An Economic Analysis of Wireless Network Infrastructure Sharing

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Abstract—**Internet service providers (ISPs) struggle to invest in upgrading their networks to catch up with growing mobile data demand, while users have to face significant data overage fees. Pooling ISPs' network infrastructures can potentially enable better user experience and lower prices. For example, Google recently launched a *cross-carrier MVNO* (mobile virtual network operator) data plan called Project Fi, where users' devices can automatically access either of two partner cellular networks or any available open WiFi network. We consider the economic impact of cross-carrier MVNOs on the mobile data market. We begin by analyzing a network selection strategy that optimizes cross-carrier users' costs. We then study ISPs' behavior, deriving the prices that partner ISPs charge the cross-carrier MVNO and that the cross-carrier MVNO charges its end users. Although the cross-carrier MVNO may lose money from selling data, it can offset this loss with side revenue, e.g., advertisement revenue when users consume more content. We derive conditions under which the cross-carrier MVNO achieves a profit and its users reduce their costs. Finally, we use a real-world network quality dataset to simulate users' network selection behavior and demonstrate the benefits of the ISP competition brought by the cross-carrier MVNO.**

I. INTRODUCTION

Facing users' ever-increasing demand in mobile data, most Internet service providers (ISPs) set a monthly data quota with a base payment and charge expensive overage fees on usage exceeding this data cap. However, this does not fully address ISP or user concerns: ISPs need to invest in technological solutions (e.g. development of 5G networks or expansions of their existing networks) for supporting a greater volume of network traffic. Users want good network quality and coverage, but may not be willing to pay for these technologies. Although a plausible approach to providing better service at low cost is for ISPs to supplement their networks with existing WiFi hotspots [1], WiFi is often available in limited areas, restricting its offloading capability [2], [3]. Thus, WiFi alone is not a sufficient solution to resolve the poor coverage and expensive charges of mobile data access.

A. The Cross-carrier MVNO

To address these challenges, we propose a new approach of combining network infrastructures as well as offloading data to WiFi. As anti-trust regulations can restrict merger efforts to pool ISPs' infrastructures [4], such infrastructure sharing is often infeasible unless done by third-party *cross-carrier MVNOs*. For example, in the U.S., Google has become such a cross-carrier MVNO, introducing a data plan called Project Fi [5], [6]. Project Fi runs on T-Mobile's, Sprint's, and US Cellular's network infrastructures, while automatically connecting users to any available open WiFi networks.[1] Unlike an MVNO's usual partnership with a single ISP, cross-carrier MVNOs lease capacity from two or more cellular networks as well as WiFi. We refer to the ISPs participating in the cross-carrier plan as "partner ISPs," and other non-participating ISPs as "non-partner ISPs."

In our study of the economic impact of the cross-carrier data plan, we assume that some users defect to the cross-carrier MVNO from partner and non-partner ISPs. Their traffic can traverse either WiFi or one of the partner ISPs' cellular networks. To reveal whether, and under what circumstances, the cross-carrier MVNO, partner ISPs, and users benefit from the cross-carrier data plan, we consider the following decisions that determine the interactions between the three parties.

- *How much should partner ISPs charge the cross-carrier MVNO, and how much should the cross-carrier MVNO charge users?* It is unclear whether the cross-carrier MVNO and partner ISPs always make a profit: the partner ISPs may lose users to the cross-carrier MVNO, decreasing the partner ISPs' revenue, and they will then charge the cross-carrier MVNO enough to make up for this revenue loss. The cross-carrier MVNO may then lose money unless it charges users more. Alternatively, the cross-carrier MVNO may operate another business that receives side benefits from offering the data plan, e.g., revenue from mobile advertisements.
- *How do users' subscription decisions impact the cross-carrier data plan?* In order to build a customer base, the cross-carrier MVNO needs to attract users who initially subscribed to a partner or non-partner ISP, for instance, by offering lower data prices. However, too many users defecting from partner ISPs increases the price charged by the partner ISPs to the cross-carrier MVNO, and jeopardizes the cross-carrier MVNO's profit. Thus, as the number of users on the cross-carrier MVNO increases,

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[1] Google does not charge users for WiFi usage, but charges them \$10 per gigabyte of cellular data usage.

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the cross-carrier MVNO's profit may not necessarily increase. A larger number of users from non-partner ISPs is more preferable, since non-partner ISPs are viewed as inactive players.

We assume that each cross-carrier user's device can switch between partner ISPs' cellular networks and open WiFi, which is a feature already implemented by the cross-carrier MVNO.[2] We briefly summarize related works in Section II before considering the following three topics:

Population of cross-carrier users on each network (Section III): Although a comprehensive design of network selection strategies is beyond this paper, the prices that the cross-carrier MVNO charges users, as well as its payments to the partner ISPs, are affected by the number of cross-carrier users (and their usage volume) on each partner ISP's network. We analyze a reasonable network selection strategy based on Google Fi's practice that minimizes individual users' data costs by prioritizing free WiFi.

Economic impact on partner ISPs (Section IV): Partner ISPs incur operational costs from cross-carrier users joining their networks, as well as revenue losses from users who defect to the cross-carrier MVNO. They can make up for these losses by charging the cross-carrier MVNO for leasing network capacity. *We derive these prices and show that if user demand decreases upon subscribing to the cross-carrier MVNO, the cross-carrier MVNO will always lose money.*

Cross-carrier MVNO behavior (Section V): Given the payments to its partner ISPs, the cross-carrier MVNO must decide how much to charge its users for cellular data. In addition to revenue from these data charges, the cross-carrier MVNO may gain side benefits from users' usage, e.g., revenue from mobile advertisements. In balancing these objectives, the MVNO must also account for changes in user demand due to different data prices. *We derive the cross-carrier MVNO's optimal price, taking into account the relative importance of revenue and side benefits. We quantify the conditions under which the cross-carrier MVNO can be profitable, while users simultaneously save money on their cross-carrier data plans.*

We simulate the population of cross-carrier users on different partner ISPs' networks, using a crowd-sourced network quality dataset from multiple ISPs in a large U.S. city [7], in Section VI. We use this data to show that the price charged by the cross-carrier MVNO to its users is stable as the number of cross-carrier subscribers increases. We then demonstrate that under realistic conditions, the cross-carrier data plan benefits both users (in terms of network quality and data cost) and the cross-carrier MVNO (in terms of profit). We conclude the paper in Section VII. All proofs are in the Appendix.

