Minimum-Cost Multicast Routing for Multi-Layered Multimedia Distribution

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Abstract. In this paper, we attempt to solve the problem of min-cost multicast routing for multi-layered multimedia distribution. More specifically, for (i) a given network topology (ii) the destinations of a multicast group and (iii) the bandwidth requirement of each destination, we attempt to find a feasible routing solution to minimize the cost of a multicast tree for multi-layered multimedia distribution. This problem has been proved to be NP-hard. We propose two adjustment procedures, namely: the tie breaking procedure and the drop-and-add procedure to enhance the solution quality of the modified T-M heuristic. We also formally model this problem as an optimization problem and apply the Lagrangean relaxation method and the subgradient method to solve the problem. Computational experiments are performed on regular networks, random networks, and scale-free networks. According to the experiment results, the Lagrangean based heuristic can achieve up to 23.23% improvement compared to the M-T-M heuristic.

1 Introduction

Multimedia application environments are characterized by large bandwidth variations due to the heterogeneous access technologies of networks (e.g. analog modem, cable modem, xDSL, and wireless access etc.) and different receivers' quality requirements. In video multicasting, the heterogeneity of the networks and destinations makes it difficult to achieve bandwidth efficiency and service flexibility. There are many challenging issues that need to be addressed in designing architectures and mechanisms for multicast data transmission [1].

Unicast and multicast delivery of video are important building blocks of Internet multimedia applications. Unicast means that the video stream goes independently to each user through point-to-point connection from the source to each destination, and all destinations get their own stream. Multicast means that many destinations share the same stream through point-to-multipoint connection from the source to every destination, thus reducing the bandwidth requirements and network traffic. The efficiency of multicast is achieved at the cost of losing the service flexibility of unicast, because in unicast each destination can individually negotiate the service contract with the source.

Taking advantage of recent advances in video encoding and transmission technologies, either by a progress coder [2] or video gateway [3] [4], different destinations can request a different bandwidth requirement from the source, after which the source only needs to transmit signals that are sufficient for the highest bandwidth destination into a single multicast tree. This concept is called single-application multiple-stream (SAMS). A multi-layered encoder encodes video data into more than one video stream, including one *base layer* stream and several *enhancement layer* streams. The base layer contains the most important portions of the video stream for achieving the minimum quality level. The enhancement layers contain the other portions of video stream for refining the quality of the base layer stream. This mechanism is similar to destination-initiated reservations and packet filtering used in the RSVP protocol [5].

The minimum cost multicast tree problem, which is the Steiner tree problem, is known to be NP-complete. Reference [6] and [7] surveyed the heuristics of Steiner tree algorithms. For the conventional Steiner tree problem, the link costs in the network are fixed. However, for the minimum cost multi-layered video multicast tree, the link costs are dependent on the set of receivers sharing the link. It is a variant of the Steiner tree problem. The heterogeneity of the networks and destinations makes it difficult to design an efficient and flexible mechanism for servicing all multicast group users.

Reference [8] discusses the issue of multi-layered video distribution on multicast networks and proposes a heuristic to solve this problem, namely: the modified T-M heuristic (M-T-M Heuristic). Its goal is to construct a minimum cost tree from the source to every destination. However, the reference provides only experimental evidence for its performance. Reference [9] extends this concept to present heuristics with provable performance guarantees for the Steiner tree problem and proof that this problem is NP-hard, even in the special case of broadcasting. From the results, the cost of the multicast tree generated by M-T-M heuristics was no more than 4.214 times the cost of an optimal multicast tree. However, no simulation results are reported to justify the approaches in [9]. The solution approaches described above are heuristic-based and could be further optimized. Consequently, for multimedia distribution on multicast networks, we intend to find the multicast trees that have a minimal total incurred cost for multi-layered video distribution.

