A Subjective and Objective Integrated Method for MAGDM Problems with Multiple Types of Exact Preference Formats

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Abstract. Group decision making with preference information on alternatives has become a very active research field over the last decade. Especially, the investigation on the group decision making problems based on different preference formats has attracted great interests from researchers recently and some approaches have been developed for dealing with these problems. However, the existing approaches can only be suitable for handling the subjective preference information. In this paper, we investigate the multiple attribute group decision making (MAGDM) problems, in which the attribute values (objective information) are given as non-negative real numbers, the information about attribute weights is to be determined, and the decision makers have their subjective preferences on alternatives. The provided subjective preference information can be represented in three well-known exact preference formats: 1) utility values; 2) fuzzy preference relations; and 3) multiplicative preference relations. We first set up three constrained optimization models integrating the given objective information and each of three preference formats respectively, and then based on these three models, we establish an integrated constrained optimization model to derive the attribute weights. The obtained attribute weights contain both the subjective preference information given by all the decision makers and the objective information. Thus, a straightforward and practical method is provided for MAGDM with multiple types of exact preference formats.

1 Introduction

Decision making is a common activity in everyday life. In many real-world situations, such as economic analysis, strategic planning, medical diagnosis, and venture capital, etc. [1], multiple decision makers are usually involved in the process of decision making, and needed to provide their preference information over a finite set of feasible alternatives. Due to that each decision maker has his/her unique characteristics with regard to knowledge, skills, experience and personality, the different decision makers may express their preferences by means of different preference representation formats, such as utility values [2], fuzzy preference relation [3], multiplicative preference relation [3], etc. The issue has attracted great attention from researchers recently, and a variety of approaches have been developed to dealing with various group decision making problems with nonhomogeneous preference information. In [4], some representation models were established for group decision making problems based on the concept of fuzzy majority for the aggregation and exploitation of the information represented by means of preference orderings, utility functions, and fuzzy preference relations. For the group decision making problem, where the information about the alternatives provided by the decision makers can be presented by means of preference orderings, utility functions, and multiplicative preference relations, Herrera [5] presented a multiplicative decision model based on fuzzy majority to choose the best alternatives, taking the multiplicative preference relation as the uniform element of the preference representation. In the case where the decision makers provide their evaluations by means of numerical or linguistic assessments, Delgado et al. [6] introduced a fusion operator of numerical and linguistic information by designing two transformation methods between the numerical and linguistic domains based on the concept of characteristic values. Based on some aggregation operators (the linguistic weighted arithmetic averaging (LWAA) operator, linguistic arithmetic averaging (LAA) operator, linguistic weighted geometric averaging (LWGA) operator and linguistic geometric averaging (LGA) operator), Xu [7] presented two procedures for group decision making with multiple types of linguistic preference relations (including additive linguistic preference relations, uncertain additive linguistic preference relations, multiplicative linguistic preference relations, and uncertain multiplicative linguistic preference relations). Ma et al. [8] constructed an optimization model to integrate the four preference structures (utility values, preference orderings, multiplicative preference relations, and fuzzy preference relations) and to assess ranking values of alternatives. The prominent characteristic of the model is that it does not need to unify different structures of preferences or to aggregate individual preferences into a collective one, and it can obtain directly the ranking of alternatives.

However, the existing approaches dealing with different preferences over alternatives can only be suitable for handling the subjective preference information. That is, they can only be used to handle group decision making problems with single attribute (or criterion) and multiple alternatives, but unsuitable for the multiple attribute group decision making (MAGDM) problems which involves finding the most desirable alternative(s) from a discrete set of feasible alternatives with respect to a finite set of attributes. The MAGDM problems with preference information on alternatives generally contain both the subjective preference information given by all the decision makers and the objective information described by attribute values. In this paper, we propose a subjective and objective integrated method for the MAGDM problems with multiple types of exact preference formats in which the attribute values (objective information) are given as non-negative real numbers, and the decision makers have their subjective preferences on alternatives. The provided subjective preference information can be represented in three well-known exact preference formats: 1) utility values; 2) fuzzy preference relations; and 3) multiplicative preference relations. To do so, we organize the paper as follows. In Section 2, we present the studied MAGDM problems. Section 3 sets up three constrained optimization models integrating the given objective information and each of three preference formats respectively. Based on these three models Section 4 establishes an integrated constrained optimization model to derive the attribute weights, and then utilizes the overall attribute values to get the ranking of alternatives, and finally, Section 5 concludes the paper.

