Centrality-Aware Machine Learning for Water Network Pressure Prediction

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Abstract—Water Distribution Networks (WDNs) frequently face operational challenges due to leakage and inefficient resource management. The inherent nonlinearities of these systems further complicate the accurate prediction of pressures and leakages. We propose a novel methodology that employs established Deep Neural Networks (DNNs) architectures, specifically Multilayer Perceptrons (MLPs) and Convolutional Neural Networks (CNNs), combined with a minimal set of carefully selected features. Our core innovation lies in the feature selection strategy rather than the models themselves: standard hydraulic parameters, such as node demand, are combined with node centrality metrics: degree, betweenness, and closeness, that quantify a node's structural importance and influence within the network. This approach provides an implicit structural representation of the WDN without requiring direct topological coordinates, allowing the DNN to remain unchanged and adaptable to network topology variations. We validate the proposed approach using simulationbased datasets generated via EPANET and Water Network Tool for Resilience (WNTR) on the Fossolo, Hanoi, Modena, and Net3 networks. Comparative analysis of MLP and CNN performance across these networks and different demand patterns demonstrates high prediction accuracy (up to $R^2 = 0.9999$). These results highlight the effectiveness of our novel feature-selection strategy and suggest potential for future work exploring the role of centrality in Machine Learning (ML)-based predictions.

Index Terms—Water distribution networks, Machine learning, Deep neural networks, Hydraulic modeling, Pressure prediction, Network centrality

I. INTRODUCTION

Water Distribution Networks (WDNs) are critical infrastructure systems that supply clean water to residential, commercial, and industrial sectors. However, these systems face significant operational challenges due to leaks, aging infrastructure, and fluctuating demand [1]. Global water losses are estimated to exceed 126 billion cubic meters annually, with a financial impact exceeding \$39 billion [2].

Pressure variability is a key factor contributing to failures in WDNs [3]. Effective monitoring and modeling of pressure states are essential for diagnosing network failures and identifying the underlying causes of pressure transients [3], [4]. Accurate pressure prediction can improve system reliability, minimize losses, and support proactive network management [5], [6]. Machine Learning (ML) methods have shown promise in water network analysis [7]. Xing et al. [8] proposed a Graph Neural Network (GNN)-based model to estimate pressure states using sensor data, while Kerimov et al. [9] examined the transferability of GNNs across different net-

work topologies, comparing their performance with traditional models such as Multilayer Perceptrons (MLPs). Deep learning methods, including Recurrent Neural Networks (RNNs) and Convolutional Neural Networks (CNNs), have also shown promise in hydraulic modeling [10], [11]. Existing approaches often rely on detailed hydraulic data, which typically necessitates the deployment of sensors within the network. However, such data may not always be available or reliable, presenting significant challenges to their implementation [12]. Furthermore, no prior work has systematically evaluated the impact of different demand patterns on model performance or compared the predictive capabilities of MLPs and CNNs under varying topological conditions and data availability.

In this paper, we address these gaps by proposing a novel deep learning-based approach for pressure prediction in WDNs that combines standard hydraulic parameters with Topological Metrics (TMs) to enhance predictive accuracy and robustness. The key innovation lies in the feature selection strategy rather than the model architecture itself, with a specific focus on *node centrality information*. Node centrality quantifies the structural importance of a node within a network based on its connectivity and positional influence, measuring its contribution to network cohesion, shortest-path routing, and flow dynamics. Simply put, centrality reflects how "important" or "well-connected" a node is within the network, indicating its influence on overall flow and communication.

We evaluate node centrality using three standard estimators: degree, betweenness, and closeness. Degree centrality measures the number of direct connections a node has; betweenness centrality reflects how often a node lies on the shortest paths between other nodes; and closeness centrality measures how easily a node can reach all other nodes in the network.

Our approach leverages a minimal yet effective set of input features, combining node demand patterns with these topological metrics. This enables an implicit structural representation without requiring direct positional information, improving the model's adaptability to topological changes.

