Robust Wireless Scheduling under Arbitrary Channel Dynamics and Feedback Delay
(Invited Paper)

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Abstract—Designing efficient scheduling algorithms is crucial to the development of modern wireless networks. In this paper, we study a wireless network model consisting of one central base-station and $K$ mobile users. Each time the base-station can simultaneously transmit to $1 \leq M \leq K$ users. The channel states change over time adversarially, and the feedback of transmission outcome can experience arbitrary delays. The objective of the base-station is to search for a policy to maximize the overall transmission success rate. We propose a scheduling algorithm named Banker-OMD-Scheduling for this setting, based on a recent banker online mirror descent technique [1]. We show that Banker-OMD-Scheduling guarantees that the total regret over a finite time horizon $T$ is $O(\sqrt{MKT} + \sqrt{D\log D})$ where $D$ is the total feedback delay.

Index Terms—wireless, scheduling, online bandit optimization, delayed feedback

I. INTRODUCTION

In the past decade, due to the rapid development of mobile technology, wireless networks have become an irreplaceable part of the world-wide communication infrastructure. Among the many technical challenges, scheduling has been one of the most fundamental problems in wireless network control and has been extensively studied, e.g., [2], [3], [4], [5] and [6]. However, existing results often focus on stochastic settings and assume instantaneous feedback for the transmissions. Thus, they do not directly apply to scenarios where the system dynamics are non-stationary.

In this paper, we study a model for wireless network under adversarial channel dynamics with delayed feedback. Specifically, we consider the following discrete-time model. There is a base-station and $K$ mobile users. In each time step, the base-station can choose $1 \leq M \leq K$ users and transmit to them. The channel states can change over time arbitrarily. The base-station has very limited knowledge on the current channel states, and needs to schedule only based on bandit feedback about the amount of data transmitted. That is, the base-station can only know the number of bits transmitted to the $M$ selected users after the decision has been made, and it gets no information about the links of the remaining $K - M$ users. Moreover, due to dynamics, the transmission feedback may arrive with delays. Furthermore, the base-station is given no knowledge about the delay length of each time step until this feedback arrives. The objective of the base-station is to find a scheduling policy to decide which users to transmit and optimize the total throughput.

This setting is very general and can be used to model many different communication processes built on different physical media. However, it is also challenging due to the arbitrary channel dynamics and feedback delay. To address the difficulties, we propose a scheduling algorithm based on the Banker-OMD framework recently developed in [1], and design Banker-OMD-Scheduling. We prove that Banker-OMD-Scheduling achieves an $O(\sqrt{T} + \sqrt{D})$ total regret under any finite time horizon length $T$.

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Online learning based scheduling algorithms have been proposed and studied in the scheduling literature, e.g., [7], [8] and [9], where regret is also adopted as the primary performance metric. Compared to these prior results, our work differs in that we allow serving multiple users simultaneously under arbitrary channel conditions, and our algorithm is robust to feedback delay. Our formulation is similar to the $M$-set adversarial semi-bandit problem [10], where there is no feedback delay and there have been algorithms achieving $O(\sqrt{MKT})$ total regret, e.g., [10]. In the terms of bandit optimization problems, each user in our setting is a bandit arm, the base-station is an agent that needs to select a subset of arms of size $M$ at each time step. Another previous online optimization problem similar to our setting is the adversarial MAB problem with feedback delays, which is a special case of our setting where $M = 1$, and there have been algorithms achieving $O(\sqrt{KT} + \sqrt{D\log K})$ total regret [11]. Hence, our setting can be viewed as a hybrid generalization of these two aforementioned online optimization problems.

 Throughout this paper, we use the $\tilde{O}$ notation to suppress poly-logarithmic factors in $T$. 

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II. NOTATION

We use $[K]$ to denote the set $\{1, 2, \ldots, K\}$. $0$ denotes the zero vector. Let $f$ be any strictly convex function defined on some convex $A \subseteq \mathbb{R}^K$. For any $x, y \in A$, if $\nabla f(x)$ exists, we use $D_f(y, x) := f(y) - f(x) - \langle \nabla f(x), y - x \rangle$ to denote the Bregman divergence between $y$ and $x$ induced by $f$. We use

$$f^*(y) := \sup_{x \in \mathbb{R}^K} \{\langle y, x \rangle - f(x)\}$$

to denote the convex conjugate of $f$. Define $\Delta^{[M, K-1]}$ to be the following set

$$\Delta^{[M, K-1]} := \left\{ x \in \mathbb{R}^K_+ : \sum_{i=1}^K x_i = M \right\}.$$

Unless stated otherwise, for any strictly convex function $f$, we define $f$ as follows: for all $x \in \mathbb{R}^K$,

$$f(x) := \begin{cases} f(x) & x \in \Delta^{[M, K-1]} \setminus \Delta^{[M, K-1]} \end{cases}$$

We say a convex function $f$ is Legendre if

1) $\text{int}(A)$ is non-empty;
2) $f$ is differentiable on $\text{int}(A)$;
3) $\lim_{n \to \infty} \|\nabla f(x_n)\|_2 \to \infty$ for any sequence $(x_n)_{n=1}^\infty$ in $\text{int}(A)$ converging to some $x \in \partial \text{int}(A)$.

