

Figure of Merit for a Multi-Generation Network

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Abstract—Many Telecommunications providers now support multiple generations of wireless standards in their network. For example, some providers simultaneously support 2G, 3G, 4G and even 5G standards. Upper management generally need a high level view of the performance being provided to consumers on a regular basis. In addition, engineers need to monitor the health of each technology in order to determine problems and be able to react quickly. We provide a framework for determination of a single Figure of Merit (FoM) that can be used for high level monitoring while at the same time providing sufficiently valuable low level indicators to assist with the isolation and detection of problems. We illustrate this framework using data from a real cellular network.

Index Terms—Figure of Merit, Performance Monitor, Data Analytics, Key Performance Indicator, Telecommunications Infrastructure

I. INTRODUCTION

The historical and current upward trend in the number of mobile wireless communication subscribers together with the increased digital content demand [1] has led to continuous progress leading to digital access generational technologies such as 2G, 3G, 4G and, more recently, 5G technologies [2], [3].

Given the swift development of new wireless communications technologies, as well as the incremental improvement of existing technologies, it is expected that the existing network infrastructure from mature telecommunication service providers will have a collection of disparate cellular technologies that serves their existing consumer base. The reason for the disparate cellular technologies can also be attributed to a combination of multiple user-derived factors, with the key one being that subscribers may be unwilling to upgrade their devices and the need to recoup the cost of past infrastructure investments.

The problem of managing migration between different generational network technologies is well researched and many insights and optimization methods have been reported over the years [4]–[6]. However, many developing countries still lack infrastructure in place for complete migration and depend on overlapping and amalgamated cross-generational systems that service areas that may have a low-subscriber density.

Traditional network management calls for Key Performance Indicators(KPIs) to be used as measures of Quality of Service (QoS) and are used for network management/troubleshooting. These quality metrics are crucial for the success and growth of telecommunication companies. However, these scores are difficult to efficiently manage in the previously mentioned

multi-technology network leading to loss in revenue and delays in network troubleshooting.

We propose a mathematical framework for a single Figure of Merit for such multi-technology networks that incorporate QoS/QoE scores that allow for a high-level of monitoring and can be further decomposed to enable a lower level view by radio access technology type (2G, 3G, 4G and 5G) and sub-categories(Voice Calls, Web and SMS). This framework focuses on QoS metrics [7]. The scores used in this model are mainly derived from network probes at the edge of the telecommunication network, near to base stations, as it is assumed that these scores would be dependent on the performance of the internal network. Also, computing KPIs at the edge of the network is more efficient in practice [8]. The FoM is calculated through assuming independence in the KPIs at the select sub-categories and then combining them using weights based on the proportion of subscribers of each radio access technology. The resultant FoM is essentially the probability that a customer uses any service on any technology and receives acceptable service and hence lies between 0 and 1. The FoM and all other calculated scores are provided through an interactive hierarchical diagram so that errors that propagate to the final FoM can be easily identified.

This paper is organized such that Section II covers traditional Performance Metrics that are used in the FoM formulation, Section III covers the mathematical formulation of the FoM, Section IV uses real gathered and simulated data to provide numerical examples, Section V discusses development and deployment of the system, Section VI focuses on how the model changes with respect to the input and Section VII provides opportunities for future work.

II. THE COMPONENT PERFORMANCE METRICS

This framework can be used in various scenarios. However, within the context of the remainder of this paper, we assume the network has these properties.

- The network consists of only digital-based radio access technologies (RAT) such as 2G, 3G, 4G and 5G.
- A Base Stations may or may not support multiple technologies.
- Web services for 2G networks are delivered via General Packet Radio Service (GPRS) and Enhanced Data GSM Evolution (EDGE).
- 4G technologies use Circuit Switched FallBack (CSFB) to facilitate voice calls and short message services (SMS) via the existing 3G network.

- 5G technologies use Evolved Packet System (EPS) Fall-back so that voice calls and SMS are delivered through the 4G CSFB protocol.

