PERFORMANCE OF SEVERAL TYPES OF MEDIAN FILTERS IN SPECTRAL DOMAIN

O. Uma Maheshwari, G.B. Vanisree, Dr. D. Ebenezer
Uma_o@annauniv.edu, *gbvanisree@yahoo.com*, Ebenez26@mailcity.com

Abstract: Median filter is well known for removing impulsive noise and preserving edges. Repeatedly filtering of any one-dimensional signal with a median filter will produce a root signal. Any impulses in the input signal will be removed by sufficient number of passes of median filter, where any root like features in the input signal will be preserved. A signal of finite length will be filtered to a root signal after a finite number of passes of a median filter of a fixed window, results in the convergence of the signal. In this paper, root signal and its properties are analyzed for One-dimensional signal. Adaptive length median filter, weighted median filter, FIR hybrid median filter and Linear combination of weighted median filter have been taken and their root signals are obtained. Their performances are analyzed by determining Power spectrum density, Mean square error and Signal to noise ratio.

Key words: Median filtering, Root signal, Power spectrum density, Mean square error, Signal to noise ratio

1. INTRODUCTION

Impulse noise occurs frequently in image processing [11]. It may be caused by transmission channel error (e.g., binary symmetric channel noise), sensor faults, edge sharpening procedures, engine sparks, ac power interference and atmospheric electrical emissions. Due to the strong amplitude of impulse noise, human visual perception is very sensitive to it and the removal of such noise is a important issue in image processing.

Linear filters have poor performance in the presence of noise that is not Additive. If a signal with sharp edges is corrupted by high frequency noise, however, as in some noisy image data, then linear filters designed to remove the noise also smooth out signal edges. In addition, impulse noise cannot be reduced sufficiently by linear filters.

A nonlinear scheme called 'median filtering' has been used with success in these situations. Some interesting results and analyses for median filters have been obtained recently [11].

The success of median filters is based on two intrinsic properties:

1. Edge preservation.
2. Efficient noise attenuation with robustness against impulsive noise.

A median filter maps a class of input signal into an associated set of root sequences.

2. ROOT SIGNAL AND ITS PROPERTIES

Repeated application of the median filter on a defined signal of finite length ultimately results in a sequence, termed a root signal, which is invariant to additional passes of the median filter [12].

The characteristics of root signals are based on the local signal structures, summarized for a median filter with window size W=2N+1, as follows:

- A *Constant neighborhood* is a region of at least N+ 1 consecutive identically valued sample.
- A *Edge* is a monotonically rising or falling set of samples surround on both sides by constant neighborhood of different values.
- An *Impulse* is a set of at most N samples whose values are different from the surrounding regions and whose surrounding are identically valued constant neighborhoods.
- An *Oscillation* is any signal structure which is not a part of constant neighborhood, an edge or an impulse
- A *root* is an appended signal which is invariant under filtering by particular median filter.

A filter is said to be idem potent if its output signal converge to a root in only one pass of the filtering process for any input signal. The root signal retains the spatial characteristics of the input signal, such as edges, while at the same time; it deletes redundant impulses and oscillations (which are defined above).

Since the output of the median filter is always one of its input samples, it is conceivable that certain signal could pass through the median filter unaltered. A filter is said to be 'idem potent' if its output signal converge to a root in only one pass of the filtering process for any input signal. The root signal retains the spatial characteristics of the input signal, such as edges, while at the same time; it deletes redundant impulses and oscillations (which are defined above).

The analysis of the root signal can be explained by taking binary signals and the theory can be extended to multilevel signals and two-dimensional image. For a window of width 2N+1, any signal of length L will converge to a root i$_3$ $\left[\frac{L-2}{2(K+2)} \right]$

It is obtained by considering binary signals first, then extending it to multi-level signals via the threshold decomposition (Peter 1986). For N>1, this bound is much lower than the

$$\left| \frac{L-1}{2} \right|$$

The inverse dependence on the window width that appears in this convergence bound allows limiting operations to be performed. Slow convergence of binary signal into a root signal.