III. PROBLEM SETTING

We consider a slotted-time system containing a central base-station and $K$ users. We assume the base-station can serve $1 \leq M \leq K$ users at a time, by performing data transmission over the $M$ channels to the users. Denote by $A$ the set of all feasible choices of subset of users to serve, i.e., $A := \{S \subseteq [K] : |S| = M\}$.

The state of each channel at each time slot is assumed to vary arbitrarily. Specifically, denote by $r_{t,i}$ the amount of data that will be transmitted to user $i$ if the base-station decides to serve user $i$ at time $t$. $r_{t,i}$ is determined by the environment at the beginning of time slot $t$, simultaneously when the base-station makes its decision (that is, to choose a subset of users of size $M$ to serve). We assume all $r_{t,i}$ values are in $[0, 1]$ and make no further assumptions on $r_{t,i}$, i.e., the environment can be arbitrary.

Denote by $S_t$ the set of users the base-station chooses to serve at time $t$, and let $A_{t,i}$ be the indicator of whether user $i$ is served at time slot $t$, i.e., $A_{t,i} := 1 [i \in S_t]$. For any time horizon of finite length $T$, the total amount of data successfully transmitted is

$$U_T := \sum_{t=1}^T \sum_{i \in S_t} r_{t,i} = \sum_{t=1}^T \sum_{i=1}^K A_{t,i} r_{t,i}.$$

For each time index $t$, the values of $r_{t,i}$ are not known to the base-station. Only values of the served users at time $t$ will be revealed to the base-station eventually. To be specific, there is a delay length sequence $(d_t)_{t=1}^\infty$, $d_t \in \mathbb{N}$, chosen by the environment beforehand (unknown to the base-station a-priori). Suppose at time $t$ the base station serves users in $S_t$, then, at the end of time slot $t+d_t$, the base-station will receive a vector

$$\tilde{r}_t = (A_{t,1} r_{t,1}, A_{t,2} r_{t,2}, \ldots, A_{t,K} r_{t,K})$$

from the environment.

The design objective of a scheduling policy is to maximize $U_T$, the total data transmitted. Since in many cases we are dealing with an indefinite time horizon rather than optimize over a fixed finite $T$, it is more convenient to introduce the classical performance metric called the pseudo-regret (referred to as regret below for convenience):

**Definition 1** (Pseudo-regret of a Scheduling Policy). We define

$$R_T := \max_{S \in A} \mathbb{E} \left[ \sum_{t=1}^T \sum_{i \in S} r_{t,i} - \sum_{t=1}^T \sum_{i \in S_t} r_{t,i} \right]$$

(1)

to be the pseudo-regret of an MAB algorithm, where the expectation is taken with respect to both the scheduling policy’s internal randomness and randomness from the environment.

It is easy to see that maximizing $U_T$ is equivalent to minimizing $R_T$. Following the convention in bandit optimization problems, we are asking for a scheduling policy achieving $o(T)$ total regret.

Our formulation can be viewed as a bandit optimization problem, where each user in our setting is a bandit arm. The base-station is an agent that needs to select a subset of arms of size $M$ at each time step, and the reward received is total amount of data transmitted. In this regard, our model generalizes the $M$-set adversarial semi-bandit problem [10], where there is no feedback delay, i.e., all $d_t$’s are equal to 0, and there have been algorithms achieving $O(\sqrt{MTK})$ total regret, e.g., [10], and the adversarial MAB problem with feedback delays, which is a special case of our setting where $M = 1$, and there have been algorithms achieving $O(\sqrt{KT} + \sqrt{T \log K})$ total regret, e.g., [11].

Follow the convention of online optimization problems with delayed feedback, we will heavily use the following two quantities in derived regret bounds, the individual delay upper bound $d := \max_{t \in [T]} d_t$ and total delay $D := \sum_{t=1}^T d_t$.

Unless stated otherwise, we will use

$$F_t = \sigma \left(S_1, \ldots, S_t, \tilde{r}_1 [1 + d_1 \leq t], \ldots, \tilde{r}_t [t + d_t \leq t] \right)$$

to denote the filtration of $\sigma$-algebra when studying random quantities indexed by time.

Below, we will present our algorithms and results. For ease of presentation, all proofs are presented in the appendix.

IV. PRELIMINARIES

Our proposed algorithm for this wireless scheduling problem is based upon a recent novel Online Mirror Descent (OMD) framework called Banker-OMD [1], which is a natural generalization of classical OMD method to robustly handle feedback delays in online optimization problems. In this section we present a brief overview of the classical OMD method and Banker-OMD, and apply them to solving the
M-set Semi-Bandit problem. The algorithms here will serve as the building block for designing the scheduling algorithm.

