

# Capacity-Fairness Performance of an Ad Hoc IEEE 802.11 WLAN with Noncooperative Stations

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**Abstract.** For an ad hoc IEEE 802.11 WLAN we investigate how the stations' incentives to launch a backoff attack i.e., to configure small minimum and maximum CSMA/CA contention windows in pursuit of a larger-than-fair bandwidth share, affect a proposed capacity-fairness index (*CFI*). We link *CFI* to the network size, "power awareness," a station's perception of the other stations' susceptibility to incentives, and the way of learning how the other stations perceive the other stations' susceptibility to incentives.

## 1 Introduction

Estimated limits of performance of an IEEE 802.11 WLAN [5] become the more realistic, the richer is the assumed network model; most existing estimates account for PHY-layer bandwidth, MAC and TCP overhead, DATA frame size, number of network stations, station mobility, and channel characteristics. We bring into the picture noncooperative behavior in the form of a backoff attack: each station  $n$  is free to configure an arbitrary  $w_n = \langle w_{n,\min}, w_{n,\max} \rangle$  (the minimum and maximum CSMA/CA contention windows) in pursuit of a larger-than-fair long-term bandwidth share [1], [3], thus engaging in a noncooperative *CSMA/CA game* [3], [7]. It can be shown that if the *greedy*  $\langle 1, 1 \rangle$  configuration is ruled out (the backoff mechanism is mandatory) then the game has a unique Nash equilibrium (NE) [4]. Otherwise, one disincentive to configure  $\langle 1, 1 \rangle$  is a certain "power awareness" i.e., fear of another station also configuring  $\langle 1, 1 \rangle$ , for all the transmission power is then spent on frame collisions. This we assume tantamount to a "penalty" bandwidth share, which leads to multiple Nash equilibria. In the absence of a compelling unique NE we introduce a simple calculus of backoff attack incentives, a form of seeking a best reply to the beliefs as to the other stations' imminent play. We propose a capacity-fairness index (*CFI*), a synthetic performance measure equal to the product of the total goodput (bandwidth utilization) and the Jain index of the stations' bandwidth shares. We link *CFI* to the network size, the stations' "power awareness," a station's perception of the other stations' susceptibility to incentives, and a station's way of learning how the other stations perceive the other stations' susceptibility to incentives. Based on the stations' bandwidth shares  $b_n$  obtained from existing models [2], [7] we demonstrate that for small enough networks and "power aware" enough stations, cooperative behavior may ultimately emerge.

## 2 CSMA/CA Game and Backoff Attack Incentives

To reflect both the total bandwidth utilization  $\sum_n b_n$  and Jain fairness [6] we use their product i.e.,  $(\sum_n b_n)^3 / (N \cdot \sum_n b_n^2)$ , that we name *capacity-fairness index (CFI)*. For an  $N$ -player CSMA/CA game with payoffs  $b_n$ , suppose that  $x$  *selfish* stations configure  $w_s = \langle 2, 2 \rangle$ ,  $y$  *greedy* stations configure  $w_g = \langle 1, 1 \rangle$  (i.e., disengage the backoff scheme), and  $N - x - y$  *honest* ones stick to a standard-prescribed  $w_h$  e.g.,  $\langle 16, 1024 \rangle$ ; let the respective payoffs be denoted by  $b_{s[g,h]}(N, x, y)$ . In existing IEEE 802.11 settings,  $b_h(N, 0, 0) > 0$  for not too large  $N$  and  $b_h(N, x > 0, y) \approx 0$ . Thus if  $x = y = 0$  then  $CFI = N \cdot b_h(N, 0, 0)$  (denote this cooperative value by  $c\text{-CFI}$ ). Note that  $b_G = b_g(N, x, 1)$  is the highest possible payoff, while  $b_{s[h]}(N, x, y > 0) = 0$ . Let  $b_c(N, x, y > 1) = b_c \leq 0$ , where  $b_c$  is a "penalty" payoff, reflecting the fact that a greedy (yet "power aware") station in this case spends all its transmission power to no effect. If  $b_c < 0$  [ $b_c = 0$ ] then any configuration profile with  $y = 1$  [ $y > 0$ ] is a NE. Thus the game has multiple Nash equilibria; to predict its outcome we calculate backoff attack incentives.