II. RELATED WORK

To alleviate network congestion, ISPs offer many variants of mobile data plans that use prices to influence user demands. Some plans allow users to trade data caps with each other, effectively averaging out users' heterogeneous data usage [8]. To further reduce users' data costs, content providers are allowed to subsidize usage on certain contents [9]. However, these works generally consider user behavior resulting from price incentives instead of network quality.

To improve network quality, many works have taken advantage of wireless radio access technologies in heterogeneous networks from a pure technological perspective [10]–[12]. Other have gone further in analyzing a single ISP's strategies on either pricing different network technologies [1], [13], [14] or incentivizing users to offload data away from congested networks [2], [3]. For instance, users can be incentivized to become WiFi hotspots for other users' data offloading [15], [16], but these connections can be highly unstable due to user mobility. Although other works consider hierarchical ISP models [17] and tiered pricing [18] for several ISPs to share capacity, user benefits remain unclear. Our work, in contrast, considers a network selection strategy by leveraging network infrastructure from multiple partner ISPs as well as opportunistic WiFi offloading, and analyzes its economic consequences for both ISPs and end users.

III. NETWORK SELECTION STRATEGIES FOR THE CROSS-CARRIER MVNO

In this section, we discuss a reasonable strategy for the cross-carrier MVNO to select (i.e., switch between) available networks for their users, and derive the corresponding numbers of users connected to the partner ISPs' cellular networks. At any given time, each of the N cross-carrier users' devices decides whether or not to switch to another network, given that the selection strategy is pre-installed on user devices to facilitate the cross-carrier data plan.[3]

A. Network Model

We suppose that devices independently switch between networks according to an algorithm pre-determined by the cross-carrier MVNO.[4] We assume that at most one user switches at each time instant, e.g., on a sub-millisecond timescale. We let $\Phi = \{\phi_1, \ldots, \phi_K\}$ and $\Psi = \{\psi_1, \ldots, \psi_L\}$ denote the available cellular base stations (BSs) and WiFi access points (APs) respectively. Each ISP may own multiple BSs. User devices make selection decisions based on their throughputs, which are in terms of the PHY rates (the average link-layer rates) $r_i(\phi_k)$ and $r_i(\psi_l)$ for each user i on ϕ_k and ψ_l respectively. We assume that the BSs communicate users' PHY rates and any other relevant information to the user devices, allowing the devices to optimize their decisions. Devices can thus avoid collectively switching to the same network, which may impair performance and result in unstable connectivity due to frequent switching between different networks [11]. We let $s_i \in \Phi \cup \Psi$ represent the network to which user i connects

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[2] Google requires its Project Fi users to use a Google-made phone, allowing such capability to be implemented on the device.

[3] Fast network selection can be implemented in device hardware, e.g., recent Google's Nexus or Pixel phones.

[4] While the cross-carrier MVNO could make these decisions centrally, allowing devices to switch independently leads to faster decision making and smoother transitions.

TABLE I
CONDITIONS FOR WIFI-PRIORITIZED SELECTION.

To From	**Cellular BS** ϕ_k	**WiFi AP** ψ_l
Cellular BS $\phi_{k'}$	$\tau_i(\phi_k) > \tau_i(\phi_{k'})$ $\forall l : \tau_i(\psi_l) < \mu$	$\exists l : \tau_i(\psi_l) \geq \mu$
WiFi AP $\psi_{l'}$	$\forall l : \tau_i(\psi_l) < \mu$	$\tau_i(\psi_{l'}) < \mu$ $\exists l : \tau_i(\psi_l) \geq \mu, l \neq l'$

and assume that users connect to only one network at a time. Users' service is always covered by one or more BSs but not necessarily by a WiFi AP.

We assume that ISPs treat their own users and cross-carrier users equally, i.e., neither is prioritized. We now define users' throughput based on cellular and WiFi networks' different MAC protocols [11], [19]:

Cellular throughput. In a cellular network with proportional fair scheduling, e.g., OFDMA, each user is allocated an equal share of the wireless medium. User i's throughput $\tau_i(\phi_k)$ on BS ϕ_k can then be written as:

$$\tau_i(\phi_k) = \frac{r_i(\phi_k)}{n_k + m_k}, \tag{1}$$

where n_k is the number of cross-carrier users who join the BS ϕ_k, and m_k is the number of the ISP's own users on ϕ_k.

WiFi throughput. WiFi APs use round-robin scheduling for downlink traffic. Thus, each user i on the AP ψ_l achieves the same throughput

$$\tau_i(\psi_l) = \frac{1}{\sum_{j=1}^{n_l} \frac{1}{r_j(\psi_l)} + \sum_{j=1}^{m_l} \frac{1}{r_j(\psi_l)}}, \tag{2}$$

where n_l is the number of cross-carrier users connected to ψ_l, and m_l is the number of other users connected to this AP. We call these $m_k + m_l$ non-cross-carrier users *regular users*.

B. WiFi-prioritized Network Selection

We now propose a network selection strategy where a device always switches to a WiFi network if its throughput in that network exceeds a given threshold μ. Unlike cellular usage, WiFi usage has no monetary cost for users, so this strategy minimizes user costs while ensuring an acceptable quality of service. The mathematical network selection conditions are given in Table I. Here, μ trades off between cost and throughput: as μ decreases, users are more likely to switch to a WiFi network, possibly experiencing lower throughput in exchange for lower data costs. Thus, μ should be large enough to not only ensure a tolerable performance, but also reduce switching costs (such as greater energy consumption or interruptions of data transmissions) that can outweigh the monetary benefit. When there is no acceptable WiFi AP, users connect to the cellular BS with the highest throughput.