A. Online Mirror Descent for Non-delayed M-set Semi-Bandit

Mirror descent is a concept originated in classical optimization [12]. To apply mirror descent in online bandit optimization problems, the typical approach is to design an unbiased estimator for the true reward (or loss) vector, which is \( r_t \) in our setting. Then, regard this loss estimator as a “gradient” and use it to take a gradient descent step, where the update is performed in the dual space induced by some Legendre function \( \Psi \) rather than in the primal space. Algorithm 1 below presents our algorithm for solving the M-set Bandit problem based on OMD.

**Algorithm 1: Online Mirror Descent for Non-delayed M-set Semi-Bandit**

**Input:** Number of arms \( K \), Number of arms to pull at each time step \( M \), time horizon length \( T \),
Legendre function \( \Psi : \mathbb{R}^K \to \mathbb{R} \), initial mixed action \( x_1 \in \Delta^{[M,K-1]} \), action scales \( \sigma_1, \ldots, \sigma_T \)

**Output:** A sequence of actions \( S_1, S_2, \ldots, S_T \in \mathcal{A} \)

1. For \( t = 1, 2, \ldots, T \) do
   2. Let \( p_t \) be a distribution on \( \mathcal{A} \) such that
      \[ x_{t,i} = \text{Pr}_{S=p_t}[i \in S] \text{ for all } i \in [K]; \]
   3. Sample \( S_t \in \mathcal{A} \) according to \( p_t \), serve the \( M \) users in \( S_t \), receive the feedback vector \( \tilde{r}_t \);
   4. Compute the importance sampling loss estimate vector \( \hat{\bar{r}}_t \) by \( \hat{\bar{r}}_{t,i} = \frac{(1-r_{t,i})A_{t,i}}{x_{t,i}} \);
   5. Select action \( x_{t+1} = \arg \min_{x' \in \Delta^{[M,K-1]}} \langle \hat{\bar{r}}_{t,i}, x' \rangle + \sigma_t D_f(x', x_t) \),
   6. end

The choice of \( \hat{\bar{r}}_t \) in Algorithm 1 (Line 4) guarantees that \( \mathbb{E}[\hat{\bar{r}}_t | F_{t-1}] = 1 - r_t \). Recall that in our problem setting, all reward values picked by the environment are \([0,1]\)-bounded, equivalently we can regard serving user \( i \) at time \( t \) as an action that incurs a loss of \( 1 - r_{t,i} \), and in this sense the \( \hat{\bar{r}}_t \) in Line 4 is an unbiased estimate for the true loss vector \( \hat{\bar{r}}_t := 1 - r_t \).

To establish the performance of Algorithm 1, we make use of the following lemma, which concerns the regret incurred in a single time-step to the OMD update happened in that time-step. This lemma is a standard result and can be found in many OMD textbooks and literatures (e.g., [13] and Chapter 28 of [14]).

**Lemma 1.** For any \( \sigma_t > 0 \), \( x, y \in \Delta^{[M,K-1]} \), \( l_t \in \mathbb{R}^K_+ \) and Legendre function \( \Psi : \mathbb{R}^K \to \mathbb{R} \)

\[
\langle l_t, x_t - y \rangle \leq \sigma_t D_f(y, x_t) - \sigma_t D_f(y, z_t) + \sigma_t D_f(x, z_t) \tag{2}
\]

where

\[
z_t = \arg \min_{x' \in \Delta^{[M,K-1]}} \langle l_t, x' \rangle + \sigma_t D_f(x', x_t), \tag{3}
\]

\[
z_t = \arg \min_{x' \in \mathbb{R}^K_+} \langle l_t, x' \rangle + \sigma_t D_f(x', x_t), \tag{4}
\]

or equivalently,

\[
z_t = \nabla \Psi^* (\nabla \Psi(x_t) - \frac{1}{\sigma_t} l_t), \quad z_t = \nabla \Psi^* (\nabla \Psi(x_t) - \frac{1}{\sigma_t} l_t). \quad \text{(5)}
\]

We see that Algorithm 1 is effectively obtained by taking an OMD step on the last mixed action \( x_t \) using \( \hat{\bar{r}}_t \) as the gradient and \( 1/\sigma_t \) as the step size, and then taking the computed \( z_t \) in Lemma 1 as the next mixed action. This procedure naturally leads to a telescoping sum regret upper-bound when all \( \sigma_t \)'s are set to the same constant. Then, by properly choosing the constant we can achieve \( O(\sqrt{T}) \) (factors in \( K \) and \( M \)) omitted) total regret.

By summing up the inequality in Lemma 1, we get the following regret bound for Algorithm 1:

**Theorem 2.** The total regret of Algorithm 1 satisfies

\[
\mathcal{R}_T \leq \sup_{\Psi \in \mathcal{A}} \left[ \sum_{t=1}^T \{ \sigma_t D_f(y, x_t) - \sigma_t D_f(y, x_{t+1}) + \sigma_t D_f(x_t, z_t) \} \right]
\]

where \( z_t = \nabla \Psi^* (\nabla \Psi(x_t) - \frac{1}{\sigma_t} l_t) \).