**Definition 1:** A selfish [greedy] backoff attack incentive is the ratio of the likely payoff upon switching from  $w_h$  to  $w_s$  [ $w_g$ ], and the cooperative payoff  $b_h(N, 0, 0)$ .

A *0-order sophisticated* approach to the "likely" part neglects similar conduct at the other stations:  $I_{g,0} = \hat{b}_g(N, 0, 1)$  and  $I_{s,0} = \hat{b}_s(N, 1, 0)$  (the hats normalize w.r.t.  $b_h(N, 0, 0)$ ). Alternatively, a station forms a model of how the other stations' play is susceptible to the calculated incentives. A *susceptibility map*  $\Phi$  returns for a  $(I_s, I_g)$  pair the probabilities  $p_s$  [ $p_g$ ] of configuring  $w_s$  [ $w_g$ ] at any other station ( $p_h = 1 - p_s - p_g$  is the probability of staying at  $w_h$ ). Intuitively,  $\Phi$  should be continuous and ensure that  $p_g$  increases in  $I_g$ ,  $p_s$  increases in  $I_s$ , and  $p_h$  decreases in both  $I_s$  and  $I_g$ . Taking  $(p_s, p_g) = \Phi(I_{s,0}, I_{g,0})$ , one can calculate the expected normalized payoffs:

$$I_{s[g],1} = \sum_{\substack{x,y,z \geq 0 \\ x+y+z \leq N-1}} \binom{N-1}{x \ y \ z} p_s^x p_g^y p_h^z \cdot \hat{b}_{s[g]}(N, x + 1[x], y[y + 1]). \quad (1)$$

This approach can be termed *1-order sophisticated*, as it does account for the other stations also calculating incentive measures, though neglects their use of  $\Phi$ . Higher-order sophistication consists in re-applying (1) to account for the other stations using  $\Phi$ , their accounting for the other stations using  $\Phi$  etc. In the limit  $\Phi$  is deemed *common knowledge* [4]. Hence,  *$\infty$ -order sophisticated* incentive measures solve the fixpoint-type equation (where  $F$  is defined by (1) with  $(p_s, p_g) = \Phi(I_{s,\infty}, I_{g,\infty})$ ):

$$(I_{s,\infty}, I_{g,\infty}) = F(I_{s,\infty}, I_{g,\infty}), \quad (2)$$

A unique solution of (2) obtains e.g., if  $\Phi$  is defined as follows:

$$p_s = \varphi^2(I_{s,\infty}) / (\varphi(I_{s,\infty}) + \varphi(I_{g,\infty})), \quad p_g = \varphi^2(I_{g,\infty}) / (\varphi(I_{s,\infty}) + \varphi(I_{g,\infty})). \quad (2)$$

Here, the function  $\varphi$  measures a station's willingness to switch from  $w_h$  to  $w_s$  [ $w_g$ ], given  $(I_s, I_g)$ ; it should be continuous and nondecreasing, with  $\varphi(0) = 0$  and  $\varphi(\infty) = 1$ .

If the CSMA/CA game is played, we use the expected value of *CFI* w.r.t. the probabilities of configuring  $w_g$ ,  $w_s$ , and  $w_h$ , determined by  $\infty$ -order incentives.

**Definition 2:** The *noncooperative CFI*, denoted *n-CFI*, is defined as

$$c\text{-CFI} \cdot p_h^N + p_g (1 - p_g)^{N-1} b_G + p_s \sum_{x=0}^{N-1} \binom{N-1}{x} p_s^x p_h^{N-1-x} (x+1) b_s(N, x+1, 0) \quad (4)$$

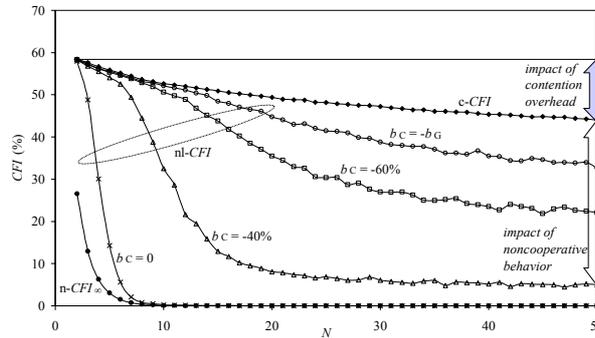
(recall that if all  $k$  nonzero payoffs out of  $N$  are equal then the Jain index is  $k/N$ ).

