# Toward Service Selection Game in a Heterogeneous Market Cloud Computing

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***Abstract*—We take the first step to study the price competition in a heterogeneous market cloud computing formed by public provider and cloud broker, all of which are also known as cloud service providers. We formulate a price competition between cloud broker and public provider as a two-stage non-cooperative game. In stage one, where cloud service providers set their service prices to maximize their revenue, we use the Nash equilibrium concept to study the equilibria for the price setting game. Cloud users can select the services (from the cloud broker or public provider) that provide them the best payoff in terms of performance (i.e., delay) and price. To that end, cloud users can adapt their service selection behavior by observing the variations in price and quality of service offered by the different cloud service providers. For the service selection game of cloud users in stage two, we use the evolutionary game model to study the evolution and the dynamic behavior of cloud users. Furthermore, the Wardrop equilibrium and replicator dynamics is applied to determine the equilibrium and its convergence properties of the service selection game. Numerical results illustrate that our game model captures the main factors behind the heterogeneous market cloud pricing and service selection, thus represents a promising framework for the design and understanding of the heterogeneous market cloud computing.**

## I. Introduction

Recently, cloud computing is becoming more and more popular in large-scale computing due to its ability to share data and computations over a network of scalable nodes. As the cloud computing market is growing, cloud users have to deal with many different service types, pricing schemes, cloud interfaces and the complexity of the cloud market. In the beginning of cloud deployment, the public cloud provider or hybrid public and private cloud infrastructure model dominates the market. However, cloud market trend shows that market share of the multi-cloud or federated clouds [1], [2], [3] are increasing. The multi-cloud model can integrate resources from different providers, which increases scalability and reliability and reduces cost while accessing to the resources is transparent to users [4], [5]. In multi-cloud architecture illustrated in Figure 1, a cloud broker is essential to transform

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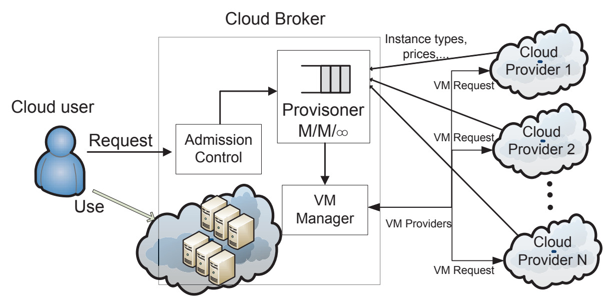
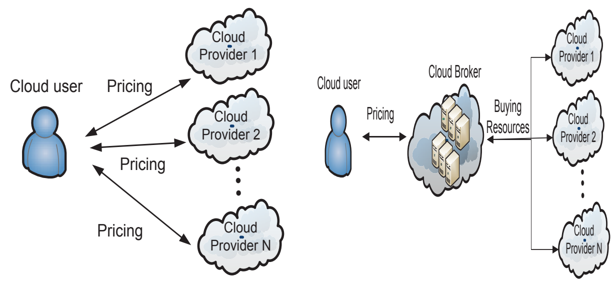



Fig. 1. Multi-cloud architecture with Cloud Broker.

the cloud market into a commodity-like service [6]. Also, the cloud broker offers a Provisioner which analyzes the workload, schedules VMs placement among multiple clouds and optimizes deployments [7]. Moreover, VM manager in the cloud broker could be used to provide a uniform interface with VM's management, independent of the particular cloud provider technology [6].

Pricing is the process of computing the exchange value of resources relative to a common form of currency. Multi-cloud model can be formed by combining public and private clouds to provide users with resizable and elastic capacities [8]. Currently, companies such as Amazon operate as standalone cloud service providers (a public provider) (Fig. 2.a). However, in a multi-cloud model (Fig. 2.b), a cloud broker offer cloud services to users as a cloud service provider [9]. The cloud broker (a third party) acts as a mediator between the cloud user and the cloud provider. Cloud users buy resources in advance from the cloud broker instead of cloud provider for getting additional benefit (e.g., compensation). The cloud brokerage model can be used to offer a commendable pricing mechanism which considers cheaper service for cloud user as well as more profit for cloud provider [9]. The cloud brokerage pricing model is somewhat similar to the price discrimination [10] (different price for the same service in different segments of the market).

In this paper, we take the first step to study the price competition in a heterogeneous market cloud formed by cloud service providers (CSPs) (i.e., cloud broker and public




(a) Standalone Cloud Service Providers. (b) Multi-cloud with a cloud broker.

Fig. 2. Pricing in Standalone and Multi-cloud.

provider) and cloud users. There are two stages of competition in this heterogeneous market cloud. In stage one, we formulate the competition among CSPs in selling the service opportunities as noncooperative games, where each CSP can set a service price such that its revenue is maximized. We use $M/M/1$ and $M/M/\infty$ queue models to show correlations among the expected task finishing times, resource capacity, and the request rates (from cloud users to the cloud broker and public provider), respectively. Since the pricing strategy of a cloud provider depends on its competitors, we take a game theoretic perspective to study the strategic situation. The Nash equilibrium is considered as the solution so that none of the service providers can improve the revenue by deviating from the equilibrium. To our knowledge, this is the first study that discusses the competition among public cloud providers (public providers) and cloud brokers.