In particular, when \( \Psi(x) = -\sum_{i=1}^K 2\sqrt{x_i}, \quad x_1 = (M/K, \ldots, M/K) \) and \( \sigma_1 = \cdots = \sigma_T = \sqrt{T} \), we have the following regret bound for Algorithm 1:

**Theorem 3.** If Algorithm 1 is running with \( \Psi(x) = -\sum_{i=1}^K 2\sqrt{x_i}, \quad x_1 = (M/K, \ldots, M/K) \) and \( \sigma_1 = \cdots = \sigma_T = \sqrt{T} \), then we have

\[
\mathcal{R}_T \leq O(\sqrt{MK^2 T}).
\]

**Remark.** Under this particular choice of \( \Psi \) and \( x_1 \), Algorithm 1 and our novel Algorithm 3 presented in Section V become identical when running on problem instances without delay. Therefore Theorem 3 is a direct corollary of Theorem 6 in Section V, our main regret bound for Algorithm 3.

B. The Banker-OMD Framework

A key observation in Theorem 2 is that when \( \sigma_1, \ldots, \sigma_T \) are all chosen to be a constant, the sum \( \sum_{t=1}^T \{ \sigma_t D_f(y, x_t) - \sigma_t D_f(y, x_{t+1}) \} \) in Theorem 2 will become a telescoping sum and equal to \( \sigma_1 D_f(y, x_1) - \sigma_1 D_f(y, x_{T+1}) \). However, in order to compute \( z_t \), we need to compute the loss estimator \( \hat{\bar{r}}_t \) first, which relies on the observed feedback vector \( \tilde{r}_t \). In the general case where there are feedback delays, i.e., \( d_t \) take a positive value, we will not be able to compute \( z_t \) in time at the end of \( t \)-th time slot, and therefore unable to directly set \( x_{t+1} \) to \( z_t \).

To tackle this drawback of classical OMD method, [1] proposed the Banker-OMD framework, which is capable of naturally proceeding even when there are absent feedback due to delay. Banker-OMD builds upon the core observation that Lemma 1 applies to any mixed action sequence \( x_1, \ldots, x_T \) and action scale sequence \( \sigma_1, \ldots, \sigma_T \) and gives a valid upper-bound for the pre-expectation total regret \( \sum_{t=1}^T \langle l_t, x_t - y \rangle \).
Under Banker-OMD, the task of getting a tight regret bound reduces to designing a policy to choose \( x_t \)'s and \( \sigma_t \)'s to make \( \sum_{t=1}^{T} \{ \sigma_tDQ(y, x_t) - \sigma_iDQ(y, z_t) \} \) small. In Banker-OMD the \( \sigma_tDQ(y, x_t) \) term is called a “withdrawal cost,” for it is a valid upper-bound for (because of the immediate cost term \( \sigma_tDQ(x_t, z_t) \)) the loss incurred due to taking the action \( x_t \), without the need of knowing the action feedback \( r_t \). After receiving \( r_t \), we can refine this upper-bound to \( \sigma_tDQ(y, x_t) - \sigma_iDQ(y, z_t) \), where the term \( -\sigma_tDQ(y, z_t) \) is called the “saving term” and is some resource we can leverage only after receiving the action feedback \( r_t \). From this reasoning, the task of getting a tight regret bound can be further reduced to properly covering the withdrawal cost of a new action with savings at hand.

The next key observation of Banker-OMD is that, suppose at some moment we have available saving terms \( \sigma_iDQ(y, z_1), \ldots, \sigma_iDQ(y, z_h) \) at hand, then a new action obtained from “convex combination in the dual space” over available \( z_i \)'s always has its withdrawal cost properly covered. This fact is formally stated in the following Lemma 4, which is an adapted version of Lemma 2 of \cite{BankerOMD}, extending the feasible action set from \( \Delta^{[I,K-1]} \) in Lemma 2 of \cite{BankerOMD} to \( \Delta^{[M,K-1]} \).

**Lemma 4.** For any \( h \geq 1, z_1, \ldots, z_h \in \Delta^{[M,K-1]} \), \( \sigma_1, \ldots, \sigma_h > 0 \) and Legendre convex function \( \Psi : \mathbb{R}^h_+ \to \mathbb{R} \), let \( \sigma = \sum_{i=1}^{h} \sigma_i \) and

\[
x = \nabla \Psi^* \left( \sum_{i=1}^{h} \frac{\sigma_i}{\sigma} \nabla \Psi(z_i) \right),
\]

then we have \( \sigma DQ(y, x) \leq \sum_{i=1}^{h} \sigma_i DQ(y, z_i) \) for any \( y \in \Delta^{[M,K-1]} \).

The Banker-OMD framework in \cite{BankerOMD} works as follows. Lemma 4 states that if at the beginning of time \( t \) we have available saving terms \( \sigma_1DQ(y, z_1), \ldots, \sigma_hDQ(y, z_h) \), i.e., feedback of actions at time \( t_1, \ldots, t_h \) have all arrived and \( z_1, \ldots, z_h \) have not been utilized in any form. Let \( \sigma = \sum_{i=1}^{h} \sigma_i \) then we are allowed to pick a new action \( x_t \) with scale \( \sigma_t = \sigma \) specified by \( \sigma_t = \nabla \Psi^* \left( \sum_{i=1}^{h} \frac{\sigma_i}{\sigma} \nabla \Psi(z_i) \right) \). The withdrawal term introduced by playing this \( x_t \) at scale \( \sigma_t \) is then guaranteed to be covered by those \( h \) saving terms.