All performance metrics are grouped by Access Technology (2G, 3G, 4G and 5G) and by sub-categories (Voice, Web, SMS) within each access technology. Note that these categories would provide sufficient insight to estimate the performance of the overall network and that any problems within the network backbone would be reflected in these metrics.

A. 2G and 3G Metrics

1) *Voice*: Call Success Rate and Call Drop Rate are known to be among the best indicators of QoS of a given cellular network [9]. Call setup delay also reflects user perceived performance. We also take into account the Voice quality rate which is based on the Mean Opinion Score.

Let r represent the radio access type, so that r is either 2G or 3G. Let $dc(r, n)$ represent the number of dropped calls that occurred during the n th measurement period. Let $sc(r, n)$ represent a count of successfully connected calls during the same period and $ac(r, n)$ be the number of attempted calls during the period. We define,

- Call Drop Rate, (P_{vm}): Probability that a successfully connected voice call is dropped.

$$P_{vm}(r, n) = \frac{dc(r, n)}{sc(r, n)} \quad (1)$$

- Call Success Rate, (P_{vc}): Probability of a successful voice call setup.

$$P_{vc}(r, n) = \frac{sc(r, n)}{ac(r, n)} \quad (2)$$

- Call Setup Delay, (P_{vd}): Probability of calls being setup within a specified delay threshold. Let $T^{cs}(r, n)$ represent a count of calls that are setup within this threshold.

$$P_{vd}(r, n) = \frac{T^{cs}(r, n)}{sc(r, n)} \quad (3)$$

- Voice Quality Rate, (P_{vq}): Probability of calls being ranked above a specified estimated Mean Opinion Score(MOS). Let $T^{vq}(r, n)$ represent a count of calls satisfying this score. Note that there are multiple ways of calculating the MOS [10]–[12].

$$P_{vq}(r, n) = \frac{T^{vq}(r, n)}{sc(r, n)} \quad (4)$$

The probability that a customer was able to complete a call with acceptable quality during the n th period is given by

$$P_v(r, n) = (1 - P_{vm}(r, n))P_{vc}(r, n)P_{vd}(r, n)P_{vq}(r, n) \quad (5)$$

which is the probability that the call setup was successful and completed within the delay threshold, the call did not drop and the quality of the call was acceptable.

- 2) *Web*: Let $ap(r, n)$ be a count of page requests attempted during period n , and $sp(r, n)$ be a count of pages that are successful during period n .

- Page Success Rate, (P_{ws}): Probability of a successful page request.

$$P_{ws}(r, n) = \frac{sp(r, n)}{ap(r, n)} \quad (6)$$

- Browsing Delay, (P_{wd}): Probability that the average page delay is below some threshold. Let $T^{wd}(r, n)$ represent the successfully loaded pages that satisfy this delay.

$$P_{wd}(r, n) = \frac{T^{wd}(r, n)}{sp(r, n)} \quad (7)$$

The probability of acceptable web service during the n th period is given by

$$P_w(r, n) = P_{ws}(r, n)P_{wd}(r, n). \quad (8)$$

- 3) *SMS*: Let $ms(r, n)$ be a count of successful messages sent during period n and $ma(r, n)$ be a count of attempted messages sent in period n .

- SMS Success Rate, (P_{ss}): Probability that SMS is successfully delivered.

$$P_{ss}(r, n) = \frac{ms(r, n)}{ma(r, n)} \quad (9)$$

- SMS Delay, (P_{sd}): Probability of successful SMS is below a specified delay threshold. Let $T^{sd}(r, n)$ represent the count of successful SMS messages within that threshold.

$$P_{sd}(r, n) = \frac{T^{sd}(r, n)}{ms(r, n)} \quad (10)$$

The probability of acceptable SMS service during the n th period is given by

$$P_m(r, n) = P_{ss}(r, n)P_{sd}(r, n). \quad (11)$$

B. 4G

- 1) *Voice*: Voice services are serviced by CSFB so successful voice calls would be quantified using metrics related to CSFB. Let $fbs(n)$ be the count of successful fallbacks , and $fb(n)$ be the number of attempted fallbacks during period n .