In stage two, on the cloud users' side, rational cloud users can select the service (from the cloud brokers and public providers) that provide them with the best payoff in terms of performance (i.e., delay) and price. We use Wardrop equilibrium to derive a steady-state equilibrium reached by cloud users in the service selection game. Then, we further focus on modeling the dynamic behavior of cloud users. We formulate this dynamic as a evolutionary game [11], which characterizes the strategic interactions among large numbers of users, whose behaviors are modeled as a dynamic adjustment process. Replicator dynamics [11], which are expressed as a set of differential equations, is used to model the evolution of the cloud users since such cloud users adapt their service provider selection based on the observed system state. We then provide equilibrium and convergence properties of the proposed game.

The remainder of this paper is organized as follows. Section II discusses the related work. Section III introduces the system models. In Section IV, we introduce the duopoly market model. Section V presents dynamic service selection game. Section VI shows the numerical results. The conclusions are drawn in Section VII.

## II. Related work

Considering prices charged by cloud providers, the authors in [12] and [13] used dynamic programming and microeconomics, respectively, to solve the resource allocation problems for cloud users. In [14], Kantere et al. proposed a price-demand model for a cloud cache and found an optimal price that maximizes the cloud provider's profit. [15] and [16] applied auction mechanism to find optimal prices in the cloud, in which cloud users had budgetary and deadline constraints, respectively. [17] studied dynamic cloud pricing by a proposed revenue management framework.

Nevertheless, most of these works focused on provider's pricing and the responses of cloud users via their demand functions. In this paper, we focus on the pricing mechanisms and their impact on the equilibrium behaviors of users in a strategic queueing system, where arriving users can take the delay and other metrics into account to make their service selection strategically, which can be traced back from the work of [18], [19], [20], [21]. Several works have addressed on this paradigm of wireless network such as [22], [23], [24], [25], [26]. Recent work on this paradigm of cloud is [27], in which optimal prices can be determined in a competitive environment with more than one cloud provider. However, all public cloud providers use the same $M/M/1$ queue to derive the expected delay of cloud users and they did not consider the dynamic behavior of users in such multiple cloud providers market. Recent works have considered evolutionary games to study the users' behavior in cognitive radio and heterogeneous wireless networks [28], [29], in which the authors focused on the equilibrium, system dynamics and price of anarchy of a cognitive radio networks.

## III. The System Model

We now present the system model by presenting preliminaries, cloud broker and public provider models.

### A. Preliminaries

We start by defining a heterogeneous market with public providers and cloud brokers that may wish to share a same market of cloud users. The cloud users arrive to the cloud market according to a Poisson process with rate $\lambda$. Upon arrival, each cloud user has to make a decision: 1) acquiring a multi-cloud service from a cloud broker for a guaranteed service; or 2) using the legacy public service from the public provider (i.e. Amazon). Here, we assume that the cloud broker can overcome the resource limitation by buying resources from cloud providers. Thus, the cloud broker can offer services with higher Quality of Service (QoS) than those of the public provider which has limited resource. This assumption implies that the cloud broker has an $M/M/\infty$ queue of virtualized instance. However, the public provider only has $M/M/1$ queue of virtualized instance due to the limitation of resource. It means that the cloud broker can provide a better QoS (i.e., delay) cloud service. There are two stages of competition in this heterogeneous market which is illustrated in Fig. 3. The competition in the first stage is among the cloud service

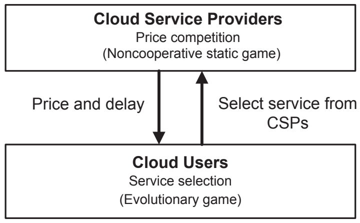



Fig. 3. Two-stage competition.

providers to sell services to the groups of cloud users. If the price and delay offered by one service provider are high, cloud user will deviate to choose the service from another service provider. Therefore, each service provider must carefully set the price so that its revenue is maximized. The competitive behavior of service providers is naturally modeled by noncooperative game model [30]. In this paper, we use noncooperative strategic game (NSG) [30] (i.e., all players' decisions are made simultaneously) and Stackelberg game (SG) [30] (i.e., one player makes a decision first in advance) to model the pricing competition between service providers. The competition in the second stage is among multiple cloud users to select cloud service offered by service providers (the cloud brokers and public providers). If many cloud users choose a cloud service provided by the same cloud provider, the corresponding service becomes congested, which may result in an performance degradation (i.e., increasing the delay). As a result, the cloud users will evolve to choose cloud service with lower price and better performance. The evolution of cloud user will stop when the cost becomes identical to the average cost of the all cloud users.

For simplicity, we consider a heterogeneous duopoly market model which has one public provider and one cloud broker. Let us denote by $\lambda_1$ the overall cloud user arrival rate at the cloud broker and by $\lambda_2$ the rate of cloud users at the public provider, so that $\lambda = \lambda_1 + \lambda_2$.