It can happen that in a time slot, savings can fall short, e.g., no feedback has arrived, but we still need to make a decision in run-time. In this case, we can use a scale \( \sigma > \sigma_t \). To do so, let

\[b_t = \sigma_t - \sigma.\]

Then, consider an \( x_0 \in \Delta^{[M,K-1]} \).

We can add two new terms \( + b_tDQ(y, x_0) - b_tDQ(y, x_0) \) (sum to zero) to the current upper-bound for the total regret. Then, we can regard the minus-signed term \( - b_tDQ(y, x_0) \) as an additional saving term, and use it together with the \( h \) available previous saving terms to form the new \( x_t \) by

\[x_t = \nabla \Psi^* \left( \sum_{i=1}^{h} \frac{\sigma_i}{\sigma} \nabla \Psi(z_i) + \frac{b_t}{\sigma} \nabla \Psi(x_0) \right),\]

Lemma 4 then asserts the net effect of playing \( x_t \) scale \( \sigma_t \) is to use up the \( h \) saving terms and add a \( + b_tDQ(y, x_0) \) term into the upper-bound. This operation can be interpreted as: when the total savings are not enough for the new action we want to form, we can invest in some \( x_0 \) to make up the difference and proceed.

Based on the above banker idea and the Banker-OMD algorithm from \cite{BankerOMD}, we design Algorithm 2 Banker-OMD-M for our delayed \( M \)-set semi-bandit problem. We emphasize that Banker-OMD-M is a general algorithm framework, it offers great flexibility in algorithm design, i.e., the regularizer \( \Psi \) and the scales \( \sigma_t, b_t, m_t \). In Banker-OMD-M, \( \Psi \) is a policy to choose action scales \( \sigma_t \), in practice \( \sigma_t \) is chosen to a function of the current time index \( t \) and statistics of experienced delays. \( \Psi \) is the policy for deciding the desired \( \sigma_t \) based on available savings and possible new investment. At the beginning of each time step \( t \), \( \Psi \) needs to output \( t \) non-negative numbers \( b_1, m_{t,1}, \ldots, m_{t,t-1} \) such that \( b_t + \sum_{i=1}^{t-1} m_{t,i} = \sigma_t \) and \( m_{t,i} \leq v_i \) for all \( 1 \leq i \leq t - 1 \).

The difference between Banker-OMD-M and Banker-OMD is that the feasible action set is \( \Delta^{[M,K-1]} \) rather than the unit simplex, and we need to appropriately sample actions (Line 13) and compute loss estimates (Line 17). Similar to the original Banker-OMD framework, the key feature of Banker-OMD-M is that it explicitly maintains an OMD-style upper-bound for cumulative pseudo-regret. At any time during the execution, its internal state variables directly give a regret upper-bound stated in the following Theorem 5, which adapts Theorem 3 of \cite{BankerOMD} and extends the feasible action set to \( \Delta^{[M,K-1]} \).

**Theorem 5 (Banker-OMD-M Regret Bound).** For Algorithm 2, at the end of any time \( T \), for any \( y \in \Delta^{[M,K-1]} \), we have

\[
\sum_{t=1}^{T} (\tilde{I}_t, x_t - y) \leq B_T \cdot DQ(y, x_0) + \sum_{t=1}^{T} \sigma_tDQ(x_t, z_t) - \sum_{t=1}^{T} v_tDQ(y, z_t)
\]

for any \( 1 \leq t \leq T \).

**V. THE BANKER-OMD-SCHEDULING ALGORITHM**

We are now ready to present our algorithm for the wireless scheduling problem with channel variation and feedback delay. The proposed algorithm Banker-OMD-Scheduling (Algorithm 3) is an application of the Banker-OMD-M framework introduced in Section IV-B using the regularizer capable to achieve \( O(\sqrt{MK}) \) total regret in the non-delayed setting, described in Section IV-A.

Comparing to the general framework Algorithm 2, Banker-OMD-Scheduling chooses a specific regularizer \( \Psi(x) = -2 \sum_{i=1}^{K} \sqrt{\alpha_i} \), which is known as the 1/2-Tsallis entropy function \cite{Tsallis}, and default investment option \( x_0 = (M/K, \ldots, M/K) \). In Banker-OMD-Scheduling, the subroutine used to make plan of integrating available savings and possible new investment to the desired new action...
Algorithm 2: Banker-OMD-M

Input: Number of arms $K$, number of arms to pull at each time step $M$, regularizer $\Psi$, default investment option $x_0 \in \Delta^{[M,K-1]}$, subroutine to pick action scales $\mathcal{S}$, subroutine to pick portfolio $\mathcal{P}$
Output: A sequence of actions $S_1, S_2, \ldots \in A$

