\ell \succeq 0 \\ \mathbf{b}_\ell \in \mathrm{R}(\lambda_\ell\mathbf{I}-\mathbf{A}_\ell) \\ -\mathbf{b}_\ell^{\text{H}}\big(\lambda_\ell\mathbf{I}-\mathbf{A}_\ell\big)^{-1}\mathbf{b}_\ell + c_\ell - \lambda_\ell\varepsilon^2 - \beta_\ell \geq 0 \end{cases}$$

where

$$\mathbf{X} = \begin{bmatrix} \lambda_\ell\mathbf{I}-\mathbf{A}_\ell & \mathbf{b}_\ell \\ \mathbf{b}_\ell^{\text{H}} & c_\ell - \lambda_\ell\varepsilon^2 - \beta_\ell \end{bmatrix}. \tag{23}$$

Proof. See Proposition 2.1 and its proof in [24]. For more details about the (generalized) trust region problem, see [24] and references thereby. ☐

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