Under this selection strategy, the total number of devices that connect to a cellular network at equilibrium is:

$$N_\Phi = N - \sum_{l=1}^{L} \left\lfloor \left(\frac{1}{\mu} - \sum_{j=1}^{m_l} \frac{1}{r_j(\psi_l)} \right) \mathbb{E}(r(\psi_l)) \right\rfloor, \tag{3}$$

where $\lfloor \cdot \rfloor$ represents the floor operation and $\mathbb{E}(r(\psi_l))$ is the expected PHY rate that each device can achieve on the WiFi AP ψ_l. In (3), $\left\lfloor \left(\frac{1}{\mu} - \sum_{j=1}^{m_l} \frac{1}{r_j(\psi_l)} \right) \mathbb{E}(r(\psi_l)) \right\rfloor$ gives the expected maximum number of devices that can use WiFi AP ψ_l while satisfying $\frac{1}{n_l \mathbb{E}\left(r(\psi_l)\right) + \sum_{j=1}^{m_l} \frac{1}{r_j(\psi_l)}} \geq \mu$.

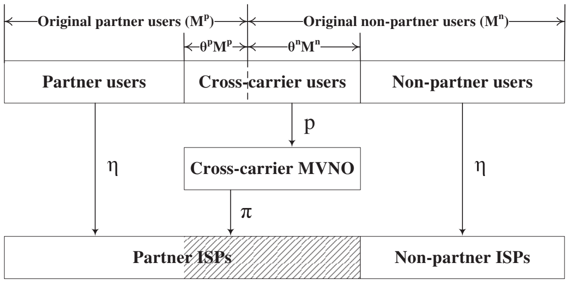

Fig. 1. Payments between users and ISPs. An arrow from A to B means that party A pays party B the data price (\$/GB) marked on the arrow. The width of each rectangle represents the number of users who pay that ISP either directly or indirectly. The shaded region indicates that these users can also use WiFi.

IV. PARTNER ISP BEHAVIOR

Based on the number of cellular users derived in (3), we now analyze the economic impact of the cross-carrier MVNO on the partner ISPs, who provide network capacity to the cross-carrier MVNO. We suppose there are K^p partner and K^n non-partner ISPs, and we denote the BSs belonging to each by ϕ_i^p, $i \in \{i_k^p, \ldots, i_{k+1}^p - 1\}$ for the partner ISP k^p, and ϕ_j^n, $j \in \{j_k^n, \ldots, j_{k+1}^n - 1\}$ for the non-partner ISP k^n. We use M_k^p and M_k^n to denote the numbers of original users (i.e., before the cross-carrier MVNO joins the market) on ISPs k^p and k^n respectively. Figure 1 shows the payments between users and the different ISPs: the partner ISPs must decide how much to charge the cross-carrier MVNO, while the cross-carrier MVNO decides how much to charge its users.

We suppose that all ISPs charge users η^d for up to d GB of data per month, with overage fees of η^o for each GB ($\eta^d/d < \eta^o$). We suppose that regular users' expected usage within a month is z, with an expected monthly charge $\hat{\eta}$. We note that $\hat{\eta} \geq \eta^d$ since every user needs to pay at least η^d per month. For simplicity, we do not consider the fact that, in reality, the ISPs' prices and data caps vary for different ISPs and different plans, for example, shared data plans. Thus, the expected revenue that each partner ISP receives from its users is $M_k^p \hat{\eta}$. We suppose that the ISP also incurs a linear operational cost of supporting the traffic in its network, so that its total profit is $M_k^p \hat{\eta} - c M_k^p z$, where $c > 0$ is the marginal operational cost.

If the cross-carrier MVNO's network quality is *better* than that of non-partner ISPs, then the cross-carrier MVNO can attract users from all ISPs with the promise of better network quality. However, if the cross-carrier MVNO's network quality is worse than that of non-partner ISPs, (e.g., Google Fi partners with Sprint, T-Mobile, and U.S. Cellular, which even when combined may not match the non-partner ISP Verizon's network quality), then the cross-carrier MVNO attracts customers by offering usage-based pricing at a lower price $p < \eta^d/d$.

It also allows users to connect to WiFi if available, and the cross-carrier MVNO does not charge its users for WiFi usage.

We use θ_k^p to denote the fraction of partner ISP k^p's users who defect to the cross-carrier MVNO, and θ_k^n for non-partner ISP k^n's users.[5] We then term θ_k^p and θ_k^n as *defection rate*. The total number of users on the cross-carrier plan is $N = \sum_{k=1}^{K^p} \theta_k^p M_k^p + \sum_{k=1}^{K^n} \theta_k^n M_k^n$, and the total number of "loyal" users on all BSs belonging to ISP k^p is $(1-\theta_k^p) M_k^n = \sum_{i=i_k^p}^{i_{k+1}^p} m_i^p$.