1. $B_0 \leftarrow 0$; // $B$ will be maintained as the total investment to $s_0$
2. for $t = 1, 2, \ldots$
   3. \quad $a_t \leftarrow 0$; // $a_t$ indicates whether time $t$'s feedback has arrived
   4. $t_0, \ldots, t_{S-1}, a_{t_0}, \ldots, a_{t_0-1}, \ldots, v_{t_1}, \ldots, v_{t_1-1} =$ (history to up time $t-1$)
   5. $\sigma_t \leftarrow \Theta(t, h_t)$; // decide the scale for the new action
   6. $v_t \leftarrow \sigma_t v_t$; // $v_t$ is the current coefficient of the minus-signed $D\Phi(y, x_t)$ term
   7. $b_t, m_{t,1}, \ldots, m_{t,t-1} \leftarrow \mathcal{P}(t, h_t, \sigma_t)$; // determine to use how much volume of available saving terms to form $x_t$
   8. $x_t \leftarrow \nabla \Psi^*\left( \frac{1}{\sigma_t} \sum_{i=1}^{t-1} m_{t,i} \nabla \Psi(z_i) + \frac{\sigma_t}{\tau} \nabla \Psi(x_0) \right)$;
   9. $B_t \leftarrow B_{t-1} + b_t$
   10. for $i = 1$ to $t-1$
       11. \quad $v_i \leftarrow v_i - m_{i,i}$; // spend $m_{i,i}$ units of the saving term $D\Phi(y, z_i)$
   12. end
   13. Let $p_t$ be a distribution on $A$ such that $x_{t,i} = \mathbb{P}_{S_{t-i}}[i \in S]$ for all $i \in [K]$;
   14. Sample $S_t \in A$ according to $p_t$, serve the $M$ users in $S_t$;
   15. for upon receiving each new feedback $(s, r_s)$ do
       16. Compute the importance sampling loss estimate vector $I_{s,t} \leftarrow \frac{1}{s_{t,i}} A_{S_{t,i}}$
       17. $z_s \leftarrow \nabla \Psi^* (\nabla \Psi(x_s) - \frac{1}{\sigma_t} l_s)$
       18. $a_t \leftarrow 1$; // the saving term $-\sigma_t D\Phi(y, x_s)$ becomes available
   19. end
   20. end

scale $\sigma_t$ is the following procedure GreedyPick (simply ported from [1]), which chooses $m_{t,i}$'s greedily as large as possible and minimizes the new $b_t$:

Algorithm 3 maintains $b_t$, the number of feedback that have not arrived at the beginning of time $t$, and computes $D_t = \sum_{s=1}^t b_s$ to keep track of the cumulative experienced delay up to time $t$ at run-time. Algorithm 3 then uses the action scale $\sigma_t = \left( \frac{1}{\tau} + b_t \frac{\ln(D_{t+1})}{2D_t} \right)^{-1}$.

Remark. The last degree of freedom in Algorithm 3 is how to construct of the distribution $p_t$ in Line 13 and how to implement a corresponding sampling scheme in Line 14. [15], [16] showed that sampling according to mean vector $x_t$ can be

Algorithm 3: Banker-OMD-Scheduling

Input: Total number of users $K$, number of users to serve at each time step $M$
Output: A sequence of actions $S_1, S_2, \ldots \in A$

1. Let $\Psi$ be the function $\Psi(x) = -2 \sum_{i=1}^K x_i$;
2. $D_0 \leftarrow 0$;
3. for $t = 1, 2, \ldots$
   4. \quad $d_t \leftarrow \text{the number of previous actions whose feedback has not arrived}$
   5. $D_t \leftarrow D_{t-1} + d_t$
   6. $\sigma_t \leftarrow \left( \frac{1}{\tau} + \frac{D_t}{\ln(D_{t+1})} \right)^{-1}$;
   7. $v_t \leftarrow \sigma_t v_t$;
   8. $b_t, m_{t,1}, \ldots, m_{t,t-1} \leftarrow \text{GreedyPick}(\sigma_t, v_1, \ldots, v_{t-1}, a_1, \ldots, a_{t-1})$;
   9. $x_t \leftarrow \nabla \Psi^* (\frac{1}{\sigma_t} \sum_{i=1}^{t-1} m_{i,t} \nabla \Psi(z_i))$; // equivalent to choosing $x_0 = \nabla \Psi^*(0) = (M/K, \ldots, M/K)$
   10. for $i = 1$ to $t-1$
       11. \quad $v_i \leftarrow v_i - m_{i,i}$
   12. end
   13. Let $p_t$ be a distribution on $A$ such that $x_{a_{t,i}} = \mathbb{P}_{S_{t-i}}[i \in S]$ for all $i \in [K]$;
   14. Sample $S_t \in A$ according to $p_t$, serve the $M$ users in $S_t$;
   15. for upon receiving each new feedback $(s, r_s)$ do
       16. Compute the importance sampling loss estimate vector $I_{s,t} \leftarrow \frac{1}{s_{t,i}} A_{S_{t,i}}$
       17. $z_s \leftarrow \nabla \Psi^* (\nabla \Psi(x_s) - \frac{1}{\sigma_t} l_s)$
       18. $a_s \leftarrow 1$;
   19. end
   20. end

Procedure GreedyPick (from [1])

Params: Time index $t$; Current available saving coefficients $v_1, \ldots, v_{t-1}$; Availability flags $a_1, \ldots, a_{t-1}$; Target scale $\sigma_t$.
Output: $b_t, m_{t,1}, \ldots, m_{t,t-1}$.