The proposed workflow involves (i) selecting the WDN topology, (ii) generating datasets through simulation, (iii) evaluating node centrality, (iv) identifying an optimal feature set for training pressure-prediction models, and (v) evaluating and comparing the performance of MLPs and CNNs across varying demand patterns and topological scenarios. We validate the approach using four well-known benchmark networks (Fossolo,

Hanoi, Net3, and Modena), which differ in complexity and size, providing a diverse testing environment. Demand patterns were simulated over periods of 300 days, capturing variations from typical daily cycles to stochastic week-long patterns.

Experimental results demonstrate that the proposed method achieves high predictive accuracy (up to $R^2=0.9999$) and remains effective across different demand and centrality patterns. Comparative analysis highlights the advantages of MLPs over CNNs under limited topological information, underscoring the effectiveness of the streamlined feature-selection strategy. The key contributions of this work are:

- A novel feature selection strategy combining hydraulic parameters and centrality metrics for pressure prediction in WDNs.
- A deep learning-based methodology that is inherently robust to topological variations.
- A comparative analysis of MLPs and CNNs under varying topological information and demand patterns.
- A feature analysis of the dataset used for pressure prediction, validated using feature importance scores from the Gradient Boosting Regressor.

II. RELATED WORK

In recent years, the application of ML in pressure prediction in WDNs has attracted increasing interest in the research community [7]. Several studies have addressed the challenge of modelling and predicting pressure variations in Water Distribution System (WDS) nodes, using ML techniques to improve the accuracy of simulations and the effectiveness of water network management [7], [13]. Supervised learning approaches using Support Vector Machine (SVM) [14] and Deep Neural Networks (DNNs) [15] have been applied to detect pressure abnormalities. More recent studies have explored deep learning techniques, including CNNs [11] and RNNs [10], to improve leak detection accuracy.

A significant example is Ashraf et al. [16], who developed a deep learning model using GNNs to overcome the limitations of traditional hydraulic simulators. Their approach exploits hydraulic laws to make state estimates based on minimal data, demonstrating superior effectiveness compared to conventional methods, especially in large networks. In another study [17], the authors address the challenge of forecasting multivariate geo-sensor time series, focusing on the complex spatial and temporal correlations present in WDSs. They propose a hybrid analysis method that integrates both spatial and temporal correlations. Experimental results demonstrate that the proposed approach outperforms existing methods in terms of accuracy for predicting flow and pressure within WDSs.

Techniques such as Gradient Correction Gated Recurrent Unit (gcGRU) and image-based deep learning have been used in studies for pressure prediction [18], [19]. For example, Kan et al. [18] combined several innovative techniques to reduce noise in the raw data and used signals extracted from the pump state by advanced techniques such as convolutional kernel

and max-pooling. This combined approach allows the gcGRU model to make accurate pressure predictions in megacities.

In addition, Zhang et al. [20] proposed a real-time pipe burst detection method for water networks using a prediction-classification approach. A Support Vector Regressor (SVR) model predicts pressure values, and a decision tree identifies bursts by analyzing multi-time-step data with statistical control rules. The method combines multiple features (absolute pressure, prediction deviation, pressure variation) to achieve a detection accuracy of 99.56%, with prediction deviation being the most significant factor. By using data from consecutive moments and dynamic thresholds, it improves burst detection within 15 minutes.

These works demonstrate how integrating ML into pressure management at WDS nodes can not only improve leakage detection, but also optimize the planning and operation of water infrastructure, paving the way for more sustainable and resilient networks. However, none of the models available in the literature employs an approach similar to ours, where models trained on different water demand patterns are compared. Furthermore, node centrality information is leveraged to achieve more reliable and robust pressure forecasts.

III. METHODOLOGY

In this section, we provide a detailed description of the methodology derived from our approach. The workflow enables a comprehensive analysis of WDNs by integrating components such as network infrastructure, topology, feature analysis, and pressure prediction algorithms. Specifically, the complete workflow includes the following steps:

- Select WDN topology;
- Dataset generation with the Water Network Tool for Resilience (WNTR)/EPANET simulation tool;
- Centrality evaluation of WDN nodes;
- Analysis and choice of features for training pressureprediction ML models;
- ML models evaluation and comparison.

A. Water Distribution Network Topologies

Table I describes four WDNs used for simulations, specifying the network name, the number of nodes, pipes, and the bibliographic reference. The networks vary in complexity: Hanoi (32 nodes, 34 pipes), Fossolo (37 nodes, 58 pipes), Net 3 (97 nodes, 119 pipes), and Modena (272 nodes, 317 pipes), offering a wide range of characteristics for comparative analysis.