2 Problem Presentation

In this section, we describe the multiple attribute group decision making (MAGDM) problems under consideration with three exact preference formats: 1) utility values; 2) fuzzy preference relations; and 3) multiplicative preference relations. For convenience, we first let $M = \{1,2,..., m\}$, $N = \{1,2,..., n\}$, and $T = \{1,2,..., t\}$.

For a MAGDM problem, let $X = \{x_1, x_2,..., x_n\}$ $(n \ge 2)$ be a discrete set of n feasible alternatives, $D = \{d_1, d_2,..., d_t\}$ be a finite set of decision makers, and $\lambda = (\lambda_1, \lambda_2,..., \lambda_t)^T$ be the weight vector of decision makers, where $\lambda_k \ge 0$, $k \in T$, $\sum_{k=1}^{t} \lambda_k = 1$, λ_k denotes the weight of decision maker d_k (Ramanathan and Ganesh [9] proposed a simple and intuitively appealing eigenvector based method to intrinsically determine the weights for group members using their own subjective opinions). Let $G = \{G_1, G_2,..., G_m\}$ be a finite set of attributes, $w=(w_1, w_2,...,w_m)^T$ be the weight vector of attributes to be determined, where w_i reflects the relative importance degree of the attribute G_i, $w_i \ge 0$, $i \in M$, and $\sum_{i=1}^{m} w_i = 1$. Let $A= (a_{ij})_{m\times n}$ be the data matrix, where a_{ij} is an attribute value, which is expressed with positive real number, of the alternative $x_j \in X$ with respect to the attribute $G_i \in G$.

In general, there are benefit attributes and cost attributes in the MAGDM problems. In order to measure all attributes in dimensionless units and to facilitate inter-attribute comparisons, we introduce the following formulas to normalize each attribute value a_{ij} in data matrix $A = (a_{ij})_{m\times n}$ into a corresponding element in data matrix $R = (r_{ij})_{m\times n}$:

$$r_{ij} = a_{ij} \Big/ \sum_{j=1}^{n} a_{ij}, \text{ for benefit attribute } G_i,\ i \in M,\ \ j \in N \tag{1}$$

$$r_{ij} = (1/a_{ij}) \Big/ \sum_{j=1}^{n} (1/a_{ij}), \text{ for cost attribute } G_i,\ i \in M,\ \ j \in N \tag{2}$$

Based on the normalized data matrix R, we get the overall attribute value of the alternative $x_j \in X$ by using the additive weighted averaging operator:

$$z_j(w) = \sum_{i=1}^{m} w_i r_{ij} \ , \ j \in N \tag{3}$$

In general, if the weight vector $w = (w_1, w_2, ..., w_m)^T$ is completely known, then (3) can be used to determine the ranking of all alternatives $x_j \, (j = 1,2,...,n)$. The greater the overall attribute value $z_j(w)$, the better the corresponding alternative x_j will be.

In addition, the decision makers also have preference information on alternatives, and the preference information provided by each decision maker is represented by one of the following exact preference formats:

1) Utility values [2]. A decision maker provides his/her preference on X as a set of m utility values, $U = \{u_1, u_2, ..., u_n\}$, where $u_j \in [0,1]$ represents the utility evaluation provided by the decision maker to the alternative x_j.

2) Fuzzy preference relation [3]. A decision maker's preference information on X is described by a fuzzy preference relation $P = (p_{ij})_{n\times n} \subset X \times X$, with

$$p_{ij} \geq 0, \ \ p_{ij} + p_{ji} = 1, \ \ p_{ii} = 0.5, \ i, j \in N \tag{4}$$

where p_{ij} indicates the preference degree of the alternative x_i over x_j. If

$$p_{ij} = p_{ik} - p_{jk} + 0.5, \ \text{for all} \ i, j, k \in N \tag{5}$$

then P is called a consistent fuzzy preference relation, which is given by [1]:

$$p_{ij} = 0.5(w_i - w_j + 1), \ \text{for all} \ i, j \in N \tag{6}$$

3) Multiplicative preference relation [3]. A decision maker's preference information on X is described by a multiplicative preference relation $B = (b_{ij})_{m\times m} \subset X \times X$ satisfying the following condition:

$$b_{ij} > 0, \ \ b_{ij} b_{ji} = 1, \ \ b_{ii} = 1, \ \ i, j \in N \tag{7}$$

where b_{ij} indicates the preference degree of the alternative x_i over x_j, it is interpreted as x_i is b_{ij} times as good as x_j. If

$$b_{ij} = b_{ik} b_{kj}, \ \text{for all} \ i, j, k \in N \tag{8}$$

then B is called a consistent multiplicative preference relation, which is given by

$$b_{ij} = w_i / w_j \ , \ \text{for all} \ i, j \in N \tag{9}$$

In the next section, we shall develop three constrained optimization models based on the objective information contained in the normalized data matrix R and each of three preference formats (utility values, fuzzy preference relations, and multiplicative preference relations) respectively.