1. $\forall 1 \leq i \leq t-1$; $m_{t,i} \leftarrow 0$;
2. $b_t \leftarrow \sigma_t$;
3. for $i = 1$ to $t-1$
   4. \quad if $a_i = 1$ then
      5. \quad \quad if $v_i \leq b_t$ then
         6. \quad \quad \quad $m_{t,i} \leftarrow v_i$; $b_t \leftarrow b_t - v_i$;
      7. \quad \quad else
         8. \quad \quad \quad $m_{t,i} \leftarrow b_t$; $b_t \leftarrow 0$;
      9. \quad \quad break;
   10. \quad end
   11. end
   12. end
achieved in \(O(K^2)\) time, [17] further proposed an \(O(K \log K)\) sampling algorithm.

The regret bound for Algorithm 3 is summarized in the following theorem.

**Theorem 6.** For the wireless scheduling problem under arbitrary channel dynamics and total feedback delay \(D\), over any time horizon length \(T\), Banker-OMD-Scheduling guarantees that

\[
\mathcal{R}_T \leq O(\sqrt{MK(\sqrt{T} + D \log D)}).
\]

To our knowledge, this is the first work achieving \(O(\sqrt{T} + \sqrt{D})\) regret in this delayed \(M\)-set semi-bandit optimization problem and our scheduling problem. In most real-world scenarios, the individual delay \(d_t\) is \(o(n^\alpha)\) for any \(\alpha > 0\), under which Algorithm 3 guarantees that the relative overhead is vanishing as \(T \to \infty\).

**VI. CONCLUSION**

In this paper, we present the Banker-OMD-Scheduling algorithm for solving the scheduling problem under arbitrary channel state dynamics and delayed feedback in downlink transmission. Banker-OMD-Scheduling is designed based on a recent banker online mirror descent framework in [1]. We prove that Banker-OMD-Scheduling achieves a sublinear regret and allows efficient implementation.

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**REFERENCES**


**APPENDIX**

**TECHNICAL PROOFS**

We present the detailed proofs in the appendix. To begin with, we review some useful properties of Legendre functions.

**Lemma 7.** Let \(C \subseteq \mathbb{R}^n\) be a convex set, \(f : C \to \mathbb{R}\) be a Legendre function. Then,

1. \(\nabla f\) is a bijection between \(\text{int}(C)\) and \(\text{int}(\text{dom}(f^*))\) with the inverse \((\nabla f)^{-1} = \nabla f^*\);
2. \(D_f(y, x) = D_f^*(\nabla f(x), \nabla f(y))\) for all \(x, y \in \text{int}(C)\);
3. the convex conjugate \(f^*\) is Legendre.

The proof for Lemma 7 can be found in many convex analysis textbook, e.g., Chapter 26 of [14]. We are now ready to give proofs for lemmas and theorems in the main text.

**A. Proof of Lemma 1**

Lemma 1 is a basic property of mirror descent update steps, the proof is a combination of some basic properties of convex functions and Bregman divergences, and it can be found in many OMD textbooks and literatures, e.g., [13] and Chapter 28 of [14]. For completeness, we also present a proof here.

**Proof of Lemma 1.** Since \(\Psi\) is Legendre, \(\nabla \Psi\) will explode on \(\partial K\), which guarantees that the minimizer \(x^*\) in the definition of \(z_t\) in (4) will lie in \(\text{int}(\mathbb{R}^K)\) and \(\frac{\partial^2}{\partial x}\left[\langle l_t, x^* \rangle + \sigma_t D_{\Psi}(x^*, x_t)\right] = 0\). The bijection property in Lemma 7 then asserts this arg min definition is equivalent to the definition in (5) using mirror maps \(\nabla \Psi\) and \(\nabla \Psi^*\). Since \(\Psi\) is a Legendre function on \(\Delta^{|M,K-1|}\), a similar argument suggests that the definitions for \(z_t\) in (3) and (5) are equivalent.

The definition of \(z_t\) in (5) implies \(l_t = \sigma_t (\nabla \Psi(x_t) - \nabla \Psi(z_t))\). The first order optimality condition of \(z_t\) in (3) implies that \(\langle \frac{1}{\sigma_t} l_t + \nabla \Psi(z_t) - \nabla \Psi(x_t), y - z_t \rangle \geq 0\) for any \(y \in \Delta^{|M,K-1|}\). Hence we have

\[
\langle l_t, x_t - y \rangle = \langle l_t, x_t - z_t \rangle + \langle l_t, z_t - y \rangle \leq \sigma_t \langle \nabla \Psi(x_t) - \nabla \Psi(z_t), x_t - z_t \rangle
\]
Lemma 8.

B. Proof of Lemma 4 and Theorem 5

Theorem 6.