TABLE I: Characteristics of water distribution systems

Network name	Node number	Pipe number	Reference
Hanoi	32	34	[21]
Fossolo	37	58	[22]
Net 3	97	119	[23]
Modena	272	317	[22]

B. Dataset Generation and Simulation Scenarios

We produce hydraulic datasets using the WNTR simulator [24]. WNTR is built on EPANET (US Environmental Protection Agency Water NETwork), an open-source program that models the hydrological and quality dynamics of a WDN. It considers the pipeline system's topological structure, as well as a set of initial conditions (e.g., pipe diameter) and operating regulations, to calculate flows and pressures throughout the network during a given time period.

Simulation datasets were produced for the four network topologies described in the previous subsection. For selected nodes within each network, demand patterns reflecting distinct consumption behaviors were applied:

- Pattern-24h: Represents a typical daily consumption cycle.
- Pattern-240h: Captures a ten-day alternating demand pattern.
- Pattern-random-01: Models a week-long demand pattern with stochastic variations characterized by a standard deviation of 10% around the average demand value.

These demand scenarios were simulated over periods of 300 days, generating hourly snapshots of network states used for training and validation. The dataset generated through EPANET included five main features associated with each node: Hour, Node ID, Base demand, Demand, and Pressure. Subsequently, three additional features were integrated into this dataset, derived from node centrality, as described in the following subsection. These additional features are intended to capture the structural properties of the network that are useful to improve the predictive capabilities of the model.

C. Centrality Measures

We evaluate the centrality of WDNs nodes using three centrality estimators, namely degree, betweenness, and closeness.

The standard *degree* centrality is given by the number of links (arcs) a node has with other nodes:

$$C_D(v) = \deg(v) \tag{1}$$

where $C_D(v)$ is the degree centrality of node v, and $\deg(v)$ is the degree of node v, i.e., the number of edges incident to v.

The measure of *betweenness* centrality sums [25] the fraction of all-pairs shortest paths that pass through a node v, regardless of their length, as shown in Formula 2:

$$C_B(v) = \sum_{\substack{s,t \in V \\ s \neq t}} \frac{\sigma(s,t \mid v)}{\sigma(s,t)}$$
 (2)

where V is the set of nodes, $\sigma(s,t)$ is the number of shortest (s,t)-paths, and $\sigma(s,t\mid v)$ is the number of those paths passing through node v other than s or t. If s=t, $\sigma(s,t)=1$, and if $v\in\{s,t\}$, $\sigma(s,t\mid v)=0$.

Closeness centrality [26] of a node v is the reciprocal of the average shortest path distance to v over all n-1 reachable nodes, computed as follows:

$$C_C(v) = \frac{n-1}{\sum_{u \in V \setminus \{v\}} d(u, v)}$$
(3)

where d(u,v) is the shortest-path distance between u and v, and n-1 is the number of nodes reachable from v. The closeness distance function computes the incoming distance to v for directed graphs, with higher values indicating greater centrality.

D. WDN Features Analysis

To validate the proposed approach, which integrates hydraulic data from EPANET simulations and centrality measures derived from network topology into a unified training dataset for pressure prediction, a preliminary feature analysis was performed. The importance of each feature was evaluated using the feature importance score calculated using gradient-boosting regression, as discussed in the following section on results.

E. Machine Learning Models for Pressure Prediction

This study employs two ML models for pressure prediction in WDNs: a MLP and a one-dimensional CNN, both trained using mean squared error loss.

- Multi-Layer Perceptron (MLP): A feedforward architecture with an input layer of 6 features, followed by four hidden layers with 128, 128, 64, and 32 neurons, respectively. Each layer uses ReLU activation and dropout (p=0.1). The output layer consists of a single neuron with sigmoid activation.
- 1D Convolutional Neural Network (CNN): A convolutional architecture with two layers using 64 and 32 filters, respectively, with kernel size 3 and 'same' padding. Each convolutional layer is followed by LeakyReLU activation (slope 0.01) and dropout (p=0.2). The flattened output is passed through a dense layer with 128 units, followed by dropout and a sigmoid-activated output neuron.