3 Constrained Optimization Models Based on Data Matrix and Each Different Exact Preference Format

In the following, we establish the relationships between the given objective information and subjective preference information. In order to do so, we make the objective decision information (the normalized data matrix) uniform respectively with each format of the subjective decision information (utility values, fuzzy preference relations, and multiplicative preference relations).

1) Model based on data matrix and utility values:

We first consider a special case where the information in the normalized data matrix $R=(r_{ij})_{m\times n}$ is consistent with the utility values u_j $(j=1,2,...,n)$, then all the overall attribute values $z_j(w)$ $(j=1,2,...,n)$ of the alternatives x_j $(j=1,2,...,n)$ should be equal to the utility values u_j $(j=1,2,...,n)$, respectively, that is,

$$\sum_{i=1}^{m} w_i r_{ij} = \mu_j, \text{ for all } j=1,2,...,n \tag{10}$$

which is equivalent to the following form:

$$R^T w = \mu \tag{11}$$

where $\mu=(u_1,u_2,...,u_n)^T$.

However, in general, the condition (10) (or (11)) does not hold. That is, the information in the normalized data matrix $R=(r_{ij})_{m\times n}$ is generally inconsistent with the utility values u_j $(j=1,2,...,n)$. Here, we introduce the deviation variable e_j :

$$e_j = \sum_{i=1}^{m} w_i r_{ij} - \mu_j, \text{ for all } j=1,2,...,n \tag{12}$$

Clearly, it is desirable that the overall attribute values $z_j(w)$ $(j=1,2,...,n)$ of the alternative x_j $(j=1,2,...,n)$ should be as closer to the utility values u_j $(j=1,2,...,n)$ as possible. Thus, we need to minimize the deviation variables e_j $(j=1,2,...,n)$, and then construct the following constrained optimization model:

(M-1) $$J_1^* = Min \sum_{j=1}^{n} e_j^2$$

$$s.t. \sum_{i=1}^{m} w_i = 1, \; w_i \ge 0, \; i=1,2,...,m$$

where $e_j^2 = (\sum_{i=1}^{m} w_i r_{ij} - \mu_j)^2$, for all $j=1,2,...,n$.

2) Model based on data matrix and fuzzy preference relations:

In order to make the information uniform, we can utilize (6) to transform all the overall values $z_j(w)$ ($j = 1,2,..., n$) of the alternatives x_j ($j = 1,2,..., n$) into a consistent fuzzy preference relation $\overline{P} = (\overline{p}_{ij})_{n\times n}$ by using the following transformation function:

$$\overline{p}_{ij} = 0.5(z_i(w) - z_j(w) + 1), \text{ for all } i, j = 1,2,..., n \tag{13}$$

i.e.,

$$\overline{p}_{ij} = 0.5(\sum_{k=1}^{m} w_k r_{ki} - \sum_{k=1}^{m} w_k r_{kj} + 1) = 0.5(\sum_{k=1}^{m} w_k (r_{ki} - r_{kj}) + 1), \text{ for all } i, j = 1,2,..., n \tag{14}$$

If the information in the normalized data matrix $R = (r_{ij})_{m\times n}$ is consistent with the fuzzy preference relation $P = (p_{ij})_{n\times n}$, then the consistent fuzzy preference relation $\overline{P} = (\overline{p}_{ij})_{n\times n}$ should be equal to $P = (p_{ij})_{n\times n}$, i.e.,

$$0.5(\sum_{k=1}^{m} w_k (r_{ki} - r_{kj}) + 1) = p_{ij}, \text{ for all } i, j = 1,2,..., n \tag{15}$$

However, the condition (15) does not always hold. That is, the information in the normalized data matrix $R = (r_{ij})_{m\times n}$ is generally inconsistent with the fuzzy preference relation $P = (p_{ij})_{n\times n}$. Then we introduce the deviation variable e_{ij} such that

$$e_{ij} = 0.5(\sum_{k=1}^{m} w_k (r_{ki} - r_{kj}) + 1) - p_{ij}, \text{ for all } i, j = 1,2,..., n \tag{16}$$

