An ILP Formulation for Partially Upgrading Elastic Optical Networks to Multi-Band

Soheil Hosseini  
Universidad de Valladolid  
Valladolid, Spain  
0000-0003-3006-5297

Ignacio de Miguel  
Universidad de Valladolid  
Valladolid, Spain  
0000-0002-1084-1159

Ramón J. Durán Barroso  
Universidad de Valladolid  
Valladolid, Spain  
0000-0003-1423-1646

Óscar González de Dios  
Telefónica I+D  
Madrid, Spain  
0000-0002-1898-0807

Noemí Merayo  
Universidad de Valladolid  
Valladolid, Spain  
0000-0002-6920-0778

Juan Carlos Aguado  
Universidad de Valladolid  
Valladolid, Spain  
0000-0002-2495-0313

Edward Echeverry  
Telefónica SA  
Madrid, Spain  
0000-0001-5520-0948

Patricia Fernández  
Universidad de Valladolid  
Valladolid, Spain  
0000-0001-8729-3085

Rubén M. Lorenzo  
Universidad de Valladolid  
Valladolid, Spain  
0000-0003-1423-1646

Evaristo J. Abril  
Universidad de Valladolid  
Valladolid, Spain  
0000-0003-4164-2467

Abstract—We propose an integer linear programming (ILP) formulation to perform a partial upgrade of conventional elastic optical networks to multi-band optical networks in which both the C-band and the L-band are activated. Under the assumption that, in dynamic network operation, the optical connections are established following precomputed paths, the formulation determines which (limited) set of fibers should be upgraded to support multi-band transmission with the objective of maximizing the number of precomputed paths that can benefit from such upgrade. We investigate the number performance of terms of bandwidth blocking ratio as a function of the traffic load and the number of links upgraded from the C-band to the C+L bands. Moving towards other spectral bands provides more spectral resources, which leads to reductions in blocking probability. However, in the case of not having enough financial resources to achieve a fully upgraded network, the selection of fibers for migration must be done efficiently, for which the proposed ILP formulation is useful. The performance of the link upgrades provided by the ILP formulation is compared with a heuristic proposed in a previous work, by analyzing the reduction on the blocking probability obtained.

Keywords—elastic optical networks, multi-band, fiber migration, integer linear programming, network simulation

I. INTRODUCTION

The continuous traffic growth rate over the Internet in the following years will cause the optical fibers to face a capacity crunch [1]. Two solutions have been proposed to increase the capacity of optical fibers and make them well adapted to the emerging bandwidth-hungry applications.

The first solution is space division multiplexing (SDM), which is realized by using novel fiber infrastructures like multi-core fibers (MCFs) or multi-mode fibers (MMF) [2, 3], or by using simpler and mature technologies like multi-fiber transmission (MFT) [4]. However, owing to the scarceness of idle fibers, MFT requires either civil works to increase the number of fibers or leasing dark fibers from an infrastructure operator. As concluded in [5], SDM technology is an excellent solution to increase the capacity of optical networks, but leasing or rolling out new dark fibers, which is required for SDM activation, is a highly costly process for the network operators [6].

The second solution to deal with the capacity increase is band division multiplexing (BDM) [7, 8]. The idea behind this solution is to exploit all the available spectrum from existing fibers. In other words, in BDM technology, the goal is set toward achieving almost 54 THz of bandwidth by extending the already installed fibers beyond the C-band [9]. In this work, we focus on this approach.

As evolving from the C-band to other spectral bands requires the deployment of different components such as amplifiers, upgrading all the fibers in the network would potentially increase the costs significantly. Therefore, in [10], we analyzed the migration of C-band elastic optical networks towards the C+L bands, when upgrading only a subset of links in the network. In that paper, we demonstrated that with a limited upgrade, significant improvements in blocking probability (in a scenario where connections are dynamically established and released over time) could be obtained. For that aim, we studied the trade-off between the decrease on blocking probability and the number of upgraded fibers. For instance, we showed that in the well-known NSFNet topology, upgrading 57% of the links from the C to the C+L bands led to a reduction of the blocking probability higher than one order of magnitude for low and medium traffic loads, and that upgrade enables to transport around twice the amount of traffic than a C-band network while guaranteeing a bandwidth blocking ratio lower than 10^{-3}.