F. Evaluation metrics and comparison

Finally, each model was tested on 4 different water distribution networks, each of which featured 3 different water demand patterns, for a total of 12 possible combinations. The accuracy of the models in predicting pressure was measured using the coefficient of determination (\mathbb{R}^2) and the calculation of the Root Mean Square Error (RMSE).

IV. EXPERIMENTAL RESULTS

For the four selected water networks, simulations with EPANET were performed to extract the hydraulic features, while centrality measures were calculated from their topology, as described in subsection III-C. Specifically, Figure 1 illustrates the betweenness centrality of the nodes for WDN of Hanoi, Fossolo, Net 3, and Modena. The centrality of the

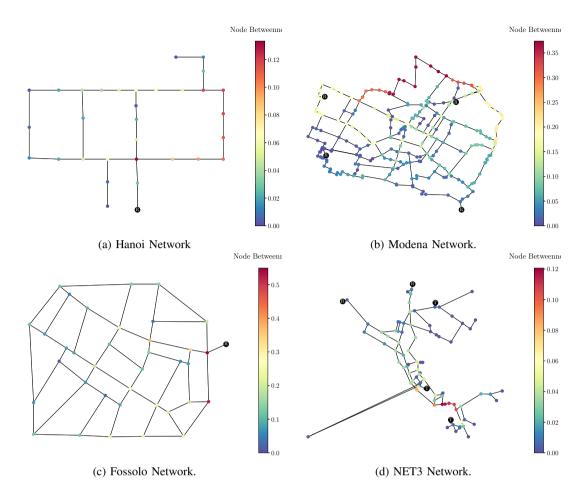


Fig. 1: Topologies of four distinct water distribution networks, including pipes, nodes and their betweenness centrality. Black nodes indicated as "R" or "T" stands for Reservoirs and Tanks respectively, and are excluded from the datasets.

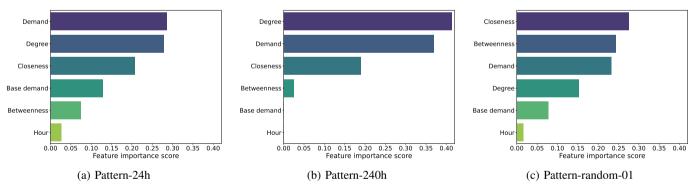


Fig. 2: Feature importance scores for pressure prediction in Modena city's WDN under different water demand patterns.

nodes is represented by their color, based on the colormap values, as measured by the betweenness algorithm. Following the same approach, we calculated closeness centrality and degree centrality, which are omitted here for brevity but are available in the GitHub repository¹. In addition, a feature analysis of the dataset used for pressure prediction was performed using the feature importance scores calculated with the gradient boosting

¹https://github.com/WITS-Restart/WDN-IoT-Dataset-Workbench/tree/main/dataset-network-out

TABLE II: CNN Pressure Prediction Performance on Various WDNs Over 300 Days

Network	Pattern	R^2	RMSE
Fossolo	24h	0.99083	0.01654
	240h	0.99893	0.00801
	random-01	0.99872	0.00711
Hanoi	24h	0.95284	0.05229
	240h	0.98475	0.01963
	random-01	0.94556	0.03225
Modena	24h	0.51215	0.09776
	240h	0.99277	0.01855
	random-01	0.76092	0.06578
Net 3	24h	0.91954	0.05082
	240h	0.98210	0.03022
	random-01	0.96196	0.03642

regression. Figure 2 illustrates the feature importance scores for predicting pressure in Modena city's WDN under three distinct water demand patterns. For Pattern-random-01, the most influential features were Closeness (0.276), Betweenness (0.244), and Demand (0.233), with Degree (0.153), Base demand (0.078), and Hour (0.016) contributing less significantly. Under Pattern-240h, Degree (0.414) and Demand (0.370) dominated the prediction, followed by Closeness (0.190) and Betweenness (0.026). In Pattern-24h, the leading predictors were Demand (0.286), Degree (0.278), and Closeness (0.207). The analysis shows that the importance of features related to WDN centrality play a significant role in predicting pressure in WDNs.