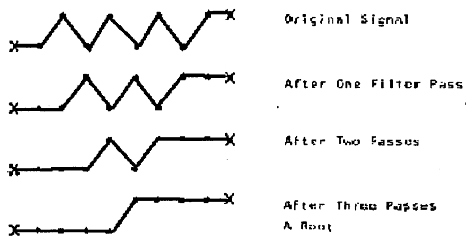

Figure 1. Slow convergence of a binary signal

3. PRINCIPLES OF SEVERAL TYPES OF MEDIAN FILTERS

In this section the principles of several types of median filters are discussed. The root signal for the filters discussed is analyzed in the next section. Their performances are analyzed by determining Power spectrum density, Mean square error and Signal to noise ratio.

Weighted median filter

The weighted median filter is a extension of median filter which gives more weights to some values within the window. For a discrete time continuous valued input vector $X = [X_1, X_2, X_3 ... X_N]$, the output Y of the WM filter of span X associated with the integer weights $W = [W_1, W_2, W_3 ... W_N]$ is given by,

$Y = MED[W_1 \Diamond X_1, W_2 \Diamond X_2, ..., W_N \Diamond X_N]$

Where MED {.} denotes the median operation and ◊ denotes duplication, i.e.,

$$K \Diamond X = (\underbrace{X, \ldots\ldots, X}_{K\ times})$$

FIR Hybrid median filter

A new class of generalized median filters, which contain linear substructures. The root signals types as well as the noise attenuation properties of these FIR median hybrids (FMH) filters are similar to those of the standard median filters. The FMH filters require, however less computations than the standard median filters [10].

The simplest FMH filter is the averaging FMH filter consisting of two identical averaging filters.

$$Y(n) = MED\left[(1/k) \sum^{k} x(n-1), s(n), (1/k) \sum^{k} x(n+1)\right]$$

i =1 i =1

Linear combination of weighted medians

A class of linear combination of weighted median (LCWM) filters that can offer various frequency characteristics including LP, BP and HP responses. The scheme is modeled on the structure and design procedure of the linear-phase FIR HP filter.

Design procedure for the LCWM filter:

Design an N-tap prototype FIR filter h using frequency specifications.

Choose a weight vector w of the M-tap SM sub filter (smoother)(M<N).

Using the row-searching algorithm, find $B_{N,M}$ and convert it into B_p.

Using SSP's and 1/Ms, transform B_p into B.

Using alpha = $h*B^{-1}$

Adaptive median filter

Median filters employing adaptive length algorithms, based on noise detection, exhibit improved performance for impulse noise removal. The detection algorithm is fundamentally different from other commonly used adaptive or threshold algorithms, which are based on statistical parameters and /or edge detection, and which seen les suitable for impulse noise smoothing. Impulse noise generally has a lower probability of occurrence and a considerably higher probability for large amplitude. A smooth region with impulse noise, and an edge with smaller amplitude, is difficult to recognize from some simple statistical parameters. To detect impulse noise deterministically thus seems a more proper procedure. The algorithm is insensitive to specific threshold values, and its realization is feasible and efficient. One dimensional median filters can be used to remove either positive or negative impulse noise of low density. Such filters can achieve quite good performance with very efficient realizations [1].

4. ALGORITHM FOR THE PERFORMANCE ANALYSIS OF THE FILTERS

1. A signal is generated which contains proper edges, constant regions and randomly variably noise. Such a signal is termed as '**test signal**'.
2. The generated test signal is allowed to pass through the designed filters like Adaptive median filter, Weighted median filter, FIR hybrid median filter and Linear combination of weighted medians .
3. The time domain and frequency domain model of the signal is plotted.
4. The output signal of the respective filters are repeatedly passed through the same filter. Root signal is determined for each type of filter.
5. Frequency domain model of the root signal is plotted for each output.
6. Next, the mean square error, power spectrum and signal to noise ratio are calculated, by varying the intensity of the noise.
7. The performance is analyzed from the values obtained above.

5. ROOT ANALYSIS OF SEVERAL TYPES OF FILTERS

The signal that is invariant to subsequent passes is said to be root signal. The performance analysis is done by determining the root signal for each type of filter. A common test signal is generated and the signal is allowed to pass through several types of filters repeatedly to get the root signal. The root properties for Adaptive median length, Weighted median filter, FIR Hybrid median filter and Linear Combination of weighted median are analyzed below.