A. Prices Charged by Partner ISPs

Suppose the partner ISP k^p charges the cross-carrier MVNO a usage-based price π_k, where $\pi_k > c$ to ensure that the partner ISP can cover its operational costs from the cross-carrier users' traffic. After losing $\theta_k^p M_k^p$ users to the cross-carrier MVNO, the partner ISP's profit becomes

$$\begin{aligned}\mathcal{R}(\pi_k) = {} & (1-\theta_k^p) M_k^p \hat{\eta} + \pi_k \big(\textstyle\sum_{i=i_k^p}^{i_{k+1}^p} n_i^p\big) y \\ & -c\Big((1-\theta_k^p) M_k^p z + \big(\textstyle\sum_{i=i_k^p}^{i_{k+1}^p} n_i^p\big) y\Big),\end{aligned} \tag{4}$$

where y is the expected monthly usage of cross-carrier users, as discussed in Section V-A. We note that n_i^p, the number of cross-carrier users on ISP k^p's ith BS, is affected by θ_k^p, θ_k^n, and also the users' network selection strategy. A larger θ_k^p or θ_k^n means more cross-carrier users and thus a larger amount of cross-carrier traffic on ISP k^p's network, leading to higher payments from the cross-carrier MVNO. However, this does not necessarily lead to profit increase for the partner ISP: a larger θ_k^p implies that the partner ISP loses more users to the cross-carrier MVNO, decreasing its revenue from its own users. On the other hand, partnering with the cross-carrier MVNO attracts traffic from cross-carrier users defecting from other non-partner ISPs, increasing the partner ISP's revenue.

The above argument shows that the partner ISP should carefully set the price π_k that it charges the cross-carrier MVNO, so as to ensure that it does not lose profit by partnering with the cross-carrier MVNO. By solving $\mathcal{R}(\pi_k) \geq M_k^p \hat{\eta} - c M_k^p z$, i.e., setting the profit with the cross-carrier MVNO to exceed that before the cross-carrier's entrance into the market, we can derive the revenue-neutral price that the partner ISP k_p^n charges the cross-carrier MVNO:

Proposition 1: If the partner ISP k^p sells data to the cross-carrier ISP at a price $\pi_k = \pi_k^\star$, where

$$\pi_k^\star = \frac{\theta_k^p M_k^p (\hat{\eta} - cz)}{\sum_{i=i_k^p}^{i_{k+1}^p} n_i^p y} + c, \tag{5}$$

then it will secure the same revenue as that achieved without the cross-carrier MVNO.

We now discuss the intuition behind four key factors affecting the price $\pi_k^\star$:

- *Operational cost.* Intuitively, as the operational cost c increases, $\pi_k^\star$ should also increase: it will be more expensive for the partner ISP to accommodate cross-carrier users' traffic. In fact, (5) shows that we always have $\pi_k^\star > c$, and as c increases, so does $\pi_k^\star$. Thus, the cross-carrier MVNO indirectly pays for operational costs, although it does not directly operate a cellular network.
- *The selection strategy.* The amount of cross-carrier users' traffic on each partner ISP k^p depends on the relative network qualities of the different partner ISPs, as well as WiFi availability. A partner ISP with better network quality will likely observe more cross-carrier usage (higher n_i^p) and therefore charge the cross-carrier MVNO a lower per-gigabyte price $\pi_k^\star$ in (5).
- *Users lost to the cross-carrier MVNO.* Since the partner ISP wishes to make up for the revenue lost by these users, the rate it charges the cross-carrier MVNO is proportional to the fraction of users lost. However, this is partially offset by the fact that these users may still have some traffic on ISP k^p's network: a larger θ_k^p may lead to a larger $\sum_{i=i_k^p}^{i_{k+1}^p} n_i^p$ and thus a lower price.
- *Users from competitors.* Since the cross-carrier MVNO does not distinguish its users by the ISPs that they come from, each partner ISP will likely observe a larger number of users than it had before the cross-carrier MVNO entered the market (larger $\sum_{i=i_k^p}^{i_{k+1}^p} n_i^p$). The partner ISP will then collect payments on a larger amount of cross-carrier traffic, thus reducing the per-gigabyte price (5) that it charges the cross-carrier MVNO.

B. Impact on the Cross-Carrier MVNO

Intuitively, the per-GB price $\pi_k^\star$ charged by partner ISP k^p increases with the amount of data $\theta_k^p M_k^p z$ that ISP k^p loses when its users subscribe to the cross-carrier data plan. However, the price also decreases with the amount of cross-carrier data $\sum_{i=i_k^p}^{i_{k+1}^p} n_i^p y$ on ISP k^p's network. Thus, the amount of cross-carrier usage affects the cross-carrier MVNO's profit:

Lemma 1: Suppose that the amount of data consumed by users that the partner ISP k^p has lost to the cross-carrier MVNO exceeds the amount of data consumed by the new cross-carrier users that access ISP k^p's network, i.e., $\theta_k^p M_k^p z > \sum_{i=i_k^p}^{i_{k+1}^p} n_i^p y$. Since $\pi_k^\star \geq p$, the cross-carrier MVNO always loses money if the partner ISP k^p sets its price as in (5).

However, if there is an influx of users from non-partner ISPs, the average price charged by the partner ISP decreases:

Proposition 2: The average per-GB price that the cross-carrier MVNO pays to all its partner ISPs is given by

$$\hat{\pi}^\star = \frac{(\hat{\eta} - cz) \sum_{k=1}^{K^p} \theta_k^p M_k^p}{y N_\Phi} + c, \tag{6}$$

which decreases if more users from non-partner ISPs subscribe to the cross-carrier MVNO, i.e., N_Φ increases.

Here, (6) is obtained by summing up the cross-carrier payments $\pi_k^\star \sum_{i=i_k^p}^{i_{k+1}^p} n_i^p y$ to all partner ISPs, where $\pi_k^\star$ is given in (5), and dividing by the total cross-carrier cellular data $y N_\Phi$.

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[5] Users decide whether to join the cross-carrier MVNO based on various factors, e.g., device availability, contracts with their current ISPs, or personal preference. Modeling these complex human preferences is outside the scope of this work, so we take θ_k^p and θ_k^n as given.

Similar to Proposition 1, the average price increases as the partner ISPs lose more users, but decreases as partner ISPs experience more cross-carrier traffic on their networks.

V. CROSS-CARRIER MVNO OPTIMIZATION

With an understanding of the price charged by partner ISPs to the cross-carrier MVNO, we now consider the remaining link: how much the cross-carrier MVNO charges the end user, and how cross-carrier users change their data usage in response to these prices.