### B. Cloud Broker Model

The average cost incurred by a cloud user consists of two components: (i) the service price of the cloud broker's service $p_1$, and (ii) an average delay cost. We assume that the cloud broker always has sufficient number of servers to serve the demand of cloud user. As illustrated in Fig. 1, whenever an arriving cloud user decides to request the service to the cloud broker, the admission control unit will send the accepted request to the Provisoner. The Provisoner finds an allocation of VM among different cloud providers which optimize the user criteria and adheres to the placement constraints. VM Manager can provide a uniform management interface for operations, e.g., to deploy, monitor, and terminate VMs, with multiple VM providers. The cloud broker (cloud service provider) is modeled by an $M/M/\infty$ queue, serving a common pool of potential cloud users with infinite server, which combines the resource capacity of multiple VM providers that the VM Manager operates. The $M/M/\infty$ queuing model has been adopted by a number of existing papers in the literature that analyzed data center or cloud provider operations. In [31], the authors represent the large multi-server system as an $M/G/\infty$ system. In [32], the authors solve the "packing" of virtual machines in physical host machines problem in a network cloud having an infinite server system. We assume that the virtual resource capacity of the cloud broker is represented by its service rate $\mu$. Let $\alpha$ be the delay cost per unit time (i.e., $\alpha$ represents its urgency). The expected cost when acquiring the multi-cloud service from the cloud broker is thus given by

$$C_1 = \frac{\alpha}{\mu} + p_1. \tag{1}$$

The revenue of the cloud broker corresponds to the total revenue obtained by pricing users. As a consequence, the broker's utility function is expressed as follows

$$U_1 = \lambda_1 p_1. \tag{2}$$

### C. Public Provider Model

If a cloud user chooses to use service from the public provider, it joins a queue of cloud users who have chosen the public provider. This queue is used in order to model the delay incurred when a few cloud users wish to use the same cloud infrastructure of the public provider. Here, public provider system is modeled by an $M/M/1$ queue, serving a common pool of potential cloud users with one "virtual" server. The $M/M/1$ queuing model has been used in cloud computing literature [27], [33] in order to analyze the response time exhibited when processing requests as a function of the computational capacity and the request arrival rate.

The PP system is modeled as an $M/M/1$ queue with a service rate $\mu$. We assume that $\lambda < \mu$ for the queue stable condition. Here, we mainly consider homogenous service rates where the service rates of the PP and CB are the same (i.e., $\mu$). Based on queuing theory [34], the expected cost when acquiring the public service from the public provider is thus given by

$$C_2 = \frac{\alpha}{\mu - \lambda_2} + p_2. \tag{3}$$

Given a service price $p_2$ of the public provider, the public provider's utility function is expressed as follows

$$U_2 = \lambda_2 p_2. \tag{4}$$

## IV. Pricing Competition in Heterogenous Duopoly Market

In this section, we derive the equilibrium points of the following games, namely: (i) the equilibrium cloud users choosing the cloud broker and public provider, (ii) the equilibrium prices set by the cloud broker and public provider in a heterogeneous duopoly market. We consider two scenarios: (1) both providers fix their price at the same time (Section IV-B *Noncooperative Strategic Game*), and (2) the cloud broker set a price before the public provider, anticipating the strategy of latter, thus exploiting his dominant position (Section IV-C *Stackelberg Game*). This paper studies duopoly market for the ease of analysis, but the duopoly scenario can still provide us enough insight into pricing and user dynamics of the heterogenous market cloud. The analysis introduced in Sections IV and V can be extended to the multi public providers and cloud brokers scenarios at the expense of the increased number of analysis and complexity according to the rising variables.

### A. Wardrop Solution of Service Selection Game

Given prices $(p_1, p_2)$, the equilibrium cloud users choosing the cloud broker and public provider is achieved by cloud users in the service selection game, since in the market a large number of cloud users must determine individually CSP they should buy cloud service. In the service selection game, we have two conditions: first, cloud users individually minimize the perceived cost, which is expressed as $C_1$ in (1) if they choose the cloud broker, and $C_2$ in (3) if they choose the public provider; second, at the equilibrium point, the cost $C_1$ is equal to the cost $C_2$. Two above conditions satisfy the two Wardrop's principles [35], that are: the total costs perceived by users on all used services are equal and the average delay/cost is minimum. Therefore, at the Wardrop equilibrium, we have $C_1 = C_2$ or:

$$\frac{\alpha}{\mu} + p_1 = \frac{\alpha}{\mu - \lambda_2} + p_2. \tag{5}$$

Then, we can compute the equilibrium cloud user request $\lambda_2^{wa}$ for the public provider as a function of the prices set by both the cloud broker and public provider:

$$\lambda_2^{wa} = \frac{(p_1 - p_2)\mu^2}{(p_1 - p_2)\mu + \alpha}. \tag{6}$$

with $0 < \lambda_2^{wa} < \lambda$. The condition $0 < \lambda_2^{wa}$ and queue stable condition $\lambda_2^{wa} < \mu$ imply the condition $p_1 > p_2$. The equilibrium cloud user request sent to the cloud broker, $\lambda_1^{wa}$, will therefore be equal to $\lambda - \lambda_2^{wa}$.