- CSFB Call Success, (P_{fbs}): Probability that a CSFB successfully falls back to UMTS.

$$P_{fbs}(n) = \frac{fbs(n)}{fb(n)} \quad (12)$$

- CSFB Delay, (P_{fdbd}): Probability that a CSFB is successful within a time threshold. Let $T_{fdbd}(n)$ represent a count of successful CSFB that satisfies this threshold.

$$P_{fdbd}(n) = \frac{T_{fdbd}(n)}{fbs(n)} \quad (13)$$

- 2) *Web & SMS*: Web and SMS service metrics are the same as 2G/3G (Eq. 5-8) where $r = 4G$.

C. 5G

1) *Voice*: 5G Voice services are provided by Evolved Packet System (EPS) Fallback, which uses UMTS to facilitate voice calls. Let $eps(n)$ be the count of successful fallbacks , and $ep(n)$ be the number of attempted fallbacks over period n .

- EPS Call Success, (P_{eps}): Probability that a CSFB successfully falls back to UMTS.

$$P_{eps}(n) = \frac{eps(n)}{ep(n)} \quad (14)$$

- EPS Delay, (P_{epd}): Probability that a EPS Fallback is successful within a time threshold. Let $T_{ebd}(n)$ represent a successful EPS Fallback that satisfies this threshold.

$$P_{epd}(n) = \frac{T_{ebd}(n)}{eps(n)} \quad (15)$$

2) *Web & SMS*: Web and SMS service metrics are the same as 2G/3G (Eq. 5-8) where $r = 5G$.

III. FIGURE OF MERIT COMPUTATION

The FoM is based on determining the probability that an action taken by a customer (e.g., making a phone call or online request or sending an SMS) is successful and is completed in a satisfactory manner (i.e., the call is not dropped, the web page was retrieved sufficiently fast, etc.). In order to do this, we assume independence between events and compute the probability of satisfactory actions for each type of service and for each supported technology. We then take the average of these values over all technologies under the assumption that the probability that a particular technology is used depends solely on the number of subscribers of that technology. Note that these technologies at some point use shared resources (e.g. the core network and International links) but poor performance of those platforms will be reflected in the statistics used in our calculation of the FoM.

Let $P_s(r, n)$ represent the probability of satisfactory completion of service s when using radio technology r during measurement period n . s can be voice calls, v , web requests w or SMS messages m . The technology, r , can be 2G, 3G, 4G or 5G.

In the case of a traditional 2G/3G (Circuit Switched) voice call, the call setup must be successful and prompt (low delay), the call must continue to completion (i.e. not dropped) and the quality must be satisfactory. For 2G/3G technologies, we define the probability of a satisfactory voice call, where r is 2G or 3G by

$$P_v(r, n) = (1 - P_{vm}(r, n))P_{vc}(r, n)P_{vd}(r, n)P_{vq}(r, n) \quad (16)$$

For 4G voice call services, we define an acceptable call as one where the CSFB is successful and it is below a certain delay threshold. This is represented by

$$P_v(4G, n) = P_{fbs}(4G, n)P_{fb}(4G, n) \quad (17)$$

Similarly, the probability of acceptable 5G voice call services is represented by

$$P_v(5G, n) = P_{eps}(5G, n)P_{epd}(5G, n) \quad (18)$$

In the case of a web request (e.g. for a web page) for all r the probability of acceptable service is