A. User Behavior Based on Price

Users gain utility by consuming more data, so we use the standard α-fairness utility function with $\alpha \in [0,1)$ to model the expected usage utility from consuming y amount of data:

$$V(y) = \frac{y^{1-\alpha}}{1-\alpha}. \tag{7}$$

Each user's expected cellular usage is $\frac{N_\Phi}{N}y$ with $\frac{N_\Phi}{N}$ being the fraction of cellular usage. Under price p, users pay $\frac{N_\Phi}{N}py$ to the cross-carrier MVNO, resulting in the overall utility:

$$\mathbb{E}\left(u(y)\,|\,p\right) = \gamma\frac{y^{1-\alpha}}{1-\alpha} - \frac{N_\Phi}{N}py, \tag{8}$$

where $\gamma > 0$ is a normalization constant. Given price p, we derive users' data usage by maximizing their utilities in (8). Since (8) is concave in y, it can be maximized by solving $\partial\mathbb{E}\left(u(y)\,|\,p\right)/\partial y = 0$. Thus, the expected amount of data that each user consumes at price p is given by

$$y(p) = \left(\frac{N_\Phi}{\gamma N}p\right)^{-1/\alpha}. \tag{9}$$

As expected, users will use more data if charged a lower price (y decreases with p). In Section V-B, we use (9) to find the optimal price p charged by the cross-carrier MVNO.

B. Optimal Cross-carrier MVNO Price

The cross-carrier MVNO's objective in choosing its price is to maximize its profit from selling data as well as an additional utility term that increases with the total amount of data consumed by its users. This additional term, for instance, might represent mobile advertisement revenue, which increases with usage.[6] The cross-carrier MVNO's profit is the income from its users minus the payment to its partner ISPs:

$$W(p,y) = N_\Phi yp - \hat{\pi}^\star N_\Phi y, \tag{10}$$

where $\hat{\pi}^\star$ is given in (6). We use the same α-fairness utility function as in (7) to represent the cross-carrier MVNO's utility from the total amount Ny of data consumed by all users. The cross-carrier MVNO thus calculates its price by solving the following maximization problem:

$$\begin{aligned} \underset{p,y}{\text{maximize}} \quad & \omega W(p,y) + (1-\omega)V(Ny) \\ \text{subject to} \quad & y = \left(\tfrac{N_\Phi}{\gamma N}p\right)^{-1/\alpha}, \end{aligned} \tag{11}$$

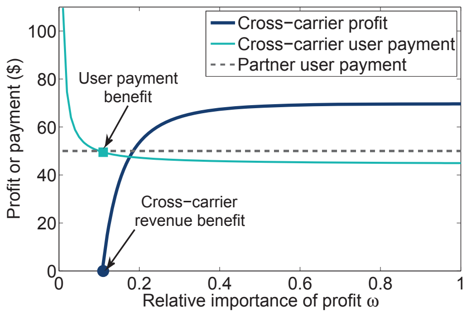

Fig. 2. A numerical example showing that both the cross-carrier MVNO and its users benefit from a larger ω. The blue dot and the green square indicate the conditions in Corollaries 1 and 2 respectively.

where the weight $\omega \in [0,1]$ trades off the data profit with user utility. We denote the optimal price to (11) by $p^\star$.

Proposition 3: The optimal price to (11) satisfies

$$p^\star = \frac{\gamma\omega c}{\gamma\omega(1-\alpha) + (1-\omega)N^{-\alpha}}. \tag{12}$$

We now wish to answer the question of when the cross-carrier MVNO and end users achieve high profit and low data costs, respectively. When the cross-carrier MVNO preferentially weights its profit, i.e., ω is larger, the optimal price in (12) increases, but users may consume less data, as shown in (9). However, as users' demand y decreases, the average unit price $\hat{\pi}^\star$ paid by the cross-carrier MVNO in (6) increases. Thus, for higher ω, the cross-carrier MVNO might increase its profit by selling less data at a higher price. The following is a necessary condition on ω under which the cross-carrier MVNO makes a profit:

Corollary 1: The cross-carrier MVNO can achieve a positive profit from selling data if ω satisfies:

$$\omega \geq \frac{N^{-\alpha}}{(2q)^{\frac{-\alpha}{2\alpha-1}}\gamma^{\frac{2\alpha}{2\alpha-1}}(cN_\Phi)^{\frac{\alpha-1}{2\alpha-1}}N^{\frac{1}{2\alpha-1}} - \gamma(1-\alpha) + N^{-\alpha}}, \tag{13}$$

where $q = (\hat{\eta} - cz)\sum_{k=1}^{K^p}\theta_k^p M_k^p$.

Even if the cross-carrier MVNO makes a positive profit, cross-carrier users may not save money as compared to their former data plans on partner and non-partner ISPs. We derive an analogous condition under which users can save money:

Corollary 2: The cross-carrier MVNO can help its users to reduce their payments compared to their expected payment $\hat{\eta}$ on the other ISPs' data plans if ω satisfies

$$\omega \geq \frac{N^{-\alpha}}{c\frac{N_\Phi}{N}\left(\frac{\hat{\eta}}{\gamma}\right)^{\frac{\alpha}{1-\alpha}} - \gamma(1-\alpha) + N^{-\alpha}}. \tag{14}$$

Figure 2 illustrates a numerical example of the cross-carrier MVNO profit and its users' costs with varying ω. We can observe that user payment decreases with ω, but the cross-carrier profit increases. From (9) and (12), we see that as ω increases, the optimal price $p^\star$ increases, so usage decreases. Users then pay less to the cross-carrier MVNO, but the cross-carrier MVNO also pays less to its partner ISPs, since the partner ISPs incur a lower cost of cross-carrier traffic due to the

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[6] Google, for instance, benefits from additional data for its Internet search engine. More traffic can also lead to higher advertisement revenue for Google and a greater use of their other products such as Gmail and YouTube.