### B. Noncooperative Strategic Game

We consider a noncooperative strategic game [30] where the cloud broker and public provider compete with each other by setting the price simultaneously to maximize their utilities. Then, given a particular service price $p_1$ of the cloud broker, the public provider will determine the best reply service price $p_2$ and vice versa. Motivated by the concept Nash equilibrium, we define equilibrium prices $(p_1^{ns}, p_2^{ns})$, from which no provider trying to maximize its own utility has any incentive to deviate unilaterally. The Nash equilibrium is obtained by using the best response function, which is the best strategy of one provider given other provider's strategies. Given a service price $p_1$, from (4) and (6), the best response function (or reaction curve) $BR_2(p_1)$ of the public provider can be expressed in terms of $p_1$ as follows

$$BR_2(p_1) = \arg\max_{p_2 \geq 0} \quad U_2(p_2), \tag{7}$$

where the utility $U_2(p_2)$ corresponding the Wardrop equilibrium is $p_2\lambda_2^{wa} = p_2 \frac{(p_1-p_2)\mu^2}{(p_1-p_2)\mu+\alpha}$. By using the first order condition $\frac{U_2}{\partial p_2} = 0$, we obtain the best response function $BR_2(p_1)$ as follows

$$BR_2(p_1) = p_1 + d - \sqrt{d^2 + p_1 d}, \tag{8}$$

where $d = \frac{\alpha}{\mu}$. Similarly, given a price $p_2$, the best response function $BR_1(p_2)$ is as follows

$$BR_1(p_2) = \arg\max_{p_1 \geq 0} \quad U_1(p_1). \tag{9}$$

Hence, we have

$$BR_1(p_2) = p_2 - d + \sqrt{\frac{(p_2 - d)\alpha}{\Delta}}, \tag{10}$$

where $\Delta = \mu - \lambda$. The Nash equilibrium prices $(p_1^{ns}, p_2^{ns})$ exist if and only if $(p_1^{ns} = BR_1(p_2^{ns}), p_2^{ns} = BR_2(p_1^{ns}))$. In other words, two reaction curves $BR_2(p_1)$ and $BR_1(p_2)$ have intersection points. In order to obtain the Nash equilibrium solution, we introduce the iterative algorithms (Algorithm 1) based on the above best response dynamics mechanism. With starting price $p_1^0$ or $p_2^0$, the algorithm iterates until the convergence condition ($th$ is a predefined threshold) is satisfied.

**Algorithm 1** Iterative Algorithm.

```
Initialize parameters: p1(0) ( p2(0)), t=1;
loop:
p2(t) = BR2(p1(t − 1)),
p1(t) = BR1(p2(t)),
ε = |p2(t) − p2(t − 1)| + |p1(t) − p1(t − 1)|,
t=t+1,
end loop: ε < th.
```

**Numerical Example:** Fig. 4 shows the best response function $BR_2(p_1)$ and $BR_1(p_2)$ when two reaction curves have one intersection point that is the unique Nash equilibrium point $(p_1^{ns}, p_2^{ns})$. Since the Nash equilibrium is defined as the set of strategies that providers adopt given other provider' strategies, it is the point at which the best response functions of two the providers intersect. This Nash equilibrium is illustrated in Fig. 4 where the best response of the public provider crosses that of the cloud broker. An example illustrates the iterative algorithm is presented in Fig. 5 (a). At first, given a starting price $p_1(0)$, the public provider reacts by setting the best response price $p_2(0)$ according to (8). At the next period, given the price $p_2(0)$, the cloud broker reacts by setting the best response price $p_1(1)$ according to (10). This process would continue until the convergence condition is satisfied. Fig. 5 (b) shows that the iterative algorithm can reach to the convergent point fastly and smoothly .

**Observation 1.** The Nash equilibrium of the above noncooperative strategic game is unique.

*Proof.* To find the Nash equilibrium, we consider each provider's profit maximization problem where each provider

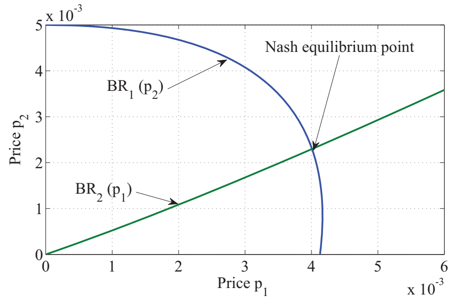
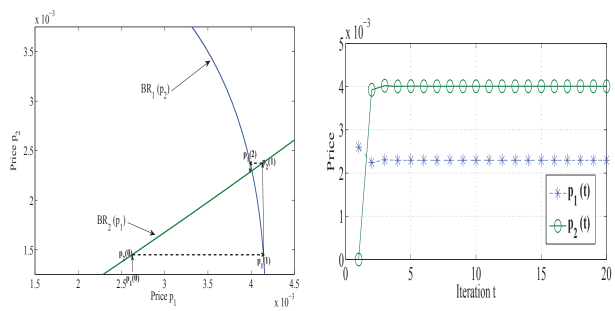



Fig. 4. Best response function with parameters: $\mu = 100; \lambda = 70; \alpha = 0.5$. These parameters imply $d = 5.10^{-3}$ and the Nash equilibrium point $(p_1^{ns}, p_2^{ns})$ = $(4.10^{-3}, 2, 3.10^{-3})$.