From the spectrum of the outputs of the test signal, the shape of the root signal is same, It is seen from the results in table 1, table 2 the information carried by the original Signal retained Adaptive length median compared to the weighted medians.

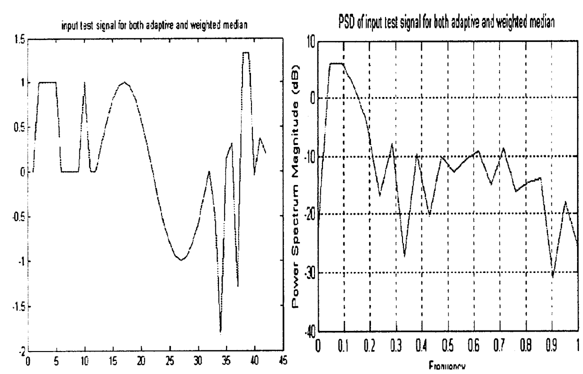

Figure 2. Figure(a),Figure(b)

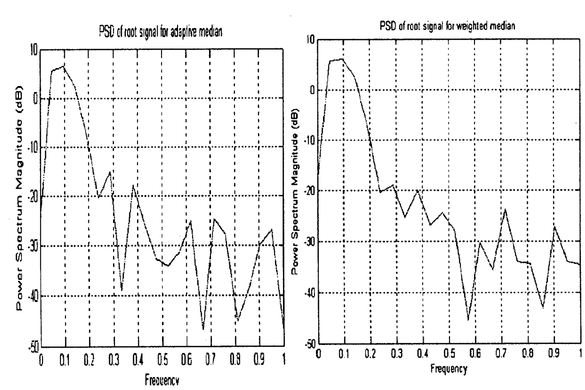

Figure 3. Figure(c),Figure(d)

Results of the test signal: (a) Model test signal. (b) Spectrum of original signal. (C). Spectrum for root of Adaptive median. (d) Spectrum for root of Weighted median

Results of the Sinusoidal signal: (a) Spectrum of original signal . (b) Spectrum of corrupted signal. (C). Specturm for root of Adaptive median. (d) Spectrum for root of Weighted median Similarly the analysis is carried for a test signal. The root signal is determined for the test signal. The analysis is extended to two dimensional image. The root signal is determined for Adaptive median filter. The results are not satisfactory, blurring of the image is seen. This can be improved by new class or some recent modification in the Adaptive median filter.

The signal is not converged to a root for the case of a LCWM and FMH filter. The reasons are discussed below. The FMH filter discussed here is a averaging type filter. Hence the impulses are not completely removed but they are reduced to a average value. Repeated filtering with a FMH filter does not removes the oscillations, but averages the oscillations. The LCWM filter discussed here is a band pass filter with a frequency range of 0.32 to 0.7. Repeated filtering of the filter continuously eliminates the frequency range other than the prescribed. So the signal does not converge to a root.

Table 1. Results of test signal for Adaptive length Median filter.

NOISE POWER	ITERATION I		ITERATION II	
	MSE	PSNR	MSE	PSNR
0	0.28330	8.4932	0.2981	8.2719
1	0.1774	9.2981	0.1776	9.2932
2	0.1158	9.3622	0.1529	8.1526
3	0.0983	10.0755	0.1017	9.99255
4	0.0612	15.0448	0.0804	13.8580

Table 2. Results of test signal for Adaptive length Median filter

NOISE POWER	ITERATION I	
	MSE	PSNR
0	0.2850	8.4680
1	0.1738	9.3876
2	0.1454	8.3745
3	0.1131	9.4636
4	0.0808	13.8770

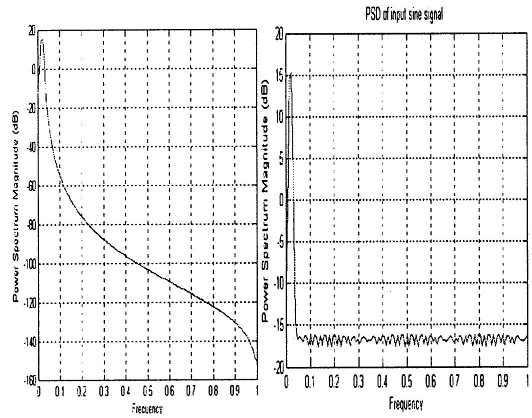

Figure 4. Figure(a),Figure(b)