$$P_w(r, n) = P_{ws}(r, n)P_{wd}(r, n) \quad (19)$$

For SMS for all r we have

$$P_m(r, n) = P_{ss}(r, n)P_{sd}(r, n) \quad (20)$$

We assume that the probability that a randomly chosen is using technology r is equal to the total number of subscribers of technology r divided by the total number of subscribers. We use α_r to represent the proportion of technology r subscribers where r is 2G, 3G, 4G or 5G. For each service $s \in \{v, w, m\}$ we define the overall probability of acceptable use of that service during period n by

$$P_s(n) = \sum_{r \in \{2,3,4,5\}G} \alpha_r P_s(r, n) \quad (21)$$

Finally we determine the Figure of Merit as a weighted combination of these service metrics. We weight each service s based on the fraction of revenue μ_s that is derived from that service. So, for example, if 60% of service revenue is derived from circuit switched voice calls then $\mu_v = 0.6$. We then determine the Figure of Merit for measurement period n as

$$P(n) = \sum_{s \in \{v,w,m\}} \mu_s P_s(n) \quad (22)$$

Note that this FoM value lies between 0 and 1.

IV. AN ILLUSTRATIVE EXAMPLE

Consider Table I which shows sample data for the input performance metrics to the Figure of Merit Framework. This data consists of a mixture of both simulated and collected data from a multi-generational network over days that have both normal performance and loss of service. There are a total of 7 measurement samples represented by indices 0-6. Each of these data entries correspond to a period of 24 hours so that counter values are retrieved and the metrics computed every 24 hours. Assume that the subscribers for each technology are equally distributed, meaning that $\alpha_r = 0.25$ for each r .

Consider the data entered for Period 0, the FoM for this instance will be calculated as follows. First the probability of a satisfactory voice call within each technology is found by finding the product of the KPIs within each voice group. $P_v(2G, 0)$ and $P_v(3G, 0)$ can be found as 0.51 and 0.76 respectively. $P_v(4G, 0)$ is found to be 0.74 and $P_v(5G, 0)$ is computed as 0.77. The overall voice experience on the network is then found by taking the product of the proportion of the users of that technology and their voice scores, and summing across all technologies as defined above to obtain $P_v(0) = 0.69$. Similar computations are performed for web access to obtain $P_w(0) = 0.74$ and SMS to obtain

TABLE I
FoM VALUES FOR GATHERED DATA

	Metric	Period							
		0	1	2	3	4	5	6	
2G	Voice	P_{vc}	0.83	0.84	0.51	0.87	0.9	0.9	0.87
		P_{vd}	0.88	0.82	0.52	0.84	0.84	0.84	0.82
		$1 - P_{vm}$	0.87	0.9	0.4	0.9	0.9	0.9	0.94
		P_{vq}	0.8	0.88	0.5	0.88	0.88	0.88	0.88
3G	Web	P_{ws}	0.72	0.77	0.77	0.77	0.8	0.8	0.77
		P_{wd}	0.74	0.7	0.7	0.7	0.79	0.79	0.7
	SMS	P_{ss}	0.79	0.89	0.89	0.74	0.74	0.74	0.9
		P_{sd}	0.77	0.95	0.95	0.79	0.79	0.79	0.95
4G	Voice	P_{vc}	0.91	0.9	0.52	0.9	0.9	0.9	0.92
		P_{vd}	0.97	0.95	0.64	0.95	0.95	0.95	0.95
	Web	$1 - P_{vm}$	0.96	0.98	0.4	0.98	0.96	0.96	0.93
		P_{vq}	0.9	0.93	0.64	0.93	0.93	0.93	0.93
5G	Web	P_{ws}	0.87	0.89	0.89	0.83	0.83	0.83	0.89
		P_{wd}	0.86	0.81	0.81	0.81	0.82	0.82	0.81
	SMS	P_{ss}	0.95	0.94	0.94	0.95	0.95	0.95	0.97
		P_{sd}	1	1	1	1	0.98	0.98	1
FoM Score		0.74	0.76	0.67	0.73	0.74	0.61	0.76	

$P_m(0) = 0.88$. For computation of the Figure of merit we assume $\mu_v = 0.3$, $\mu_w = 0.6$ and $\mu_m = 0.1$. This provides a FoM value of $P(0) = 0.74$. This process is repeated for each of the other periods resulting in the values in Table I.