(a) Illustration of The Iterative Algorithm.

(b) Convergence speed of The Iterative Algorithm.

Fig. 5. Best response dynamic with parameters: $\mu = 100; \lambda = 70; \alpha = 0.5$. These parameters imply $d = 5.10^{-3}$ and the Nash equilibrium point $(p_1^{ns}, p_2^{ns})$ = $(4.10^{-3}, 2, 3.10^{-3})$.

takes each other's action as given parameters, but which are resolved simultaneously

$$\begin{cases} \max_{p_1 \ge 0} \quad U_1(p_1,p_2) = p_1\left[\lambda - \frac{(p_1-p_2)\mu^2}{(p_1-p_2)\mu+\alpha}\right], \\ \max_{p_2 \ge 0} \quad U_2(p_1,p_2) = p_2\frac{(p_1-p_2)\mu^2}{(p_1-p_2)\mu+\alpha}. \end{cases} \tag{11}$$

By solving the first order condition $\frac{U_1}{\partial p_1} = 0$ and $\frac{U_2}{\partial p_2} = 0$, we obtain

$$\begin{cases} p_1 + d - p_2 = \sqrt{(d-p_2)\frac{\alpha}{\mu-\lambda}}, \\ p_1 + d - p_2 = \sqrt{d^2 + p_1 d}. \end{cases} \tag{12}$$

Solving (12), we have two solution for $p_1$ as follows

$$\begin{cases} {p_1}^{(1)} = -\frac{d\left(-a^2+2ad+2d^2+a^{3/2}\sqrt{5a+4d}\right)}{2(a+d)^2}, \\ {p_1}^{(2)} = \frac{d\left(a^2-2ad-2d^2+a^{3/2}\sqrt{5a+4d}\right)}{2(a+d)^2}, \end{cases} \tag{13}$$

where $a = \frac{\alpha}{\mu-\lambda}$. Since $\lambda < \mu$, we can observe that ${p_1}^{(1)}{p_1}^{(2)} = \frac{d^2(-a+d)}{a+d} < 0$ due to $a = \frac{\alpha}{\mu-\lambda} > d = \frac{\alpha}{\mu}$. Thus, only solution ${p_1}^{(2)}$ is positive. The corresponding solution $p_2^{(2)}$ is given as follows

$$ {p_2}^{(2)} = \frac{d^2\left(3a + 2d + \sqrt{a}\sqrt{5a+4d}\right)}{2(a+d)^2}. \tag{14}$$

□

### C. Stackelberg Game

We now investigate the duopoly market where we model the strategic interaction between the cloud broker and public provider as a Stackelberg competition [36], [37]. We assume that the cloud broker is the game leader and the public provider is the game follower. In the Stackelberg game, the cloud broker has the so-called first-move advantage, which means that the public provider adapts its decisions to maximize its revenue by anticipating the cloud broker's response. Then, we use backward induction to derive the Stackelberg equilibrium of the prices, which are denoted by $(p_1^{sg}, p_2^{sg})$, in a duopoly as follows.

**Follower public provider's Revenue Maximization**: First, given the cloud broker's service price $p_1$, the public provider aims to determine an optimal price $p_2$ by solving the following problem:

$$\max_{p_2 \ge 0} \quad U_2 = \lambda_2^{wa} p_2 \tag{15}$$

By using the first order condition $\frac{U_2}{\partial p_2} = 0$, we obtain an express optimal price $p_2^*$ as a function of $p_1$ as the best response procedure described in (8) as follows

$$p_2^* = p_1 + d - \sqrt{d^2 + p_1 d}. \tag{16}$$

**Leader cloud broker's Revenue Maximization**: Knowing the public provider's best-response price $p_2^*$, the cloud broker determines its price $p_1$ by solving the following problem

$$\begin{aligned} \max_{p_1 \ge 0} \quad U_1 &= \lambda_1^{wa} p_1 \qquad (17) \\ &= p_1\left[\lambda - \frac{(p_1 - p_2^*)\mu^2}{(p_1 - p_2^*)\mu + \alpha}\right]. \\ &= p_1\left[\lambda - \frac{(\sqrt{d^2 + p_1 d} - d)\mu^2}{(\sqrt{d^2 + p_1 d} - d)\mu + \alpha}\right]. \end{aligned}$$

**Stackelberg Equilibrium Summary**: The maximization (17) can be solved by finding the root of the first derivation $U_1'(p_1) = 0$. The root (i.e., the Stackelberg solution $p_1^{sg}$) can be found by using a standard root-finding algorithm such as the bisection method with logarithmic complexity [38]. Using (16), we have the Stackelberg equilibrium price $p_2^{sg} = p_1^{sg} + d - \sqrt{d^2 + p_1^{sg} d}$. Thus, the Stackelberg equilibrium of prices is obtained by using backward induction.