In [10], in order to determine which links should be upgraded from the C to the C+L-bands, we proposed a simple heuristic. For each source-destination pair of nodes in the network, the shortest path in terms of hops was computed. Then, the optical fibers were prioritized for band upgrade depending on the number of times that they were used in those shortest paths (under the condition that if the fiber between nodes i and j was upgraded, the fiber between j and i should also be upgraded). However, since the main objective in [10] was to analyze the potential advantages of a partial upgrade of the network, the
quality of the proposed heuristic was not analyzed in detail nor compared with other options.

Therefore, in this paper, we extend that work by proposing an integer linear programming (ILP) formulation which determines the set of fibers that should be upgraded to the C+L-band line system. As in most of previous works, the spectrum continuity constraint is imposed and, therefore, each optical connection (or lightpath) should use the same slice of the spectrum in all the fibers of the path from the source to the destination node. Hence, an optical connection can be established using L-band spectral resources only if all the fibers traversed by that connection have been upgraded. Therefore, the aim of the formulation is to determine which fibers should be upgraded so that the number of precomputed paths (used by the connections) that can benefit from the upgrade is maximized. However, maximizing the number of paths that can benefit from the upgrade (which is the metric that optimizes the formulation) does not necessarily translate into a minimization of the blocking probability. Therefore, the ILP formulation should be considered as an alternative (although more informed and better) heuristic for network migration, and not as an optimal method in terms of blocking probability reduction. Note that we are trying to get good dynamic operation by means of a good network planning. Thus, the performance in terms of blocking probability of the migration proposal obtained when solving the ILP formulation, and when using the heuristic proposed in [10], will be compared using three different network topologies, the North-American NSFNet [11], the Japanese JPNI2 [12], and the European Deutsche Telekom (DT) network [13].

The rest of this paper is organized as follows. In Section II, we propose the ILP formulation for partial network upgrade. Then, Section III discusses the simulation setup and the performance evaluation. Finally, the work ends by providing a conclusion in Section IV.

II. ILP FORMULATION FOR NETWORK UPGRADE

In this section, an integer linear programming (ILP) formulation to determine the set of optical fibers that should be upgraded (i.e., equipped with multi-band devices) in an optical network is proposed. The formulation takes as inputs the network topology, the maximum number of fibers that the network operator desires to upgrade, and a set of precomputed paths for each source-destination (s-d) pair of nodes in the network (which will be used for establishing dynamic optical connections when operating the network). The objective of the formulation is to determine which links should be upgraded with the aim of maximizing the number of those precomputed s-d paths that can benefit from the upgrade. Note that this approach is equivalent to minimizing the number of precomputed s-d paths that cannot benefit from the upgrade.

A. Inputs for the ILP formulation

The first input for the ILP formulation is the network topology to be partially upgraded, which is represented by a connected graph \( G = (V,E) \), where \( V \) denotes the set of nodes and \( E \) the set of bidirectional links. Then, the \( K \) shortest paths are precomputed between each source-destination (s-d) pair of nodes in the network. These \( K \) shortest paths will be eventually used for establishing end-to-end optical connections, so we focus our attention on them. In order to represent these precomputed paths, the binary variables \( r_{ij}^{sd} \) are introduced. If the precomputed path \( k \) from node \( s \) to \( d \) traverses fiber \((i,j)\), then \( r_{ij}^{sd} \) is set to 1, otherwise, it will be 0.

Moreover, let \( F \) denote the maximum number of unidirectional fibers that the network operator wants to upgrade by equipping them with multi-band devices. Finally, we also introduce a big constant in the formulation, \( U \), which represents an upper bound on the length of the network paths, i.e., a number at least equal to the number of unidirectional fibers in the topology (or higher), and \( M \), a very small constant. Additionally, a set of constants \((\alpha_k)\) are introduced as positive weighting factors in the objective function, but their meaning will be explained later, when the objective function is described.

B. Decision Variables

The output of the ILP formulation, i.e., the decision variables are as follows:

- \( f_{ij} \) are the main variables to be found, as they indicate whether a fiber should be upgraded or not by equipping it with multi-band devices. Thus, \( f_{ij} \) is a binary variable which will be 1 if the fiber \((i,j)\) is equipped with multiband devices; otherwise, it will be 0.
- \( \Delta_{sd} \) is an auxiliary variable which represents the number of fibers in the precomputed path \( k \) between nodes \( s \) and \( d \) which have not been upgraded. Thus, it is an integer number (0 or positive). The value 0 means that all fibers in the precomputed path \( k \) between nodes \((s,d)\) have been upgraded, and therefore the end-to-end path (and thus connections using that path) can benefit from L-band resources. In contrast, a higher number means that end-to-end connections using that path cannot benefit from the L-band (since at least one of the fibers has not been upgraded) and thus must employ the C-band (to comply with the spectrum continuity constraint).
- \( \delta_{sd} \) is another auxiliary variable. It is a clipped version of \( \Delta_{sd} \) becoming a binary variable. The value 0 means (as before) that all fibers in precomputed path \( k \) between nodes \((s,d)\) have been upgraded with multi-band devices, and 1 otherwise. If it is 0, the precomputed path can benefit from the upgrade. Otherwise, it cannot.