C. Technical Lemmas for Theorem 6

Remark. This upper-bound is $\sqrt{M}$ times larger than the classical MAB setting (i.e., $M = 1$), and the argument need is also similar. To prove Lemma 8, we adapt the proof from Lemma 11 of [1] to fit our feasible action set $\Delta^{[M,K-1]}$ and the choice $x_0 = (M/K, \ldots, M/K)$.

Proof of Lemma 8. The first bound is quite immediate, for $x_0 = (M/K, \ldots, M/K)$, we have

$$D_\Psi(y, x_0) = \Psi(y) - \Psi(x_0) - \langle \nabla \Psi(x_0), y - x_0 \rangle \leq 2 \sum_{i=1}^{K} \sqrt{x_{0,i}} \leq 2\sqrt{MK}$$

for any $y \in \Delta^{[M,K-1]}$.

For the second bound, in fact, for any choice of $\Psi$ and $t \geq 1$ we have

$$\sigma_t D_\Psi(x_t, \hat{z}_t) = \sigma_t D_\Psi(\nabla \Psi(\hat{z}_t), \nabla \Psi(x_t))$$

$$= \sigma_t D_\Psi(\nabla \Psi(x_t) - \frac{\bar{l}_t}{\sigma_t} \nabla \Psi(x_t))$$

$$\leq \Psi^*(\nabla \Psi(x_t) - \frac{\bar{l}_t}{\sigma_t}) - \Psi^*(\nabla \Psi(x_t)) - \left\| \frac{\bar{l}_t}{\sigma_t} \right\|_{\Psi^*}$$

where in the last step we write the Bregman divergence into a second order Lagrange remainder, $\theta_t$ is some element inside the line segment connecting $\nabla \Psi(x_t) - \frac{\bar{l}_t}{\sigma_t}$ and $\nabla \Psi(x_t)$.

Note that under this particular $\Psi(x) = -2 \sum_{i=1}^{K} \sqrt{x_i}$, we have

- $\nabla^2 \Psi^*(\cdot) = \text{diagonal}$
- The diagonal elements of $\nabla^2 \Psi^*$ are non-decreasing on the line segment $[\nabla \Psi(x) - \frac{1}{\sigma_t} \bar{l}_t, \nabla \Psi(x_t)]$.

and we can further upper-bound (8) by $\frac{1}{2\sigma_t} \| \nabla \Psi(x_t) \|_{\Psi^*} = \frac{1}{2\sigma_t} \| \nabla \Psi(x_t) \|_{\Psi^*}$. Therefore,

$$\mathbb{E}[\sigma_t D_\Psi(x_t, \hat{z}_t) \mid F_{t-1}] \leq \frac{\sqrt{MK}}{\sigma_t}$$

for any $y \in A$.

In Algorithm 3, for any $t \geq 1$, we have

$$\mathbb{E}[\sigma_t D_\Psi(x_t, \hat{z}_t) \mid F_{t-1}] \leq \frac{\sqrt{MK}}{\sigma_t}$$

where $\hat{z}_t = \nabla \Psi^*(\nabla \Psi(x_t) - \frac{1}{\sigma_t} \bar{l}_t)$. 

Proof.

\begin{align*}
+ \sigma_t (\nabla \Psi(z_t) - \nabla \Psi(x_t), y - z_t) \\
\overset{(a)}{=} \sigma_t (D_\Phi(x_t, z_t) + D_\Phi(x_t, z_t) - D_\Phi(y, z_t)) \\
- \sigma_t (D_\Phi(y, z_t) + D_\Phi(y, x_t) - D_\Phi(y, x_t)) \\
= \sigma_t D_\Phi(y, x_t) - \sigma_t D_\Phi(y, z_t) + \sigma_t D_\Phi(x_t, z_t) - \sigma_t D_\Phi(x_t, \hat{z}_t) \\
\leq \sigma_t D_\Phi(y, x_t) - \sigma_t D_\Phi(y, z_t) + \sigma_t D_\Phi(x_t, \hat{z}_t)
\end{align*}

where (a) uses the following “three-point identity” of Bregman divergences:

$$D_\Phi(a, b) + D_\Phi(b, c) - D_\Phi(a, c) = \langle \nabla \Psi(c) - \nabla \Psi(b), a - b \rangle.$$
where the last step is due to Cauchy-Schwatz inequality. □

The second result we need is an upper-bound of the sum of \( \sigma_t^{-1} \) and the final value of \( B_T \), both from [1]:

**Lemma 9** (Theorem 6 of [1]). *For any time horizon length \( T \geq 1 \), let \( D = \sum_{t=1}^{T} d_t \) denote the total delay during the first \( T \) time steps, then we have

\[
\sum_{t=1}^{T} \sigma_t^{-1} \leq O(\sqrt{T} + \sqrt{D \log D}).
\]

*Furthermore, at the end of the \( T \)-th time slot, we have

\[
B_T \leq O(\sqrt{T} + \sqrt{D \log D}).
\]

To obtain Theorem 6, we plug the bounds in Lemma 8 and Lemma 9 into Theorem 5 and then take expectation.*