## V. Service Selection Game of Cloud Users

In this section we introduce the evolutionary game in order to study the dynamic behavior of cloud users who decide which service (i.e., from the public provider or cloud broker) to use based on the observed state of the system (i.e., delay and prices). First, we provide some preliminaries of the evolutionary game. Then we formulate the service selection of cloud user as a evolutionary game in which we apply replicator dynamics to study the dynamic behavior of cloud users. Finally, we use population evolution approach to implement a service selection algorithm of cloud users.

### A. Preliminaries of Population Game

The population is a group of individuals (i.e., players) in which the number of individuals can be finite or infinite. The individuals from one population may choose strategies against individuals in another population. An evolutionary game defines a foundation to obtain the equilibrium solution for the game of the populations. In this section, we briefly introduce theoretic concepts of evolutionary games and replicator dynamics [11] which are recently used in network selection game [29], [39], [28].

In a dynamic evolutionary game, an individual from a population (i.e., a player in the game), who is able to reproduce (i.e., replicate) itself through the process of mutation and selection, is called a replicator. In this case, a replicator with a higher payoff (or lower cost) can reproduce itself faster. When the reproduction process takes place over time, this can be modeled by using a set of ordinary differential equations called replicator dynamics. This replicator dynamics is important for an evolutionary game since it can capture the essence of selection (e.g., proportion of individuals who choose different strategies), given a particular point in time.

In replicator dynamics, it is assumed that an individual chooses pure strategy $i$ from a finite set of strategies where the total number of available strategies in this set is $I$. Let $n_i$ denote the number of individuals choosing strategy $i$, and let the total population size $N=\sum_{i=1}^{I} n_i$. The proportion of individuals choosing strategy $i$ is $x_i=\frac{n_i}{N}$, and it is referred to as the population share. The population state can be denoted by the vector $x=[x_1,\dots,x_i,\dots,x_I]$. The replicator dynamics can be defined as follows:

$$\dot{x}_i(t)=x_i(t)\sigma\left[\pi_i(t)-\bar{\pi}(t)\right], i=1,\dots,I, \qquad (18)$$

where $\pi_i(t)$ is the payoff (or cost) of the individuals choosing strategy $i$ at time $t$, and $\bar{\pi}(t)$ is the average payoff of the entire population and $\sigma$ is the rate of strategy adaptation. Based on the replicator dynamics, the evolutionary equilibrium is defined as the set of fixed points of the replicator dynamics that are stable. This evolutionary equilibrium is a desirable solution to the evolutionary game since when the population of players evolves over time (i.e., based on replicator dynamics), it will converge to the evolutionary equilibrium. Furthermore, at this evolutionary equilibrium, none of the individuals wants to change its strategy since its payoff (or cost) is equal to the average payoff of the population.

### B. Formulation of Population Game

The evolutionary game for the service-selection problem in a heterogeneous market can be described as follows.

We consider a evolutionary game $G$ with $Q$ non-atomic set of players, which is defined by a strategy set denoted by $S=\{s_1,s_2\}$, identical for all players; $s_1$ means that the player chooses the service from the cloud broker, and $s_2$ means that the player chooses the service from the the public provider. Corresponding to the strategy, the proportion of individuals choosing the cloud broker's service $x_1$ is equal to $x_1=\frac{\lambda_1}{\lambda}$ and the proportion of individuals choosing the public provider's service is $x_2=\frac{\lambda_2}{\lambda}=\frac{\lambda-\lambda_1}{\lambda}$.

The cost of the individuals choosing the cloud broker's service $C_1$ is equal to $C_1=\left[\frac{\alpha}{\mu}+p_1\right]$, and the cost of the individuals choosing the public provider's service is $C_2=\left[\frac{\alpha}{\mu-\lambda_2}+p_2\right]$. Thus, the average cost of the entire population $\bar{C}$ is equal to

$$\bar{C}=\frac{\lambda_1}{\lambda}C_1+\frac{\lambda_2}{\lambda}C_2. \qquad (19)$$

### C. Replicator Dynamics of Service Selection Game

The service selection game is repeated, and in each period (i.e., in each generation), the user observes the cost of other cloud users in the same area. Then, in the next period, the user adopts a strategy that gives a lower cost. The proposed replicator dynamics provides a means to analyze how players can "learn" about their environment, and converge towards an equilibrium choice. Replicator dynamics is also useful to investigate the speed of convergence of strategy adaptation to reach a stable solution in the game [11], [29], [28].

Here, the aim of each cloud user is to minimize his cost. Hence, we can formalize the service selection game as follows:

$$\dot{x}_1(t)=x_1(t)\sigma\left[\bar{C}(t)-C_1(t)\right], \qquad (20)$$

where $\dot{x}_1(t)$ represents the derivative of $x_1$ with respect to time. A similar equation can be written for cloud user choosing the public provider, thus we can express the replicator dynamics for such cloud users as follows:

$$\dot{x}_2(t)=x_2(t)\sigma\left[\bar{C}(t)-C_2(t)\right]. \qquad (21)$$

Based on this replicator dynamics of the users, the number of users choosing service from either the public provider or cloud broker increases if their cost is below the average cost. We can observe that if we have $\dot{x}_1(0)+\dot{x}_2(0)=0$ at starting time, then at every time $t$ we obtain $\dot{x}_1(t)+\dot{x}_2(t)=0$.