C. ILP Formulation

The ILP formulation to determine the set of fibers that should be upgraded is as follows:

Minimize

\[
\sum_{sd} \alpha_k \delta_{sd} - M \cdot \sum_{ij} w_{ij} f_{ij}
\]
Subject to:

\[ \sum_{ij} f_{ij} \leq F \]  

\[ f_{ij} = f_{ji}, \quad \forall i \in V, \forall j \in V \]  

\[ \Delta^{sdk} = \sum_{ij} r_{ij}^{sk} - \sum_{ij} r_{ij}^{sk} f_{ij}, \quad \forall s, d \in V, \forall k \in K \]  

\[ \Delta^{skd} \leq U^{skd}, \quad \forall s \in V, \forall d \in V, \forall k \in K \]  

\[ f_{ij} \in \{0, 1\} \]  

\[ \delta^{skd} \in \{0, 1\} \]  

We will start by explaining the constraints, and finally the objective function. Equation (3) guarantees that the number of fibers to be upgraded does not exceed the maximum number of fibers that the network operator wants to equip with multi-band devices. Equation (4) forces the simultaneous upgrade of the fibers composing a link. That is, whenever the fiber \((i, j)\) is upgraded to C+L bands, the fiber \((j, i)\) must also be upgraded. Then, Equation (5) defines the auxiliary variable \(\Delta^{sdk}\), which computes the number of fibers that are not equipped with multiband devices over the precomputed path \(k\) between nodes \(s\) and \(d\). Note that the first term in the right-hand side of that equation is the length in hops of the path \(k\) between \((s, d)\). The second term is the number of fibers of that path that have been upgraded, so the difference (i.e., the value of \(\Delta^{sdk}\)) is the number of fibers of the path which have not been upgraded. Equation (6) determines the value of \(\delta^{skd}\) (a clipped, binary version of \(\Delta^{skd}\)). Since \(U\) is a big positive constant, note that if \(\Delta^{skd}\) is higher than 0, constraint (6) forces the binary variable \(\delta^{skd}\) to be set to 1. If \(\Delta^{skd} = 0\), \(\delta^{skd}\) could be either 0 or 1. However, when considering how the objective function is set in Equation (2), the value 0 will be preferred, as it leads to minimizing the objective function. In sum, \(\delta^{skd}\) works as a clipped (binary) version of \(\Delta^{skd}\). Finally, constraints (7) and (8) set the binary nature of \(f_{ij}\) and \(\delta^{skd}\) variables.

The objective function is defined by Equation (2). The aim is to minimize the number of precomputed paths that cannot benefit from the upgrade in the links. As we have just described, \(\delta^{skd}\) is 1 when path \(k\) between nodes \(s\) and \(d\) cannot benefit from the upgrade (and 0 otherwise), so adding these variables constitute the objective function. The \(\alpha_k\) in the path that are weighting factors (constants) set to values between 0 and 1, to model the relevance of the primary paths \((k = 1)\), secondary paths \((k = 2)\) and so on, when determining the set of fibers to be upgraded. When the network is operated dynamically, a usual strategy consists in using the first precomputed path \((k = 1)\) between nodes \(s\) and \(d\) for establishing a connection between those nodes if possible, and only resort to higher order paths if there are no resources on the first path. Therefore, it seems reasonable to set a higher weight \(\alpha_k\) for \(k = 1\) than for higher order paths. The second term of the objective function is introduced in order to break ties following the spirit of the heuristic in [10], i.e., in case of ties, updating fibers with higher values of \(w_{ij}\) is preferred. As the aim is to break ties, \(M\) is a very small constant (e.g., set to \(10^{-6}\) in the following tests).

As previously mentioned, it is very important to note that this ILP formulation finds the optimal set of links to be upgraded with the aim of maximizing the number of precomputed paths that fully benefit from the upgrade. Nevertheless, that does not necessarily mean that the blocking probability will be minimized when operating the network dynamically and using those paths for establishing the optical connections. The performance of the upgraded network in a dynamic scenario will be studied in the following section.