In this paper, we extend the idea of [8] for minimizing the cost of a multi-layered multimedia multicast tree and propose two more precise procedures (tie-breaking procedure and drop-and-add procedure) to improve the solution quality of M-T-M heuristic. Further, we formally model this problem as an optimization problem. In the structure of mathematics, they undoubtedly have the properties of linear programming problems. We apply the Lagrangean relaxation method and the subgradient method to solve the problems [10][11]. Properly integrating the M-T-M heuristics and the results of Lagrangean dual problems may be useful to improve the solution quality. In addition, the Lagrangean relaxation method not only gets a good feasible solution, but also provides the lower bound of the problem solution which helps to verify the solution quality. We name this method Lagrangean Based M-T-M Heuristics.

The rest of this paper is organized as follows. In Section 2, we describe the detail of the M-T-M heuristic and present the evidence that the M-T-M heuristic does not perform well under some often seen scenarios. We propose two procedures to improve the solution quality. In Section 3, we formally define the problem being studied, as well as a mathematical formulation of min-cost optimization is proposed. Section 4 applies Lagrangean relaxation as a solution approach to the problem. Section 5, illustrates the computational experiments. Finally, in Section 6 we present our conclusions and the direction of future research.

2 Heuristics of Multi-Layered Multimedia Multicasting

Reference [12] proposes an approximate algorithm named T-M heuristic to deal with the Steiner tree problem, which is a min-cost multicast tree problem. The T-M heuristic uses the idea of minimum depth tree algorithm (MDT) to construct the tree. To begin with, the source node is added to the tree permanently. At each iteration of MDT, a node is temporarily added to the tree until the added node is a receiver of the multicast group. Once the iterated tree reaches one of the receivers of the multicast group, it removes all unnecessary temporary links and nodes added earlier and marks the remaining nodes permanently connected to the tree. The depth of the permanently connected nodes is then set to zero and the iterations continue until all receivers are permanently added to the tree. In [8], the author gives examples of the performance of the T-M heuristic and shows that in some cases the T-M heuristic does not achieve the optimum tree.

Reference [8] modified the T-M heuristic to deal with the min-cost multicast tree problem in multi-layered video distribution. For multi-layered video distribution, which is different from the conventional Steiner tree problem, each receiver can request a different quality of video. This means that each link's flow of the multicast tree is different and is dependent on the maximum rate of the receiver sharing the link. The author proposes a modified version of the T-M heuristic (M-T-M heuristic) to approximate the minimum cost multicast tree problem for multi-layered video distribution.

The M-T-M heuristic separates the receivers into subsets according to the receiving rate. First, the M-T-M heuristic constructs the multicast tree for the subset with the highest rate by using the T-M heuristic. Using this initial tree, the T-M heuristic is then applied to the subsets according to the order of receiving rate from high to low. For further details of the M-T-M heuristic, please refer to reference [8].

2.1 Some Scenarios of the Modified T-M Heuristic

In most networks, the performance of the Modified T-M heuristic is better than the T-M heuristic in multi-layered video multicasting. But, in some scenarios, we have found that the M-T-M does not perform well.

Consider the network in Figure 1 with node 1 as the source and nodes 3 and 4 as the destinations requiring rates 2 and 1, respectively. Assume the base costs of all links are the same, which is 1. First, the M-T-M heuristic separates the receivers into two subsets, one for rate 1 and the other for rate 2. It then runs a MDT algorithm such as Dijkstra algorithm to construct the tree with the highest rate subset. At Step 4, the T-M heuristic reaches the destination with the highest rate and removes all unnecessary intermediate links. After setting the depth of the permanently connected nodes to zero, it continues the search process for the other destinations. At Step 5, the M-T-M heuristic tree is found and the sum of the link costs is 5. But the sum of the link costs for the optimum tree shown is 4.

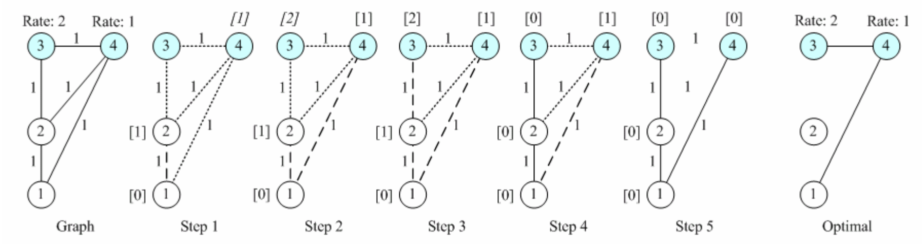

Fig. 1. Example of the M-T-M heuristic for multi-layered distribution with constant link cost.