### D. Implementations of The Service Selection Algorithm

We present population evolution approach for dynamic evolutionary game-based service selection by each individual user in a heterogenous market cloud. This approach is based on population evolution in which cost information of cloud users using different providers is exchanged between two group of cloud users (e.g, by a third party who collect cost information of all cloud users or by an information exchanging mechanism). The service-selection decision of each user is based on its current cost and the average cost of all users. This service-selection algorithm based on population evolution approach can be described in Population Evolution Algorithms (Algorithm 2). In future work, we will propose a reinforcement-learning-based approach, in which the cloud users learn the performances and prices of different cloud providers by interaction to make the optimal decision for the service selection without information exchanging between cloud user groups.

**Algorithm 2** Population Evolution Algorithms

Choose randomly the rate of strategy adaptation $\sigma > 0$.
All cloud users choose randomly the service from the cloud broker or public provider.
*loop*:
  A user computes cost $C_i$ ($i$ = 1,2 ) from the average delay and price. This cost information is informed to the other user group.
  Based on the exchanging cost information, the average cost $\bar{C} = \frac{\lambda_1 C_1 + \lambda_2 C_2}{\lambda}$ is calculated in each user group.
  **if** $C_i < \bar{C}$ **then**
    **if** *rand* $< \sigma(\bar{C} - C_i)/\bar{C}$ **then**
      Choose service $j$, where $j \neq i$.
    **else**
      Keep service $i$.
*end loop*: for all cloud user in two groups (who are choosing service from the cloud broker or public provider).

## VI. Performance Evaluation

In this section, we analyze and discuss the numerical results obtained from solving pricing and service selection games in different scenarios. At first, we measure the sensitivity of the provider' utilities and prices, as well as cloud users' equilibrium arrival rate and costs, for different service rates $\mu$. Then, we evaluate the convergence to evolutionary equilibrium of the service selection game with Population Evolution Algorithms.

### *A. Pricing and Service Selection Games*

We first consider a heterogenous cloud system with the parameters as follows: $\alpha = 0.5$, total cloud user arrival rate $\lambda = 100$ and service rates $\mu$ are given in range of 110 to 190. Figure 6 shows the equilibrium utility of the cloud broker ($U_1$) and the public provider ($U_2$), respectively, in the two scenarios (the cloud broker and public provider participate in Noncooperative Strategic Game (NSG) and Stackelberg Game (SG)). It can be seen that in NSG the equilibrium utilities of the cloud broker are less than the equilibrium utilities of the public provider. However, in SG, since the cloud broker takes first-move advantage, the equilibrium utilities of the cloud broker are higher than the equilibrium utility of the public provider.

In addition, the difference between the equilibrium prices set by the cloud broker and public provider in these two scenarios can be showed in Fig. 7. It shows that the gap between two competitive prices in NGS are less than those in SG. It can imply that the price competition in NSG is fairer than that in SG. Fig. 8 (a) and (b) provide better appreciation of the difference between the prices. Beside prices, the intensive competition between the cloud broker and public provider in SG makes the cost of the cloud user increase as depicted in Fig. 9. Further more, we can observe that when the service rate $\mu$ increases, both the cost of cloud users and the prices set by the providers decrease. The reason is that the increasing of service rate $\mu$ implies the reducing of the delay cost of users at both the cloud broker and public provider.

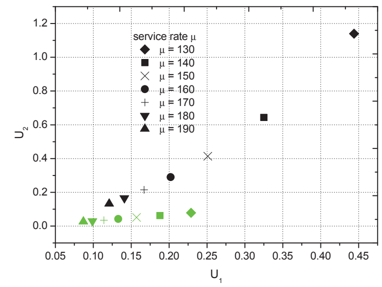



Fig. 6. Comparison of the utility of the provider in Stackelberg Game(black color) and Noncooperative Strategic Game (green color).

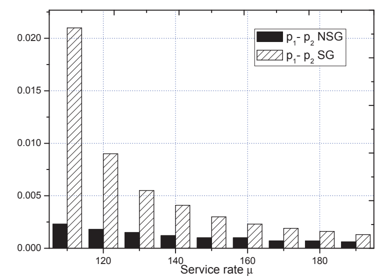
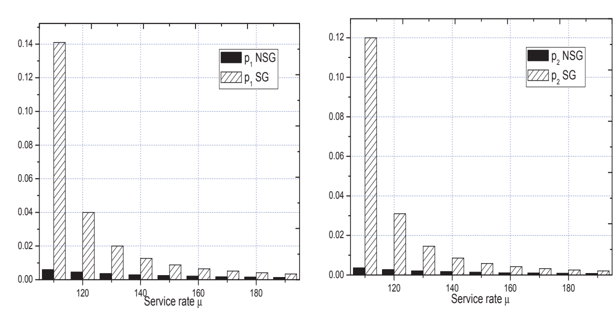



Fig. 7. Difference in the equilibrium prices $p_1$ and $p_2$ in the NSG and SG scenarios.