Fig. 3. Locations of cellular BSs and public WiFi APs [20].

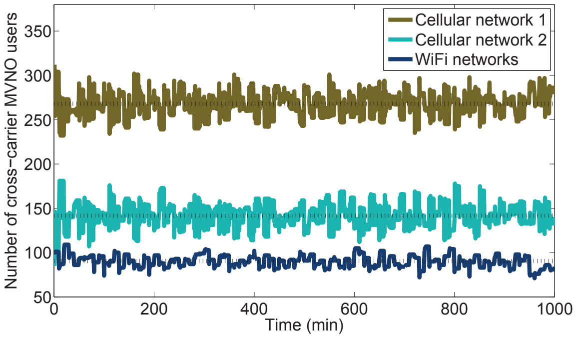


Fig. 4. The number of cross-carrier users on each network remains stable over time if users switch networks according to Table I.

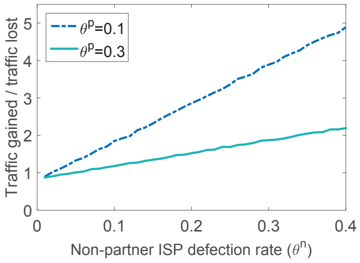
reduction in cross-carrier usage. Since the decrease in payment offsets the cross-carrier MVNO's decrease in revenue, its profit in fact increases with ω, and the conditions in Corollary 1 and 2 can both hold. Thus, *both the cross-carrier MVNO and users can monetarily benefit from the cross-carrier data plan*. In contrast, the cross-carrier MVNO's data utility decreases with ω due to the decreasing usage, leading to less side revenue.

The cross-carrier MVNO can choose ω, large or small, to balance its profit and side revenue. However, when the number of users $N \to \infty$, $p^\star \to \frac{c}{1-\alpha}$, which is independent of ω; thus, selecting ω has less effect for a large number of users.

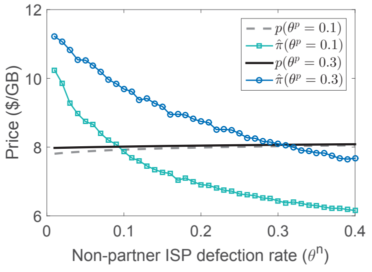
VI. TRACE-BASED SIMULATIONS

In this section, we use real-world data to examine the prices derived in Sections IV and V. By comparing the user price charged by the cross-carrier MVNO, users' expected data consumption, and their final costs on the cross-carrier and partner ISP data plans, we numerically illustrate when the users and cross-carrier MVNO both benefit from the presence of the cross-carrier network service.

Simulation setup. We use data from public, crowd-sourced databases to estimate cellular BS locations, cellular signal strength, and public WiFi locations from a 10 km × 10 km square area in downtown San Francisco [7], [20], [21]. The locations of the cellular BSs and public WiFi APs are shown in Figure 3. To calculate cellular user throughput, we assume that users connect to the nearest cellular BS, and we convert the signal strength to the PHY rate (assuming noise floor -115 dBm) [22], [23] and user throughput using (1). We use (2) to calculate WiFi user throughput, assuming that users have uniformly distributed PHY rates [24].



(a) Increase of partner ISP traffic.



(b) Changes of prices.

Fig. 5. The cross-carrier MVNO's per-GB payment to partner ISPs decreases as more users defect to the cross-carrier plan from non-partner ISPs, but increases as more users defect from partner ISPs. The per-GB price that the cross-carrier MVNO charges users remains stable, as the number of users increases. The cross-carrier MVNO makes a profit when $p > \hat{\pi}$.

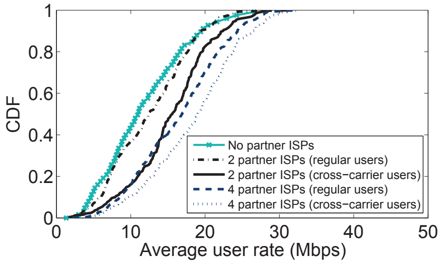
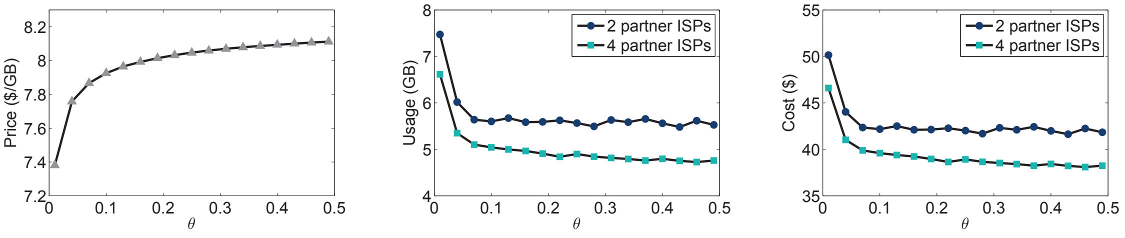


Fig. 6. Both cross-carrier and regular users' average rates increase with the number of partner ISPs.

Each cross-carrier user's device runs the WiFi-prioritized selection introduced in Section III-B every minute. We consider two partner and two non-partner ISPs as our baseline experiment, e.g., T-Mobile and Sprint are partner ISPs of Google, while the other two large U.S. ISPs, Verizon and AT&T, are non-partners. We show in Figure 4 that the user population dynamics of this selection average out to a stable number over time. To assess the impact of random variations, we simulate a mobility model where each user performs a random walk with step size one kilometer every ten minutes, re-associating to a new cellular BS or WiFi AP if necessary.

Parameter setting. With no cross-carrier MVNO, we assume that 1000 and 1200 users subscribe to the two partner and the two non-partner ISPs respectively. The operational cost c incurred at each ISP is \$5/GB, which is half the unit price that Google charges its Google Fi users. After the cross-carrier



(a) Unit price.