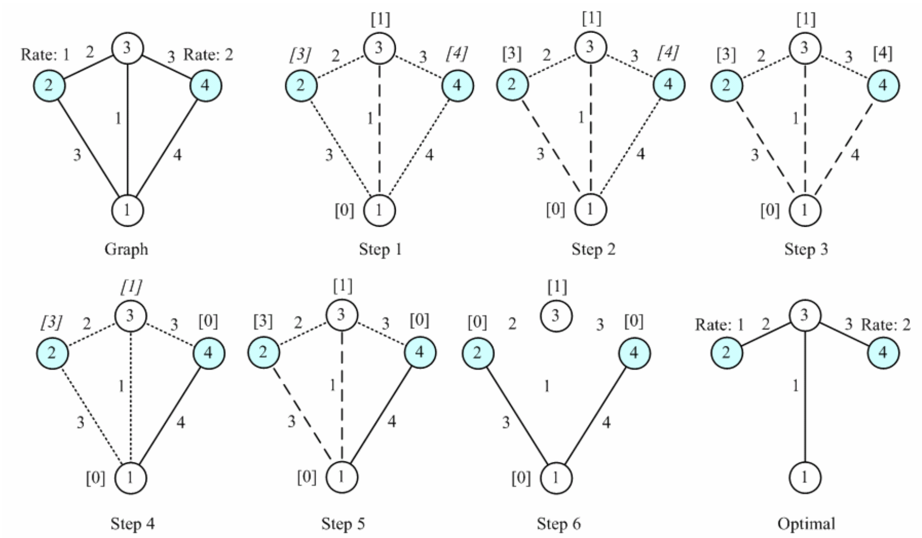

Fig. 2. Example of the M-T-M heuristic for multi-layered distribution with arbitrary link cost.

Consider the other network in Figure 2 with node 1 as the source and nodes 2 and 4 as the destinations requiring rates 1 and 2, respectively. The link costs are indicated by the side of the links. At Step 6, the M-T-M heuristic tree is found and the sum of the link costs is 11. But, the sum of the link costs for the optimum tree shown is 10.

2.2 Enhanced Modified T-M Heuristic

With reference to the above scenarios, we propose two adjustment procedures to improve the solution performance. The first one is the *tie breaking procedure*, which is used to handle the node selection when searching the nearest node within the M-T-M heuristic. The second is the *drop and add procedure*, which is used to adjust the multicast tree resulting from the M-T-M heuristic in order to reach a lower cost.

Tie Breaking Procedure. For the MDT algorithm, ties for the nearest distinct node may be broken arbitrarily, but the algorithm must still yield an optimal solution. Such ties are a signal that there may be (but need not be) multiple optimal solutions. All such optimal solutions can be identified by pursuing all ways of breaking ties to their conclusion. However, when executing the MDT algorithm within the M-T-M heuristic, we found that the tie breaking solution will influence the cost of the multicast tree. For example in Figure 2, the depth of nodes 2 and 4 is the same and is minimal at Step 1. The tie may therefore be broken by randomly selecting one of them to be the next node to update the depth of all the vertices. In general, we choose the node with the minimal node number within the node set of the same minimal depth for implementation simplicity. Although we choose node 1 as the next node to relax, node 2 is the optimal solution.

We propose a tie breaking procedure to deal with this situation. When there is a tie, the node with the largest requirement should be selected as the next node to join the tree. The performance evaluation will be shown in section 5.

Drop and Add Procedure. The drop and add procedure we propose is an adjustment procedure to adjust the initial multicast tree constructed by M-T-M heuristic. Nevertheless, redundantly checking actions may cause a serious decline in performance, even if the total cost is reduced. Therefore, we consider the most useful occurrence to reduce the total cost and control the used resources in an acceptable range. The details of procedures are:

1. Compute the number of hops from the source to the destinations.
2. Sort the nodes in descending order according to {incoming traffic/its own traffic demand}.
3. In accordance with the order, drop the node and re-add it to the tree. Consider the following possible adding measures and set the best one to be the final tree. Either adds the dropping node to the source node, or to other nodes having the same hop count, or to the nodes having a hop count larger or smaller by one.