Clearly, it is desirable that the consistent fuzzy preference relation $\overline{P} = (\overline{p}_{ij})_{n\times n}$ should be as closer to the fuzzy preference relation $P = (p_{ij})_{n\times n}$ as possible. Thus, we need to minimize the deviation variables e_{ij} $(i, j = 1,2,..., n)$, and then construct the following constrained optimization model:

(M-2) $J_2^* = Min \sum_{i=1}^{n} \sum_{j=1}^{n} e_{ij}^2$

$$s.t. \sum_{i=1}^{m} w_i = 1, \; w_i \geq 0, \; i = 1,2,..., m$$

where $e_{ij}^2 = [0.5(\sum_{k=1}^{m} w_k (r_{ki} - r_{kj}) + 1) - p_{ij}]^2$, for all $i, j = 1,2,..., n$.

3) Model based on data matrix and multiplicative preference relations:

To integrate the decision information in the normalized data matrix $R=(r_{ij})_{m\times n}$ and the multiplicative preference relation $B=(b_{ij})_{n\times n}$, we utilize (9) to transform all the overall values $z_j(w)$ ($j=1,2,...,n$) of the alternatives x_j ($j=1,2,...,n$) into a consistent multiplicative preference relation $\overline{B}=(\overline{b}_{ij})_{n\times n}$ by using the following transformation function:

$$\overline{b}_{ij}=z_i(w)/z_j(w), \text{ for all } i,j=1,2,...,n \tag{17}$$

i.e.,

$$\overline{b}_{ij}=\sum_{k=1}^{m}w_k r_{ki}\Big/\sum_{k=1}^{m}w_k r_{kj}, \text{ for all } i,j=1,2,...,n \tag{18}$$

If the information in the normalized data matrix $R=(r_{ij})_{m\times n}$ is consistent with the multiplicative preference relation $P=(p_{ij})_{n\times n}$, then the consistent multiplicative preference relation $\overline{B}=(\overline{b}_{ij})_{n\times n}$ should be equal to the multiplicative preference relation $B=(b_{ij})_{n\times n}$, i.e.,

$$\sum_{k=1}^{m}w_k r_{ki}\Big/\sum_{k=1}^{m}w_k r_{kj}=b_{ij}, \text{ for all } i,j=1,2,...,n \tag{19}$$

For the convenience of calculation, (19) can be transformed as:

$$\sum_{k=1}^{m}w_k r_{ki}=b_{ij}\sum_{k=1}^{m}w_k r_{kj}, \text{ for all } i,j=1,2,...,n \tag{20}$$

However, the condition (20) does not always hold. That is, the information in the normalized data matrix $R=(r_{ij})_{m\times n}$ is generally inconsistent with the multiplicative preference relation $B=(b_{ij})_{n\times n}$. Then we introduce the deviation variable f_{ij} such that

$$f_{ij}=\sum_{k=1}^{m}w_k r_{ki}-b_{ij}\sum_{k=1}^{m}w_k r_{kj}=\sum_{k=1}^{m}w_k(r_{ki}-b_{ij}r_{kj}), \text{ for all } i,j=1,2,...,n \tag{21}$$

Clearly, it is desirable that the consistent multiplicative preference relation $\overline{B}=(\overline{b}_{ij})_{n\times n}$ should be as closer to the multiplicative preference relation $B=(b_{ij})_{n\times n}$ as possible. Thus, we need to minimize the deviation variables f_{ij} $(i,j=1,2,...,n)$, and then construct the following constrained optimization model:

(M-3) $J_3^*=Min\sum_{i=1}^{n}\sum_{j=1}^{n}f_{ij}^2$

$s.t.\ \sum_{i=1}^{m}w_i=1,\ w_i\geq 0,\ i=1,2,...,m$

where $f_{ij}^2 = (\sum_{k=1}^{m} w_k (r_{ki} - b_{ij} r_{kj}))^2$, for all $i, j = 1,2,..., n$.

4 Constrained Optimization Models Integrating Data matrix and All Three Different Preference Structures

Now we consider the MAGDM problem with three different exact preference structures, namely, utility values, fuzzy preference relations, and multiplicative preference relations. Without loss of generality, we suppose that:

1) The decision makers $d_k (k = 1,..., t_1)$ provide their preference information on n alternatives x_j $(j = 1,2,...,n)$ by means of the utility values $u_j^{(k)}$ $(j = 1,2,...,n;\ k = 1,..., t_1)$.

2) The decision makers $d_k (k = t_1 + 1,..., t_2)$ provide their preference information on n alternatives x_j $(j = 1,2,...,n)$ by means of the fuzzy preference relations $P^{(k)} = (p_{ij}^{(k)})_{n \times n}$, $k = t_1 + 1,..., t_2$.