(b) Monthly usage.

(c) Total cost.

Fig. 7. As more users from partner and non-partner ISPs join the cross-carrier plan ($\theta^p = \theta^n = \theta$), (a) the users' cross-carrier data price increases slightly, leading to (b) a decrease in their monthly usage. However, (c) users' expenditures on the cross-carrier data plan decrease as more users join the data plan: without the cross-carrier MVNO, users would pay $50 for an expected monthly data usage of 1.8 GB, for a unit price of $27.78/GB.

MVNO enters the market, we assume that all partner ISPs see a rate of θ^p, and that all non-partner ISPs see a defection rate of θ^n. We use a relatively small $\omega = 0.1$, indicating that the cross-carrier MVNO preferentially weights utility instead of profit. We also set $\alpha = 0.5$ and $\gamma = 15$. In addition, users are expected to consume 1.8 GB of data with the partner or non-partner ISPs at an expected monthly cost of $50 [25].

Impact of user defection. We first examine how the prices change as more users defect from non-partner ISPs, versus partner ISPs, to the cross-carrier MVNO (θ^n increases). In Figure 5, we fix θ^p of users from the two partner ISPs and vary θ^n. Figure 5(a) shows the ratio of the cross-carrier traffic on partner ISPs to the traffic lost from users defecting to the cross-carrier MVNO: we see that this ratio increases nearly linearly with the influx of users from non-partner ISPs.

In Figures 5(b), the unit price p increases slightly as θ^n increases (Proposition 3), since there are more cross-carrier users with a higher θ^n. Since new users from non-partner ISPs consume more data on the partner ISPs' networks, partner ISPs do not lose any additional revenue with the same θ^p, leading to a lower per-gigabyte price charged from partner ISPs to the cross-carrier MVNO (Proposition 2). The cross-carrier MVNO then benefits more as more users join its data plan from non-partner ISPs; for sufficiently high θ^n, we have $\hat{\pi} < p$, allowing the cross-carrier MVNO to earn a profit.

User benefit. We now analyze how users would benefit from the ISP competition by comparing the scenarios where just two ISPs or all four ISPs partner with the cross-carrier MVNO. Figure 6 shows the cumulative distribution function (CDF) of users' achievable rates for these scenarios. Having only two partner ISPs, the cross-carrier MVNO users obtain better network performance than the partner ISPs' users, due to WiFi on the cross-carrier MVNO. Even the partner ISPs' regular users benefit compared to the scenario without a cross-carrier MVNO: they no longer compete with cross-carrier traffic, which has been offloaded to WiFi. If all four ISPs become partner ISPs, these effects are exacerbated, and both the cross-carrier and other ISP users obtain even better performance.

In Figure 7, we show that the cross-carrier users reduce their costs; in fact, they consume more data at a lower price compared to the non-MVNO scenario. As the number of cross-carrier users increases in Figure 7(a), the price remains relatively stable, particularly once $\theta = \theta^p = \theta^n > 0.1$ and the cross-carrier plan reaches a critical mass of users. As this price increases, both user usage (Figure 7(b)) and total cost (Figure 7(c)) stabilize. We can therefore conclude that $\omega = 0.1$ is sufficiently high for users to save money (Corollary 2).

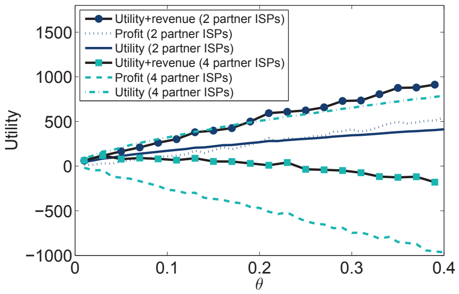


Fig. 8. The cross-carrier MVNO's utility and profit increases with the number of users ($\theta^p = \theta^n = \theta$) when it partners with only two ISPs, but its profit can decrease if it partners with all four ISPs.

Cross-carrier MVNO benefit. Finally, we show that as the number of users increases, the cross-carrier MVNO does not necessarily increase its profit. Figure 8 shows that with only two partner ISPs, the cross-carrier MVNO's utility and profit both increase as more users join the cross-carrier data plan. Thus, ω is sufficiently high for Corollary 1 to hold and the cross-carrier MVNO to earn a profit. However, when all four ISPs partner with the cross-carrier MVNO, its profit decreases with the number of users, as does its total utility. This result contrasts with that in Figure 6, where more partner ISPs resulted in better network service for end users. Thus, the cross-carrier MVNO may wish to restrict either the number of users joining its data plan or number of ISPs that it partners with. Partnering with more ISPs forces the MVNO to make higher payments to its partners, deceasing its profit.

VII. CONCLUSION

A cross-carrier MVNO increases ISP competition in the mobile data market, creating a set of complex interactions among users, partner ISPs, non-partner ISPs, and the cross-carrier MVNO. We model these interactions, first by considering a reasonable network selection strategy for users on the cross-carrier MVNO. This strategy in turn affects the amount of cross-carrier traffic on each partner ISP, and thus the payments of the cross-carrier MVNO to the partner ISPs.

We then analyze the price that the partner ISP charges the cross-carrier MVNO and the price that the cross-carrier MVNO charges its users. We find that if the cross-carrier

MVNO chooses its prices with sufficient consideration for its profit, then both the cross-carrier MVNO and users can benefit (i.e., the cross-carrier MVNO has positive profit, and users can save money on their data plans). Numerical simulations with real network quality data confirm these mutual benefits. However, if too many partner ISP users join the cross-carrier data plan or the cross-carrier MVNO partners with too many ISPs, then its payments to the partner ISPs may become too high, and the cross-carrier MVNO may not be profitable.