3 Problem Formulation

3.1 Problem Description

The network is modeled as a graph where the switches are depicted as nodes and the links are depicted as arcs. A user group is an application requesting transmission in this network, which has one source and one or more destinations. Given the network topology, the capacity of the links and bandwidth requirement of every destination of a user group, we want to jointly determine the following decision variables: (1) the routing assignment (a tree for multicasting or a path for unicasting) of each user group; and (2) the maximum allowable traffic rate of each multicast user group through each link.

By formulating the problem as a mathematical programming problem, we intend to solve the issue optimally by obtaining a network that will enable us to achieve our goal, i.e. one that ensures the network operator will spend the minimum cost on constructing the multicast tree. The notations used to model the problem are listed in Table 1.

Table 1. Description of Notations

Notation	Description
	Given Parameters
Notation	**Description**
a_l	Transmission cost associated with link *l*
α_{gd}	Traffic requirement of destination *d* of multicast group *g*
G	The set of all multicast groups
V	The set of nodes in the network
L	The set of links in the network
D_g	The set of destinations of multicast group *g*
h_g	The minimum number of hops to the farthest destination node in multicast group *g*
I_v	The incoming links to node *v*
r_g	The multicast root of multicast group *g*
I_{r_g}	The incoming links to node r_g
P_{gd}	The set of paths destination *d* of multicast group *g* may use
δ_{pl}	The indicator function which is 1 if link *l* is on path *p* and 0 otherwise
	Decision Variables
Notation	**Descriptions**
x_{gpd}	1 if path *p* is selected for group *g* destined for destination *d* and 0 otherwise
y_{gl}	1 if link *l* is on the subtree adopted by multicast group *g* and 0 Otherwise
m_{gl}	The maximum traffic requirement of the destinations in multicast group *g* that are connected to the source through link *l*

3.2 Mathematical Formulation

According to the problem description in pervious section, the min-cost problem is formulated as a combinatorial optimization problem in which the objective function is to minimize the link cost of the multicast tree. Of course a number of constraints must be satisfied.

Objective function (IP):

$$Z_{IP} = \min \sum_{g\in G}\sum_{l\in L} a_l \; m_{gl} \tag{1}$$

subject to:

$$\sum_{p\in P_{gd}} x_{gpd}\alpha_{gd}\delta_{pl} \le m_{gl} \qquad \forall g \in G, d \in D_g, l \in L \tag{2}$$

$$m_{gl} \in [0, \max_{d\in D_g} \alpha_{gd}] \qquad \forall l \in L, g \in G \tag{3}$$

$$y_{gl} = 0 \text{ or } 1 \qquad \forall l \in L, g \in G \tag{4}$$

$$\sum_{l\in L} y_{gl} \ge \max\{h_g, |D_g|\} \qquad \forall g \in G \tag{5}$$

$$\sum_{d\in D_g}\sum_{p\in P_{gd}} x_{gpd}\delta_{pl} \le |D_g| y_{gl} \qquad \forall g \in G, l \in L \tag{6}$$

$$\sum_{l\in I_v} y_{gl} \le 1 \qquad \forall g \in G, v \in V - \{r_g\} \tag{7}$$

$$\sum_{l\in I_{r_g}} y_{gl} = 0 \qquad \forall g \in G \tag{8}$$

$$\sum_{p\in P_{gd}} x_{gpd} = 1 \qquad \forall d \in D_g, g \in G \tag{9}$$

$$x_{gpd} = 0 \text{ or } 1 \qquad \forall d \in D_g, g \in G, p \in P_{gd} \tag{10}$$

The objective function of (1) is to minimize the total transmission cost of servicing the maximum bandwidth requirement destination through a specific link for all multicast groups *G*, where *G* is the set of user groups requesting connection. The maximum bandwidth requirement on a link in the specific group m_{gl} can be viewed so that the source would be required to transmit in a way that matches the most constrained destination.