We simulated poor voice performance for the 2G and 3G technologies in period 2 by simulating poor voice metrics. Since this occurred on two different technologies then it represents an issue in the core network. In period 5 we simulate poor web service for 5G. In Figure 1 we plot the FoM as a function of time to illustrate how these simulated failures show up as drops in the FoM score.

The score at Period 2 is lower than the previous entries (Periods 0 and 1) and therefore reflects that there is a change in performance experienced on the network. This drop in FoM score is not as severe as the drop at period 5 due to the weighting factors since the voice score has an overall weight of 0.3 while data has an overall weight of 0.6. This plot reflects the changes that are made to the FoM given varying network conditions.

V. SENSITIVITY ANALYSIS OF THE FOM MODEL

Sensitivity Analysis is the analysis of the behavior of the model based on changes to its input [13]. In this section we examine how the output of the FoM model changes with both:

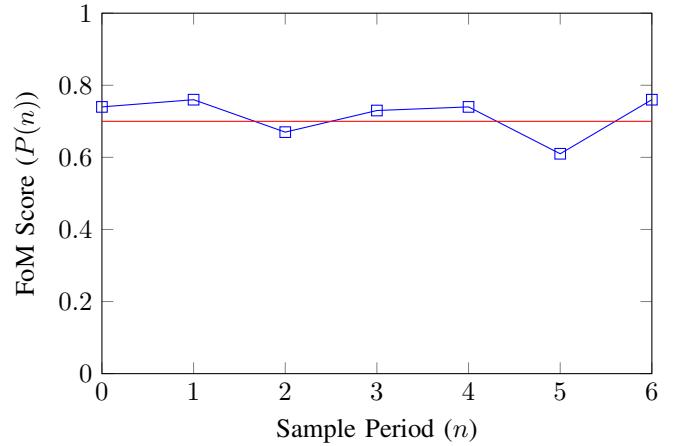


Fig. 1. Simulated FoM Score

1) the variables used for weights in both equations 21 and 22, denoted by α_r and μ_s in Section III and 2) The pool of calculated metrics within each service, denoted by the Metrics described in Section II.

Consider equation 22 which models the final score for the FoM. Let us assume that the weights encapsulated in this model, denoted by α_r and μ_s , are constant. We can then measure the rate of change of the model for each sub-service, i.e. $\frac{dP(n)}{dS_{(r,s)}}$, where $S_{(r,s)}$ is the combined score for a service $s \in \{v, w, m\}$, and a subscriber base $r \in \{2G, 3G, 4G, 5G\}$.

For any $S_{(r,s)}$, the following result holds true:

$$\frac{dP(n)}{dS_{(r,s)}} = \mu_s \alpha_r \quad (23)$$

The result from equation 23 shows that the rate of change of the FoM and any service $\frac{dP(n)}{dS_{(r,s)}}$ is directly dependent on the amount of revenue and subscribers in that service.

Next we can consider the one at a time(OAT) approach to determine the impact each metric has on the final model. To do this each metric P_z would be removed entirely from the model, which is also equivalent to having a maximum value of 1 for that metric, and the FoM score would be recalculated without this metric as $P(n)^{P'_z}$. The difference, Δ , between this score and a baseline score would be determined by the below equation.

$$\Delta = \frac{P(n)^{P'_z} - P(n)}{P(n)} \quad (24)$$

Table II shows the Δ values when using the metrics provided by Period 0 of Table I as a baseline. From this Table, we can see that web metrics currently has the highest percentage change when compared to the baseline and will therefore give an insight on what needs attention in the network. In this case, the 2G web services would need the most attention from the service provider. This is due to the result obtained in equation 23 and the weights used in the example in Section IV.