### III. Simulation Setup and Performance Evaluation

In order to evaluate the performance of the ILP formulation to determine the set of links to upgrade, and to compare with the heuristic proposed in [10], we have considered three different network topologies of similar size, the 14-node NSFNet [11], the 12-node JPN12 [12], and the 14-node Deutsche Telecom (DT) topology [13]. The characteristics of the analyzed topologies can be found in Table I.

<table>
<thead>
<tr>
<th>Topology</th>
<th>Number of Nodes</th>
<th>Number of Links</th>
</tr>
</thead>
<tbody>
<tr>
<td>NSFNet</td>
<td>14</td>
<td>21</td>
</tr>
<tr>
<td>JPN12</td>
<td>12</td>
<td>17</td>
</tr>
<tr>
<td>DT</td>
<td>14</td>
<td>23</td>
</tr>
</tbody>
</table>

We have assumed that the available spectrum in these networks is divided into 12.5 GHz slots, and that for every migrated fiber, a guardband of 400 GHz must be allocated between the C-band and the L-band. In particular, we have assumed those 400 GHz to be located at the beginning of the L-band. In this way, the C-band consists of 320 frequency slots and the L-band consists of 516 frequency slots. Moreover, three different modulation formats have been considered: BPSK, QPSK and 16QAM.

We have solved the ILP formulation using IBM ILOG CPLEX, in order to determine the fibers to upgrade in different scenarios, from 3 bidirectional links \((F = 6)\) to 18 bidirectional links \((F = 36)\), for each of the topologies (except for JPN12, where a maximum of 15 links have been upgraded). When solving the ILP formulation, only the shortest path between each source-destination pair has been considered, i.e., \(K = 1\) (or equivalently, \(\alpha_1 = 1\) and \(\alpha_k = 0\) for \(k > 1\)). This approach is consistent with the heuristic proposed in [10], which only considers the shortest path when determining the set of fibers to upgrade (i.e., \(K = 1\)). Despite being an ILP formulation, it can be solved very quickly in these scenarios. Each instance of the formulation has been solved in less than 1 minute in a laptop with an Intel(R) Core(TM) i7-4720HQ CPU processor, 2.60 GHz, and 16 GB RAM.

In order to analyze the performance of the different upgrades under dynamic traffic, we have considered the following additional assumptions. The arrival of connection requests is modeled as a Poisson process with arrival rate \(\lambda\). The selection of the source and the destination node of every connection request is done based on a uniform distribution. Moreover, the service time (or duration) of each established connection follows an exponential distribution with average \(T\). The requested traffic rate for the connections is randomly selected (according to a uniform distribution) in the range of \(C_{\text{min}} = 12.5\ \text{Gb/s}\) to \(C_{\text{max}} = 312.5\ \text{Gb/s}\) in steps of 12.5 Gb/s. This translates into a demand
of 1 to 25 frequency slots if the BPSK modulation format is used for the subcarriers. Nevertheless, a more spectrally efficient modulation format (QPSK or even 16QAM) is used if the route employed for the connection does not exceed the maximum optical reach for that modulation format considering the selected spectral band (Table II).

<table>
<thead>
<tr>
<th>Modulation Level</th>
<th>Multi-band Optical Reach (km)</th>
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<tbody>
<tr>
<td></td>
<td>C-band</td>
</tr>
<tr>
<td>QPSK</td>
<td>1800</td>
</tr>
<tr>
<td>16QAM</td>
<td>370</td>
</tr>
</tbody>
</table>

The traffic load is defined as in [14], which normalizes the classical definition in Erlangs ($\lambda T$), by considering the average data rate ($C_{avg}$), the maximum data rate of the connections ($C_{max}$), and the number of nodes in the network, $N$. In particular, the traffic load is calculated by equation (8),

$$ Load = \frac{\lambda T}{N(N-1)} \times \frac{C_{avg}}{C_{max}} $$

(8)

where $C_{avg} = (C_{min} + C_{max})/2$.

For each connection request, to determine how to establish that connection, the routing, band, modulation level, and spectrum assignment (RBMLSA) hop-based algorithm proposed in [10] is used. As in [10], the $K$-shortest paths strategy (with $K = 3$) is used for routing. Regarding band selection, if all the fibers of the selected path for the connection have been upgraded, the L-band is prioritized over the C-band. Otherwise, due to the spectrum continuity constraint, the connection is restricted to use the C-band. Then, the most spectrally efficient modulation format complying with the maximum optical reach is selected, and the Best-Fit strategy [14] is used for spectrum assignment. Additional details on the RBMLSA hop-based algorithm can be found in [10].