Constraint (2) is referred to as the capacity constraint, where the variable m_{gl} can be interpreted as the "estimate" of the aggregate flow. Since the objective function is strictly an increasing function with m_{gl} and (1) is a minimization problem, each m_{gl} will equal the aggregate flow in an optimal solution. Constraint (3) is a redundant constraint which provides upper and lower bounds on the maximum traffic requirement for multicast group *g* on link *l*. Constraints (4) and (5) require that the number of links on the multicast tree adopted by the multicast group *g* be at least the maximum of h_g and the cardinality of D_g. The h_g and the cardinality of D_g are the legitimate lower bounds of the number of links on the multicast tree adopted by the multicast group *g*. Constraint (6) is referred to as the tree constraint, which requires that the union of the selected paths for the destinations of user group *g* forms a tree. Constraints (7) and (8) are both redundant constraints. Constraint (7) requires that the number of selected incoming links y_{gl} to node is 1 or 0, while constraint (8) requires that there are no selected incoming links y_{gl} to the node that is the root of multicast group *g*. As a result, the links we select can form a tree. Finally, constraints (9) and (10) require that only one path is selected for each multicast source-destination pair.

4 Solution Approach

4.1 Lagrangean Relaxation

Lagrangean methods were used in both the scheduling and the general integer programming problems at first. However, it has become one of the best tools for optimization problems such as integer programming, linear programming combinatorial optimization, and non-linear programming [10][11].

The Lagrangean relaxation method permits us to remove constraints and place them in the objective function with associated Lagrangean multipliers instead. The optimal value of the relaxed problem is always a lower bound (for minimization problems) on the objective function value of the problem. By adjusting the multiplier of Lagrangean relaxation, we can obtain the upper and lower bounds of this problem. The Lagrangean multiplier problem can be solved in a variety of ways. The subgradient optimization technique is possibly the most popular technique for solving the Lagrangean multiplier problem [10] [13].

By using the Lagrangean Relaxation method, we can transform the primal problem (IP) into the following Lagrangean Relaxation problem (LR) where Constraints (2) and (6) are relaxed. For a vector of non-negative Lagrangean multipliers, a Lagrangean Relaxation problem of (1) is given by

Optimization problem (LR):

$$Z_D(\beta,\theta) = \min \sum_{g\in G}\sum_{l\in L} a_l m_{gl} + \sum_{g\in G}\sum_{d\in D_g}\sum_{l\in L}\sum_{p\in P_{gd}} \beta_{gdl} x_{gpd} \alpha_{gd} \delta_{pl} - \sum_{g\in G}\sum_{d\in D_g}\sum_{l\in L} \beta_{gdl} m_{gl} + \sum_{g\in G}\sum_{l\in L}\sum_{d\in D_g}\sum_{p\in P_{gd}} \theta_{gl} x_{gpd} \delta_{pl} - \sum_{g\in G}\sum_{l\in L} \theta_{gl} \left|D_g\right| y_{gl} \quad (11)$$

subject to: (3) (4) (5) (7) (8) (9) (10).

Where β_{gdl}, θ_{gl} are Lagrangean multipliers and β_{gdl}, $\theta_{gl} \geq 0$. To solve (11), we can decompose (11) into the following three independent and easily solvable optimization subproblems.

Subproblem 1: (related to decision variable x_{gpd})

$$Z_{Sub1}(\beta,\theta) = \min \sum_{g\in G}\sum_{d\in D_g}\sum_{p\in P_{gd}} [\sum_{l\in L} \delta_{pl}(\beta_{gdl}\alpha_{gd} + \theta_{gl})] x_{gpd} \quad (12)$$

subject to: (9) (10).

Subproblem 1 can be further decomposed into $|G||D_g|$ independent shortest path problems with nonnegative arc weights. Each shortest path problem can be easily solved by Dijkstra's algorithm.

Subproblem 2: (related to decision variable y_{gl})

$$Z_{Sub2}(\theta) = \min \sum_{g\in G}\sum_{l\in L} (-\theta_{gl}\left|D_g\right|) y_{gl} \quad (13)$$

subject to: (4) (5) (7) (8).