We further investigate the changing of the cloud broker in two mentioned game scenarios. As shown in Fig. 6, the utilities of the cloud broker in the NSG are higher than those in the SG, however, the difference between utilities in two scenarios decreases while the service rate $\mu$ increases. Fig. 10 brings a interesting fact that in NSG, the service rate $\mu$ does not affect the equilibrium arrival rate $\lambda_1$ of user at the cloud broker. However in SG scenarios, when the service rate $\mu$ rises up, the equilibrium rate $\lambda_1$ also increases.

### *B. Population Game Numerical Results*

**Impact of Delay in Population Evolution Algorithms**: At the time when a user makes the decision on service selection, current information at a certain time $t$ about an average cost (i.e., $\bar{C}$ in ) may not be available. Therefore, a user must rely




(a) Stackelberg Game(SG). (b) Noncooperative Strategic Game (NSG).

Fig. 8. Comparison of the equilibrium prices of the provider in Stackelberg Game(SG) and Noncooperative Strategic Game (NSG).

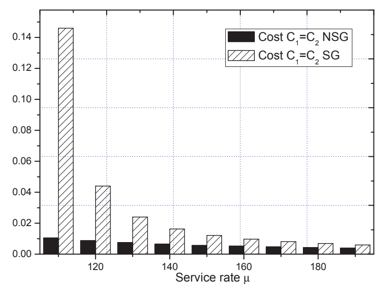



Fig. 9. Difference in the equilibrium cost of the cloud user in the NSG and SG scenarios.

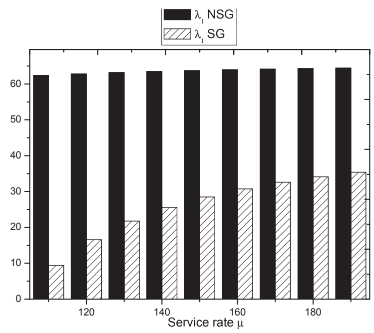



Fig. 10. Comparison of the equilibrium user arrival rate at the cloud broker in SG and NSG.

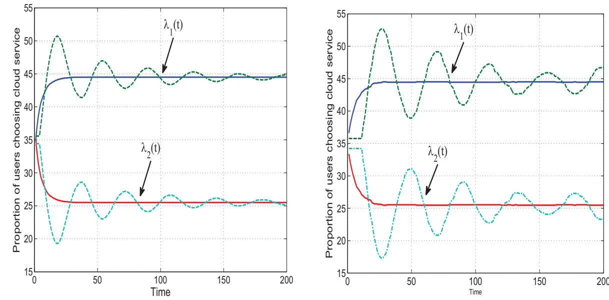
on historical information, which again, may be delayed for a certain period. This delay can occur due to the information exchange latency among user groups. Thus, we assume that a user make a service selection at time $t$ bases on the information at time $t$-$\tau$ (i.e., delay for $\tau$ units of time). In this case, the replicator dynamics can be modified as follows

$$\dot{x}_i(t) = x_i(t-\tau)\sigma\left[\bar{C}(t-\tau) - C_i(t-\tau)\right], i = 1,2. \quad (22)$$

The convergence of Population Evolution Algorithms with different values of $\tau$ is shown in Fig. 11. We investigate the impact of $\tau$ on the dynamics of strategy adaptation. When $\tau \geq 1$, we observe a fluctuating dynamics of strategy adaptation. For small of value $\tau = 3$, the difference between dynamics of strategy adaptation without delay and that with delay is very little as time increases. The larger the delay is, the more the fluctuation there will be. We can observe that if $\tau > 10$, the dynamics of strategy adaptation of users never reaches the evolutionary equilibrium as presented in Fig. 11 (b). The reason is that the decisions of users tend to be inaccurate when information is out-of-date.

## VII. Conclusion

In this paper, we study the price competition in a heterogenous market cloud computing formed by public providers and cloud brokers (all also known as cloud service providers) with two stages of competition. In pricing competition between the cloud service providers, we derive the equilibrium prices in two game models: Noncooperative Strategic Game and Stackelberg Game. At the same time, we study the dynamic of




(a) Delay time $\tau = 3$. (b) Delay time $\tau = 10$.

Fig. 11. The convergence of Population Evolution Algorithms with different values of $\tau$. The solid line is Population Evolution Algorithms without delay (i.e., $\tau = 0$).

cloud users in service selection game by using the evolutionary game model. We use the Wardrop equilibrium concept and replicator dynamics to compute the equilibrium and characterize its convergence properties in the service selection game. Performance evaluation demonstrates that our game model can represent the main characteristics of the heterogenous market cloud computing pricing and service selection. Throughout the numerical results, we found that the advantage for the cloud broker to set the price firstly is significant by observing the revenue comparison in the Stackelberg Game and Noncooperative Strategic Game. We also have proposed an population evolution approach to implement the evolution of cloud user to enforce them to converge to the equilibrium choice. Here, the service selection algorithm based on population evolution utilizes information from all users to achieve fast convergence. In future work, we will implement the reinforcement-learning-based approach, in which the cloud users learn the performances and prices of different cloud providers by interaction to make the optimal decision for the service selection without the centralized controller.

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