A simulator has been implemented in Python and a total of $10^5$ connection requests have been generated (after warming up the network with $10^4$ initial connection requests). The performance in terms of bandwidth blocking ratio has been then analyzed. Thus, the bandwidth blocking ratio versus traffic load is depicted in Fig. 1 for the three topologies, NSFNet (Fig. 1.a), JPN12 (Fig. 1.b), and DT network (Fig. 1.c). In these figures, different colors represent different numbers of upgraded links. For each color, the continuous lines with filled circles represent the results obtained with the upgrade provided by the ILP formulation, and the dashed lines and hollow squares represent the results with the upgrade provided by the heuristic in [10].

As shown in these figures, the dynamic performance when upgrading the network according to the solution provided by the ILP formulation generally outperforms (or gets equal results) than the solution provided when using the heuristic in [10], i.e., lower or similar blocking probabilities are usually obtained.

When just a few bidirectional links (3) or many links (18 for NSFNet and DT, or 15 in the case of JPN12) are upgraded, the dynamic performance of both approaches is very similar. It is for an intermediate number of upgraded links when the difference between the two methods is more significant.

Fig. 1. Bandwidth blocking ratio depending on the network traffic load in (a) NSFNet, (b) JPN12, and (c) DT-network topologies.
The type of topology also has an impact on the results. For the NSFNet topology (Fig. 1.a), the ILP formulation provides better or at least equal results than the heuristic in [10] for all the upgrade scenarios. The JPN12 topology is the one that shows the major difference between the two methods, with a remarkable decrease on blocking probability (around one order of magnitude for a traffic load of 0.8) when 6 or 9 bidirectional links are upgraded following the ILP solution compared with the heuristic in [10]. Finally, for the DT-network topology, the results obtained with both techniques are similar.

In summary, a total of 17 upgrade scenarios have been analyzed (6 for NSFNet, 5 for JPN12, and 6 for the DT network). The configuration provided by the ILP formulation leads to better dynamic performance in 8 of those cases and provides similar performance in other 8, while the configuration provided by the heuristic in [10] is better in only 1 of the cases. The ILP formulation generally provides better results as it maximizes the number of precomputed end-to-end paths that can benefit from the upgrade, i.e., it takes into account that due to the spectrum continuity constraint all links in a path should be upgraded so that a connection following that path can use the L-band. In contrast, the heuristic in [10] does not take this issue into account and simply prioritizes the upgrades in those individual fibers used by a higher number of precomputed paths. Nevertheless, the simple heuristic proposed in [10] shows comparable performance in many cases, so being also a valuable technique. The heuristic is obviously quicker than the ILP formulation, although the ILP formulation can also be solved in a short time, in less than 1 minute in the analyzed scenarios, which is insignificant for a planning (offline) method.

IV. CONCLUSION

In [10], we demonstrated that a partial upgrade of the network towards the use of C+L bands can bring significant improvements in blocking probability (and thus in increasing the supported traffic load) without the need of fully upgrading the whole network, and allowing the operator to perform gradual upgrades. In this paper, we have presented a novel integer linear programming (ILP) formulation which determines the set of fibers that should be upgraded to support both C and L spectral bands, so that the number of precomputed paths that fully benefit from the upgrade (and which will be used when establishing optical connections) is maximized. However, it should be noted that that strategy does not necessarily imply that the blocking probability will be minimized when operating the network dynamically.

This ILP approach has been compared with the heuristic proposed in [10] to determine which fibers to upgrade. We have demonstrated that the ILP formulation generally leads to better (or at least similar) results in terms of blocking ratio when the upgraded network is operated dynamically, and in some cases the improvement is very significant. For instance, for a 6-link upgrade of the JPN12, the network can support up to 60% more traffic when using the upgrade provided by the ILP formulation instead of using the solution provided by the heuristic in [10], while ensuring a blocking probability lower than 10^{-3}. Although solving the ILP formulation is slower than using the heuristic proposed in [10], the solution can be obtained in less than one minute in a regular laptop in all the scenarios analyzed (which include topologies with 12-14 nodes, and 17-23 links, where between 3 and 18 of those links are upgraded).

Future work will include analyzing the performance in bigger topologies and studying different configuration options (like considering more than one shortest path, i.e., \( K > 1 \), in the formulation, and the impact of different weighting strategies, i.e., different values for \( \delta_k \) in those cases). On the other hand, we also plan to address migration strategies which not only consider the impact of the upgrade in optical fibers but also on switching nodes, as well as the use of multifiber links.

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