The algorithm to solve Subproblem 2 is:

Step 1 Compute max$\{h_g, |D_g|\}$ for multicast group g.

Step 2 Compute the number of negative coefficients ($-\theta_{gl}$ $|D_g|$) for all links in the multicast group g.

Step 3 If the number of negative coefficients is greater than max$\{h_g, |D_g|\}$ for multicast group g, then assign the corresponding negative coefficient of y_{gl} to 1 and 0 otherwise.

Step 4 If the number of negative coefficients is no greater than max$\{h_g, |D_g|\}$ for multicast group g, assign the corresponding negative coefficient of y_{gl} to 1. Then, assign [max$\{h_g, |D_g|\}-$ the number of positive coefficients of y_{gl}] numbers of the smallest positive coefficient of y_{gl} to 1 and 0 otherwise.

Subproblem 3: (related to decision variable m_{gl})

$$Z_{Sub3}(\beta) = \min \sum_{g\in G}\sum_{l\in L}(a_l - \sum_{d\in D_g}\beta_{gdl})m_{gl} \quad (14)$$

subject to: (3).

We decompose Subproblem 3 into $|L|$ independent problems. For each link $l \in L$:

$$Z_{Sub3.1}(\beta) = \min \sum_{g\in G}(a_l - \sum_{d\in D_g}\beta_{gdl})m_{gl} \quad (15)$$

subject to: (3).

The algorithm to solve (15) is:

Step 1 Compute $a_l - \sum_{d\in D_g}\beta_{dgl}$ for link l of multicast group g.

Step 2 If $a_l - \sum_{d\in D_g}\beta_{dgl}$ is negative, assign the corresponding m_{gl} to the maximum traffic requirement in the multicast group, otherwise assign the corresponding m_{gl} to 0.

According to the weak Lagrangean duality theorem [13], for any β_{gdl}, $\theta_{gl} \geq 0$, $Z_D(\beta_{gdl}, \theta_{gl})$ is a lower bound on Z_{IP}. The following dual problem (D) is then constructed to calculate the tightest lower bound.

Dual Problem (D):

$$Z_D = \max Z_{\mathrm{D}}(\beta_{gdl}, \theta_{gl}) \quad (16)$$

subject to:

$$\beta_{gdl}, \theta_{gl} \geq 0$$

There are several methods for solving the dual problem (16). The most popular is the subgradient method, which is employed here [14]. Let a vector s be a subgradient of $Z_D(\beta_{gdl}, \theta_{gl})$. Then, in iteration k of the subgradient optimization procedure, the multiplier vector is updated by $\omega^{k+1}=\omega^k+t^k s^k$. The step size t^k is determined by $t^k=\delta(Z^h_{IP} - Z_D(\omega^k))/\|s^k\|^2$. Z^h_{IP} is the primal objective function value for a heuristic solution (an upper bound on Z_{IP}). δ is a constant and $0< \delta \leq 2$.

4.2 Getting Primal Feasible Solutions

After optimally solving the Lagrangean dual problem, we get a set of decision variables. However, this solution would not be a feasible one for the primal problem since some of constraints are not satisfied. Thus, minor modification of decision variables, or the hints of multipliers must be taken, to obtain the primal feasible solution of problem (IP). Generally speaking, the better primal feasible solution is an upper bound (UB) of the problem (IP), while the Lagrangean dual problem solution guarantees the lower bound (LB) of problem (IP). Iteratively, by solving the Lagrangean dual problem and getting the primal feasible solution, we get the LB and UB, respectively. So, the gap between UB and LB, computed by (UB-LB)/LB*100%, illustrates the optimality of problem solution. The smaller gap computed, the better the optimality.

To calculate the primal feasible solution of the minimum cost tree, the solutions to the Lagrangean Relaxation problems are considered. The set of x_{gpd} obtained by solving (12) may not be a valid solution to problem (IP) because the capacity constraint is relaxed. However, the capacity constraint may be a valid solution for some links. Also, the set of y_{gl} obtained by solving (13) may not be a valid solution because of the link capacity constraint and the union of y_{gl} may not be a tree.