3) The decision makers $d_k (k = t_2 + 1,..., t)$ provide their preference information on n alternatives x_j $(j = 1,2,..., n)$ by means of the multiplicative preference relations $B^{(k)} = (b_{ij}^{(k)})_{n \times n}$, where $k = t_2 + 1,..., t$.

In the previous section, we have established three constrained optimization models based on the data matrix and each of the three different exact preference structures, namely, utility values, fuzzy preference relations, and multiplicative preference relations. Based on these three constrained optimization models, in the following we establish an integrated model to reflect both the objective decision information contained in the normalized data matrix $R = (r_{ij})_{m \times n}$ and the group opinion of all the decision makers $d_k (k = 1,..., t)$:

(M-4) $$\bar{J}_1^* = Min \sum_{k=1}^{t_1} \sum_{j=1}^{n} \lambda_k \left(\sum_{i=1}^{m} w_i r_{ij} - \mu_j^{(k)} \right)^2$$

$$\bar{J}_2^* = Min \sum_{k=t_1+1}^{t_2} \sum_{i=1}^{n} \sum_{j=1}^{n} \lambda_k \left[0.5 \left(\sum_{l=1}^{m} w_l (r_{li} - r_{lj}) + 1 \right) - p_{ij}^{(k)} \right]^2$$

$$\bar{J}_3^* = Min \sum_{k=t_2+1}^{t} \sum_{i=1}^{n} \sum_{j=1}^{n} \lambda_k \left[\sum_{l=1}^{m} w_l (r_{li} - b_{ij}^{(k)} r_{lj}) \right]^2$$

$$s.t. \sum_{i=1}^{m} w_i = 1,\ w_i \geq 0,\ i = 1,2,..., m$$

where $\lambda = (\lambda_1, \lambda_2, ..., \lambda_t)^T$ be the weight vector of the decision makers d_k $(k = 1,..., t)$, with $\lambda_k \geq 0, k \in T$, and $\sum_{k=1}^{t} \lambda_k = 1$.

By the linear equal weighted summation method [10], the model (M-4) can be transformed into the following single objective constrained optimization model:

$$\textbf{(M-5)} \quad \overline{J}^* = Min(\overline{J}_1 + \overline{J}_2 + \overline{J}_3)$$

$$s.t. \sum_{i=1}^{m} w_i = 1, \; w_i \geq 0, \; i = 1,2,..., m$$

where

$$\overline{J}_1 = \sum_{k=1}^{t_1} \sum_{j=1}^{n} \lambda_k \left(\sum_{i=1}^{m} w_i r_{ij} - \mu_j^{(k)} \right)^2$$

$$\overline{J}_2 = \sum_{k=t_1+1}^{t_2} \sum_{i=1}^{n} \sum_{j=1}^{n} \lambda_k \left[0.5 \left(\sum_{l=1}^{m} w_l (r_{li} - r_{lj}) + 1 \right) - p_{ij} \right]^2$$

$$\overline{J}_3 = \sum_{k=t_2+1}^{t} \sum_{i=1}^{n} \sum_{j=1}^{n} \lambda_k \left[\sum_{l=1}^{m} w_l (r_{li} - b_{ij} r_{lj}) \right]^2$$

which has m weight variables, a linear equality constraint, m linear inequality constraints, and a nonlinear objective function which is to be minimized.

Solving the model (M-5) by the well-known optimization software Lingo 9.0, we get the optimal objective function value $\overline{J}^*$, the optimal attribute weight vector $w^* = (w_1^*, w_2^*, ..., w_m^*)^T$. After that, by (3), we calculate the overall attribute values $z_j(w^*)$ $(j = 1,2,..., n)$, by which we can rank all the alternatives x_j $(j \in N)$ and then select the best one(s).

5. Concluding Remarks

In this paper, we have established an integrated constrained optimization model to solving the multiple attribute group decision making (MAGDM) problems with preference information on alternatives. The model integrates all the given objective information contained in the data matrix and the subjective preferences given by the decision makers over alternatives represented by three well-known exact preference formats: 1) utility values; 2) fuzzy preference relations; and 3) multiplicative preference relations. Structurally, the model consists of a linear equality constraint and a system of linear inequality constraints, and a nonlinear objective function which is to be minimized. We can solve the model easily by using some existing optimization software packages such as the well-known optimization software Lingo 9.0. On the basis of the attribute weights derived from the established model, we have utilized the overall attribute values of alternatives to achieve the final ranking of the given alternatives so as to get the desirable decision result.

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