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APPENDIX

A. Proof of Lemma 1

Proof: To prove Lemma 1, we first prove that $\hat{\eta} > \eta^d z/d$. Suppose that each user's realized usage for a month is a random variable with distribution f, so we have $z = \int_0^\infty x f(x)\mathrm{d}x$ and the expected monthly payment that a users pays to the ISP is given by $\hat{\eta} = \eta^d + \eta^o \int_d^\infty (x-d) f(x)\mathrm{d}x$. Due to $\frac{\eta^d}{d} < \eta^o$, we have $\hat{\eta} > \left(\eta^d \int_0^d \frac{x}{d} f(x)\mathrm{d}x + \eta^d \int_d^\infty f(x)\mathrm{d}x\right) + \frac{\eta^d}{d}\int_d^\infty (x-d) f(x)\mathrm{d}x = \frac{\eta^d}{d}\int_0^\infty x f(x)\mathrm{d}x = \frac{\eta^d z}{d}$.

Then, if $\theta_k^p M_k^p z / \sum_{i=i_k^p}^{i_{k+1}^p} n_i^p y > 1$, we can derive $\pi_k^\star > \left(\frac{\eta^d}{d} - c\right) \theta_k^p M_k^p z / \sum_{i=i_k^p}^{i_{k+1}^p} n_i^p y + c > \frac{\eta^d}{d}$. Since $p < \eta^d/d$, the proof completes. ■

B. Proof of Proposition 3

Proof: Expanding the objective function in (11) leads to $\omega W(p,y) + (1-\omega) V(Ny) = \omega p N_\Phi y - \omega\left((\hat{\eta} - cz)\sum_{k=1}^{K^p} \theta_k^p M_k^p + c N_\Phi y\right) + (1-\omega)\frac{(Ny)^{1-\alpha}}{1-\alpha}$. By substituting $p = \frac{\gamma N}{N_\Phi} y^{-\alpha}$ into (11) and omitting the constant terms, solving (11) is equivalent to maximizing the concave function: $g(y) = \omega(\gamma N y^{1-\alpha} - c N_\Phi y) + (1-\omega)\frac{(Ny)^{1-\alpha}}{1-\alpha}$. Thus, after taking the first-order derivative of $g(y)$ and setting $\partial g(y)/\partial y = 0$, we have $y^{\star -\alpha} = \frac{\omega c N_\Phi}{\gamma\omega(1-\alpha)N + (1-\omega)N^{1-\alpha}}$. Since $p^\star = \frac{\gamma N}{N_\Phi} y^{\star -\alpha}$, we have also obtained the optimal price. ■

C. Proof of Corollary 1

Proof: By replacing $q = (\hat{\eta} - cz)\sum_{k=1}^{K^p} \theta_k^p M_k^p$, we can rewrite (6) as $\hat{\pi}^\star = \frac{q}{y N_\Phi} + c$. When $p^\star \geq \hat{\pi}^\star$, the income is larger than the payment. This leads to

$$
\begin{aligned}
& \frac{\gamma\omega c}{\gamma\omega(1-\alpha)+(1-\omega)N^{-\alpha}} \geq \frac{q}{y N_\Phi} + c \\
\overset{(a)}{\Rightarrow}\quad & \frac{\alpha - \frac{1-\omega}{\gamma\omega}N^{-\alpha}}{(1-\alpha)+\frac{1-\omega}{\gamma\omega}N^{-\alpha}} \left(\frac{N_\Phi}{\gamma N}\right)^{-\frac{1}{\alpha}} \left(\frac{c}{(1-\alpha)+\frac{1-\omega}{\gamma\omega}N^{-\alpha}}\right)^{-\frac{1}{\alpha}} \geq \frac{q}{c N_\Phi} \\
\Rightarrow\quad & q (c N_\Phi)^{\frac{1}{\alpha}-1} (\gamma N)^{-\frac{1}{\alpha}} \left((1-\alpha) + \frac{1-\omega}{\gamma\omega}N^{-\alpha}\right)^{-\frac{1}{\alpha}+1} \\
& \qquad + \left((1-\alpha) + \frac{1-\omega}{\gamma\omega}N^{-\alpha}\right) \leq 1 \\
\overset{(b)}{\Rightarrow}\quad & \left((1-\alpha) + \frac{1-\omega}{\gamma\omega}N^{-\alpha}\right)^{-\frac{1}{2\alpha}+1} \leq \frac{1}{\left(2q(cN_\Phi)^{\frac{1}{\alpha}-1}(\gamma N)^{-\frac{1}{\alpha}}\right)^{1/2}} \\
\Rightarrow\quad & \omega \geq \frac{N^{-\alpha}}{(2q)^{\frac{-\alpha}{2\alpha-1}} \gamma^{\frac{2\alpha}{2\alpha-1}} (cN_\Phi)^{\frac{\alpha-1}{2\alpha-1}} N^{\frac{1}{2\alpha-1}} - \gamma(1-\alpha) + N^{-\alpha}},
\end{aligned}
$$

where (a) is due to $y^\star = \left(\frac{N_\Phi}{\gamma N} p^\star\right)^{-1/\alpha}$ and (b) is due to the arithmetic and geometric inequality. ■

D. Proof of Corollary 2

Proof: If user payment to the cross-carrier is less than that to their former ISPs, we have $\frac{N_\Phi}{N} p y \leq \hat{\eta}$. By substituting $y = \left(\frac{N_\Phi}{\gamma N} p\right)^{-1/\alpha}$ and $p^\star = \frac{\gamma\omega c}{\gamma\omega(1-\alpha)+(1-\omega)N^{-\alpha}}$, we can obtain $\frac{c}{(1-\alpha)+\frac{1}{\gamma}(\frac{1}{\omega}-1)N^{-\alpha}} \geq \frac{N}{N_\Phi}\gamma^{\frac{1}{1-\alpha}} \hat{\eta}^{\frac{\alpha}{\alpha-1}}$ due to $\frac{\alpha}{\alpha-1} < 0$. This leads to (14) after inequality transformation. ■