Here we propose a comprehensive, two-part method to obtain a primal feasible solution. It utilized a Lagrangean based modified T-M heuristic, followed by adjustment procedures. While solving the Lagrangean relaxation dual problem, we may get some multipliers related to each OD pair and links. According to the information, we can make our routing more efficient. We describe the Lagrangean based modified T-M heuristic below.

[Lagrangean based modified T-M heuristic]

Step 1 Use $a_l - \sum_{d \in D_g} \beta_{dgl}$ as link l's arc weight and run the M-T-M heuristic.

Step 2 After getting a feasible solution, we apply the *drop-and-add procedure* described earlier to adjust the result.

Initially, we set all of the multipliers to 0, so we will get the same routing decision as the M-T-M heuristics followed by the drop-and-add procedure at the first iteration.

5 Computational Experiments

In this section, computational experiments on the Lagrangean relaxation based heuristic and other primal heuristics are reported. The heuristics are tested on three kinds of networks- regular networks, random networks, and scale-free networks. Regular networks are characterized by low clustering and high network diameter, and random networks are characterized by low clustering and low diameter. The scale-free networks, which are power-law networks, are characterized by high clustering and low diameter. Reference [15] shows that the topology of the Internet is characterized by power laws distribution. The power laws describe concisely skewed distributions of graph properties such as the node degree.

Two regular networks shown in Figure 3 are tested in our experiment. The first one is a grid network that contains 100 nodes and 180 links, and the second is a cellular network containing 61 nodes and 156 links. Random networks tested in this paper are generated randomly, each having 500 nodes. The candidate links between all node pairs are given a probability following the uniform distribution. In the experiments, we link the node pair with a probability smaller than 2%. If the generated network is not a connected network, we generate a new network.

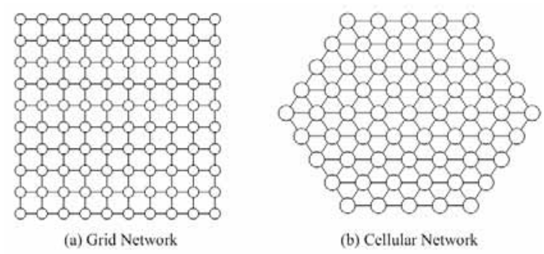

Fig. 3. Regular Networks

Reference [16] shows that the scale-free networks can arise from a simple dynamic model that combines incremental growth with a preference for new nodes to connect to existing ones that are already well connected. In our experiments, we applied this preferential attachment method to generate the scale-free networks. The corresponding preferential variable (m_0, m) is (2, 2). The number of nodes in the testing networks is 500.

For each testing network, several distinct cases, which have different pre-determined parameters such as the number of nodes, are considered. The traffic demands for each destination are drawn from a random variable uniformly distributed in pre-specified categories {1, 2, 5, 10, 15, 20}. The link costs are randomly generated between 1 and 5. The cost of the multicast tree is decided by multiplying the link cost and the maximum bandwidth requirement on a link. We conducted 2,000 experiments for each kind of network. For each experiment, the result was determined by the group destinations and link costs generated randomly. Table 2 summaries the selected results of the computational experiments.

In general, the results of LR are all better than the M-T-M heuristic (MTM), the M-T-M heuristic with tie breaking procedure (TB), and the M-T-M heuristic followed by drop-and-add procedure (DA). This is because we get the same solution as the M-T-M heuristic at the first iteration of LR. For each testing network, the maximum improvement ratio between the M-T-M heuristic and the Lagrangean based modified T-M heuristic is 16.18 %, 23.23%, 10.41 %, and 11.02%, respectively. To claim optimality, we also depict the percentile of gap in Table 2. The results show that 60% of the regular and scale free networks have a gap of less than 10%, but the result of random networks show a larger gap. However, we also found that the M-T-M heuristic perform well in many cases, such as the case D of grid network and case D of random network.