Tables II and III summarize the performance of the CNN and MLP models for pressure prediction across different networks and demand patterns, with all patterns observed over a 300-day period. The CNN model exhibits excellent performance on Fossolo and Net 3, with $R^2 > 0.99$ and a very low RMSE, while the Modena network shows lower accuracy, particularly with the 24h pattern ($R^2 = 0.51215$). Overall, the 240h pattern improves prediction accuracy compared to 24h, suggesting that longer temporal sequences allow the model to better capture pressure dynamics. Indeed, for the MLP model, 240h pattern consistently achieves near-perfect predictions across all networks, with R^2 values above 0.98 and minimal RMSE. In contrast, the 24h pattern shows variability, performing well for Hanoi ($R^2 = 0.992$) but significantly worse for Modena ($R^2 = 0.654$), suggesting sensitivity to shorter time windows. The random-01 pattern shows mixed results, with Hanoi retaining high performance ($R^2 = 0.953$), while Fossolo and Modena demonstrate lower predictive accuracy, as reflected in higher RMSE values. Overall, the MLP models performed better than the CNNs, however, the variability in performance across networks indicates that hydraulic configuration and demand profile significantly affect the effectiveness of predictions, highlighting the importance of adapting the model to the specific characteristics of each water distribution system.

Figure 3 shows at a glance the meaning of the coefficient

TABLE III: MLP Pressure Prediction Performance on Various WDNs

Network	Pattern	R^2	RMSE
Fossolo	24h	0.999624	0.003353
	240h	0.999978	0.000897
	random-01	0.732528	0.073501
Hanoi	24h	0.992475	0.022595
	240h	0.999995	0.000379
	random-01	0.953054	0.030377
Modena	24h	0.654850	0.097890
	240h	0.999552	0.002468
	random-01	0.898213	0.045443
Net 3	24h	0.961738	0.035365
	240h	0.980191	0.026033
	random-01	0.974648	0.029597

of determination \mathbb{R}^2 , representing the ground truth values of pressures computed by EPANET versus the values of pressures predicted by the MLP model. The better the predictions, the more the data lies along the red dashed line y=x, and the closer \mathbb{R}^2 is to 1, as in Fig. 3(a). Less precise results produce a more spread graph, as in Fig. 3(b). Moreover, Fig. 3(c) shows the effects of not including Topological Metrics in the training, resulting in predictions completely uncorrelated with the testing values. In the smaller Fossolo network, predictions are nearly perfect, whereas in Modena, data points are scattered above and below the red line. Despite this variability, no significant bias towards overestimation or underestimation can be observed.

V. CONCLUSIONS

This study proposed a deep ML approach for pressure prediction in WDNs by integrating standard hydraulic parameters with network centrality metrics. Leveraging DNNs, in particular MLP and CNN architectures, we demonstrated that incorporating Topological Metrics, such as degree, betweenness and closeness, improves prediction accuracy without requiring explicit topological information. Furthermore, the TM approach reduces the need for explicit water-related data, which often requires the installation of additional sensors. The proposed methodology was validated on benchmark networks, Fossolo, Hanoi, Modena and Net3, using simulationbased datasets generated with EPANET and WNTR. The models achieved high predictive accuracy, with coefficient of determination (R^2) values reaching up to 0.9999. These results confirm the effectiveness of centrality-aware feature selection in improving ML-based hydraulic modeling. The proposed approach not only improves the adaptability of the model to different network configurations but also reduces the dependency on water data from sensors installed within the WDNs, making it a scalable and flexible solution for realworld applications.

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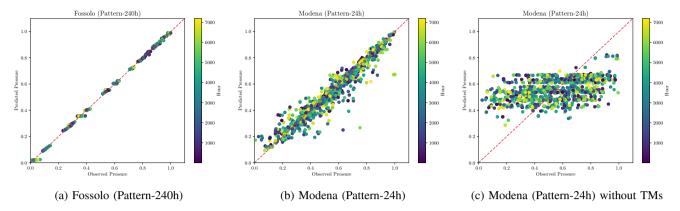


Fig. 3: A linear plot comparing the values of nodal pressures calculated by EPANET with those predicted by the MLP model. (a) represent the best-performing experiment, while (b) the worst-performing one. Moreover, (c) highlights that excluding Topological Metrics during training leads to predictions entirely uncorrelated in the test values.

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