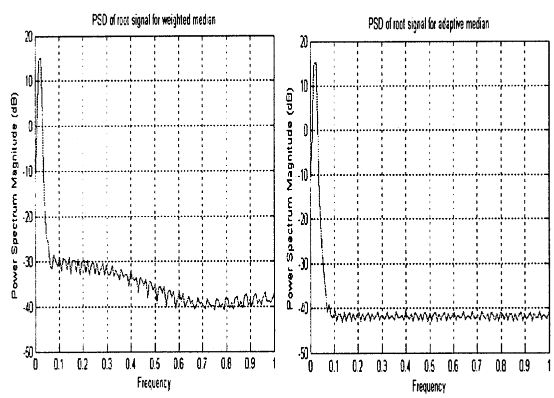

Figure 5. Figure(c),Figure(d)

6. CONCLUSION

In this project the root analysis for several types of filters like adaptive length median filter, fir hybrid median filter, weighted median filter and linear combination of weighted medians is performed. The root signals are obtained for one-dimensional signal of adaptive median filter and weighted median filter. For the case of FIR Hybrid median and Linear combination of weighted median, the input signal is not converged to root signal, the reasons were discussed. The root performance are compared from the results of mean square error, signal to noise ratio and power spectrum density.

REFERENCES

1. Ho-Ming Lin, Alan N. Willson, (1988), "Median Filter with Adaptive Length", *IEEE Transaction action on Circuits and systems*, Vol. 35, No. 6.
2. Jaakko Astola and Pauli Kuosmanen, (1977), "*Fundamentals of nonlinear digital Filtering*", CRC Press, New York.
3. Jaakko Astola and Pekka Heinonen, (1987), "On Root Structures of Median and Median – Type Filters", *IEEE Transaction on Acoustic Speech Signal Processing*, vol. ASSP 35, No. 8.
4. Kang-Sun Choi, Student Member, Aldo W. Morales, Senior Member, IEEE, and Sung-Jea Ko, Senior Member, IEEE, (2001), " Design of linear combination of weighted medians", *IEEE Transaction on Signal processing*, Vol. 49.
5. Neal G. Gallagher, (1988), "Median Filters: A Tutorial", Jr. School of Electrical Engineering, Purdue University, West Lafayette, USA.
6. Neal G. Gallagher, Jr. Member, IEEE, Gary L. Wise, Member, IEEE (1981), "A Theoretical Analysis of Properties of Median filters", *IEEE Transaction on Acoustic Speech Signal Processing*, Vol. ASSP 29, No. 6.
7. Olli yli- Harja, Heikki Huttunen, Antti Niemisto & karan Egiazarian, "Design of Recursive weighted median filters with negative weights", Signal processing laboratory, Tampere University of Technology, Finland.
8. Patrick fitch, Edward J. Coyle and Neal C. Gallegher, (1985), "Root Properties and Convergence Rates of Median Filters", *IEEE Transaction on Acoustic Speech Signal Processing*, vol. ASSP 33, No. 1.
9. Peter D. Wendt, Edward J. Coyle and Neal C. Gallagher, (1986) " Some Convergence Properties of Median filters", *IEEE Transaction on Circuits and systems*, Vol. 33, No. 3.
10. Pekka Heinonen, Yrjo Neuvo, (1987), " FIR-Median Hybrid Filters", *IEEE Transaction on Acoustic Speech Signal Processing*, Vol. 35, No. 6.
11. Pitas. I and Venetsanopoulos. A.N. (1990), " *Nonlinear Digital Filters: Principles and Applications*". Boston, MA. Kluwer Academic.
12. Thomas A. Nodes, Student Member, IEEE and Neal. C. Gallagher, Jr, Member IEEE (1982), "Median filters: Some Modifications and Their Properties", *IEEE Transaction on Acoustic Speech Signal Processing*, vol. ASSP 30, No.5.
13. Yin. L, Yang.R, Gabbouj.M, and Neuo.Y, (1996), "Weighted median filter: A tutorial," *IEEE Transaction on Circuits Syst.* Vol.4.