According to the experiments results, we found that the tie breaking procedure we proposed is not uniformly better than random selection. For example, the case H of cellular network, the performance of M-T-M (1517) is better than TB (1572). Consequently, we suggest that in practice we can try both tie breaking methods (randomly select or the method we proposed), and select the better result. The experiments results also show that the drop and add procedure does reduce the cost of the multicast tree.

Table 2. Selected Results of Computational Experiments

CASE	Dest. #	M-T-M	TB	DA	UB	LB	GAP	Imp.
Grid Network						Max Imp. Ratio: 16.18 %		
A	5	332	330	332	290	286.3714	1.27%	14.48%
B	5	506	506	506	506	503.6198	0.47%	0.00%
C	10	158	153	148	136	123.1262	10.46%	16.18%
D	10	547	547	547	547	541.8165	0.96%	0.00%
E	20	522	507	502	458	397.8351	15.12%	13.97%
F	20	1390	1405	1388	1318	1206.235	9.27%	5.46%
G	50	2164	2229	2154	1940	1668.448	16.28%	11.55%
H	50	759	700	759	693	588.3226	17.79%	9.52%
Cellular Network						Max Imp. Ratio: 23.23 %		
A	5	182	167	172	167	160.4703	4.07%	8.98%
B	5	119	119	119	109	105.9671	2.86%	9.17%
C	10	194	185	190	180	156.9178	14.71%	7.78%
D	10	174	174	170	150	138.0774	8.63%	16.00%
E	20	382	349	382	310	266.1146	16.49%	23.23%
F	20	815	800	811	756	689.6926	9.61%	7.80%
G	50	602	595	602	567	479.9626	18.13%	6.17%
H	50	1517	1572	1503	1357	1187.332	14.29%	11.79%
Random Networks						Max Imp. Ratio: 10.41 %		
A	5	107	107	107	107	94.70651	12.98%	0.00%
B	5	88	88	88	86	74.63349	15.23%	2.27%
C	10	170	170	170	170	134.6919	26.21%	0.00%
D	10	123	125	123	123	97.90988	25.63%	0.00%
E	20	317	317	317	284	221.2635	28.35%	10.41%
F	20	226	216	226	216	168.0432	28.54%	4.42%
G	50	850	860	850	806	558.5077	44.31%	5.18%
H	50	702	715	702	690	446.9637	54.37%	1.71%
Scale-Free Networks						Max Imp. Ratio: 11.02 %		
A	5	82	82	82	82	78.35047	4.66%	0.00%
B	5	79	75	75	75	73.70663	1.75%	5.33%
C	10	210	210	210	208	196.3969	5.91%	0.96%
D	10	528	528	528	506	505.4039	0.12%	4.35%
E	20	886	896	886	854	770.9776	10.77%	3.75%
F	20	1068	1050	1022	962	920.2371	4.54%	11.02%
G	50	1869	1871	1869	1754	1502.061	16.77%	6.56%
H	50	1911	1946	1911	1891	1598.817	18.27%	1.06%

TB: The result of the modified T-M heuristic with the tie breaking procedure
DA: The result of the modified T-M heuristic followed by the drop-and-add procedure
UB and LB: Upper and lower bounds of the Lagrangean based modified T-M heuristic
GAP: The error gap of the Lagrangean relaxation
Imp.: The improvement ratio of the Lagrangean based modified T-M heuristic

6 Conclusions

In this paper, we attempt to solve the problem of min-cost multicast routing for multi-layered multimedia distribution. Our achievement of this paper can be expressed in terms of mathematical formulation and experiment performance. In terms of formulation, we propose a precise mathematical expression to model this problem well. In terms of performance, the proposed Lagrangean relaxation and subgradient based algorithms outperform the primal heuristics (M-T-M heuristic). According to the experiment results, the Lagrangean based heuristic can achieve up to 23.23% improvement compared to the M-T-M heuristic. We also propose two adjustment procedures to enhance the solution quality of the M-T-M heuristic.

Our model can also be easily extended to deal with the constrained multicast routing problem for multi-layered multimedia distribution by adding capacity and delay constraints. Moreover, the min-cost model proposed in this paper can be modified as a max-revenue model, with that objective of maximizing total system revenues by totally, or partially, admitting destinations into the system. These issues will be addressed in future works.

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