In reality, the other stations' susceptibility to incentives may be learned by playing the CSMA/CA game repeatedly and observing successive configuration profiles. We model this intuition by taking a sigmoid  $\varphi(I) = (1 - e^{-4I}) / (1 + e^{-4(I-a)})$  and manipulating its center  $a$ . In the  $i^{\text{th}}$  instance of the game, station  $n$ 's perception of the other stations' susceptibility is reflected by  $a_n^i$ , with the dynamics  $a_n^{i+1} = \max\{0, a_n^i + \delta_n(X^i, Y^i)\}$ . Here, the function  $\delta_n$  describes the learning process at station  $n$ , and  $X^i$  [ $Y^i$ ] is the number of selfish [greedy] stations in the  $i^{\text{th}}$  instance, distributed according to  $p_s$ ,  $p_g$ , and  $p_h = 1 - p_s - p_g$  as calculated from (3), with the sigmoid  $\varphi$  centered at  $a_n^i$ .

Let  $\delta_n(x > 0, 0) = -\Delta_n$ ,  $\delta_n(0, 0) = -2\Delta_n$ , and  $\delta_n(x, y > 0) = \Delta_n$ , where  $\Delta_n$  is proportional to station  $n$ 's initial  $a_n^0$  through a constant  $\Delta$  (thus relative changes of  $a$  are the same at each station).  $((a_1^i, \dots, a_N^i), i = 1, 2, \dots)$  is an  $N$ -dimensional random walk with an absorbing state  $(0, \dots, 0)$  corresponding to cooperative behavior (the solution of (2) then yields  $p_s = p_g = 0$ ) and another absorbing state  $(\infty, \dots, \infty)$ , with  $p_s = p_g = 1/2$  and  $p_h = 0$ . The capacity-fairness indices at these absorbing states are *c-CFI* and  $n\text{-CFI}_\infty = (N \cdot b_s(N, N, 0) + b_G) / 2^N$ , respectively. Define  $a_{\max}$  so that the corresponding solution of (2) yields  $p_s$  and  $p_g$  close to  $1/2$ .

**Definition 3:** The *noncooperative learning CFI*, denoted *nl-CFI*, is defined as  $\pi_N \cdot c\text{-CFI} + (1 - \pi_N) \cdot n\text{-CFI}_\infty$ ,  $\pi_N$  being the probability of reaching the absorbing state  $(0, \dots, 0)$  given that each station  $n$  selects  $a_n^0 \in [0..a_{\max}]$  at random.

Assuming  $\Delta = 0.2$ , Fig. 1 depicts *c-CFI* and *nl-CFI*. The latter turns out to be distinctly closer to *c-CFI* than *n-CFI* for "power aware" enough stations, as confirmed by numerical experiments whose details are omitted.



**Fig. 1.** *c-CFI* and *nl-CFI* for various "power awareness" levels

### 3 Conclusion

The introduction of  $w_g$  and "power awareness" changes the CSMA/CA game into one with multiple Nash equilibria i.e., without a compelling outcome. We envisage that each station then calculates common-knowledge incentives to configure  $w_s$  and  $w_g$ , and the corresponding probability distribution of imminent configuration profiles. Our study quantitatively illustrates a few intuitions:

- the network's ability to provide high and fair bandwidth shares to all stations diminishes as  $N$  increases, partly on account of growing contention overhead, but mostly because of the stations' limited willingness to behave cooperatively; these two factors are illustrated for the  $b_C = -40\%$  curve at  $N = 50$ ,
- incentive calculus dictates that the willingness to behave cooperatively grow with "power awareness" for fear of spending all the transmission power without getting any bandwidth share; accordingly,  $CFI$  improves as  $b_C$  goes more negative,
- the predictions depend on a station's perception of the other stations' susceptibility to incentives, reflected by  $\Phi$ , and the learning process, reflected by  $\delta$ ,
- each of the nl- $CFI$  curves lies between the n- $CFI_\infty$  and c- $CFI$  ones; its bias towards the latter measures the chance  $\pi_N$  of emergence of cooperative behavior; this is almost certain for small enough  $N$  assuming enough "power awareness."

Although the "penalty" bandwidth share  $b_C$  was assumed constant across the stations, it is relatively easy to generalize to nonuniform "power awareness" in order to study the coexistence of devices with diverse battery lifetimes.

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