TABLE II
 Δ FOR METRICS USING OAT APPROACH

		P_z	Δ
2G	Voice	P_{vc}	0.0099
		P_{vd}	0.0064
		P_{vm}	0.0071
		P_{vq}	0.0123
	Web	P_{ws}	0.0411
		P_{wd}	0.0371
	SMS	P_{ss}	0.0049
		P_{sd}	0.0056
3G	Voice	P_{vc}	0.0076
		P_{vd}	0.0024
		P_{vm}	0.0032
		P_{vq}	0.0085
	Web	P_{ws}	0.0225
		P_{wd}	0.0245
	SMS	P_{ss}	0.0017
		P_{sd}	0
4G	Voice	P_{fbs}	0.0127
		P_{fdbd}	0.0086
	Web	P_{ws}	0.0201
		P_{wd}	0.0161
	SMS	P_{ss}	0.0007
		P_{sd}	0
	Voice	P_{eps}	0.0085
		P_{epd}	0.0136
5G	Data	P_{ws}	0.0134
		P_{wd}	0.0094
	SMS	P_{ss}	0
		P_{sd}	0

VI. THE FOM PLATFORM

The deployed FoM platform can be separated into three distinct sections: 1) Data input and processing from network probes, 2) Data warehousing and 3) Front end usage. The entire platform was done entirely using Python and external Python libraries. Raw data from network probes are stored in an external database in real-time. KPI values as shown in Figure 2, are calculated every 24 hours for the past 24 hours. All data processing and calculations were done primarily using dataframe oriented packages such as Pandas and RAPIDS [14]. After calculation, these values are stored in a database. Given the hierarchical architecture of the FoM, a NoSQL(MongoDB) database was used.

The created MongoDB schema allowed "Status" Flags to be included in the captured data. These flags allowed for the calculation of the higher-level FoM values, even if the KPI scores were not able to be captured by dropping them, from the overall FoM equations. If a KPI value could not be calculated, then the score reported for that value by the front-end would be 0, and the remaining calculations of the FoM would ignore this KPI.

This process of data capture, calculation and storage was automated so that it would be performed every 24 hours. The end-user interaction point was a color-graded TreeMap [15], which allows for viewing of the FoM score and the sub-components for each category and their respective scores. The TreeMap was created using the Plotly library and placed in a Dash interactive application to be served in a traditional client-server configuration. Upon every successful connection to the server, the latest complete FoM document from the NoSQL database is used to populate the TreeMap.

Figure 3 uses dummy values to show the front-end view of the interactive FoM when 3G(UMTS) scores are selected. The colorbar helps with an initial visual estimation of the score at that block and when hovered over with the cursor, the actual score is displayed. The hierarchical nature of the TreeMap, along with the color-coding, allows for the user to track scores through the RAT type and sub-category to determine the root cause of poor performance in the network.

VII. CONCLUSIONS AND FUTURE WORK

In this paper we presented a mathematical framework for a Figure of Merit based on a multi-technology cellular network and an interactive platform which allows to see all calculated scores that contributes to the overall FoM. This interactive platform allows for ease of use by upper management to make business decisions such as resource allocation and also by engineers to detect and troubleshoot problems that may arise within their network. Various QoS and QoE scores for differing sub-categories within each digital generational cellular network was used (2G,3G,4G,5G). The sub-categories used were Voice, Web and SMS. The probabilistic metrics within these categories were selected based on the assumption that they should provide sufficient insight on the performance of the parent network. These metrics were assumed independent within each sub-category thus simplifying the probability computations. The usefulness of this platform was also demonstrated through the use of a mixture of real and simulated data which consists of normal operational data and also instances of voice service failure in 2G/3G and web service failure in 5G. By doing this the lack of performance in these services was reflected by the FoM score.

In the future, scores representing other services such as video streaming can be considered in the model. The platform also serves as a data collection point which means that, over a length of time, certain insights and even predictive models can be developed so that network performance problems can be detected proactively.

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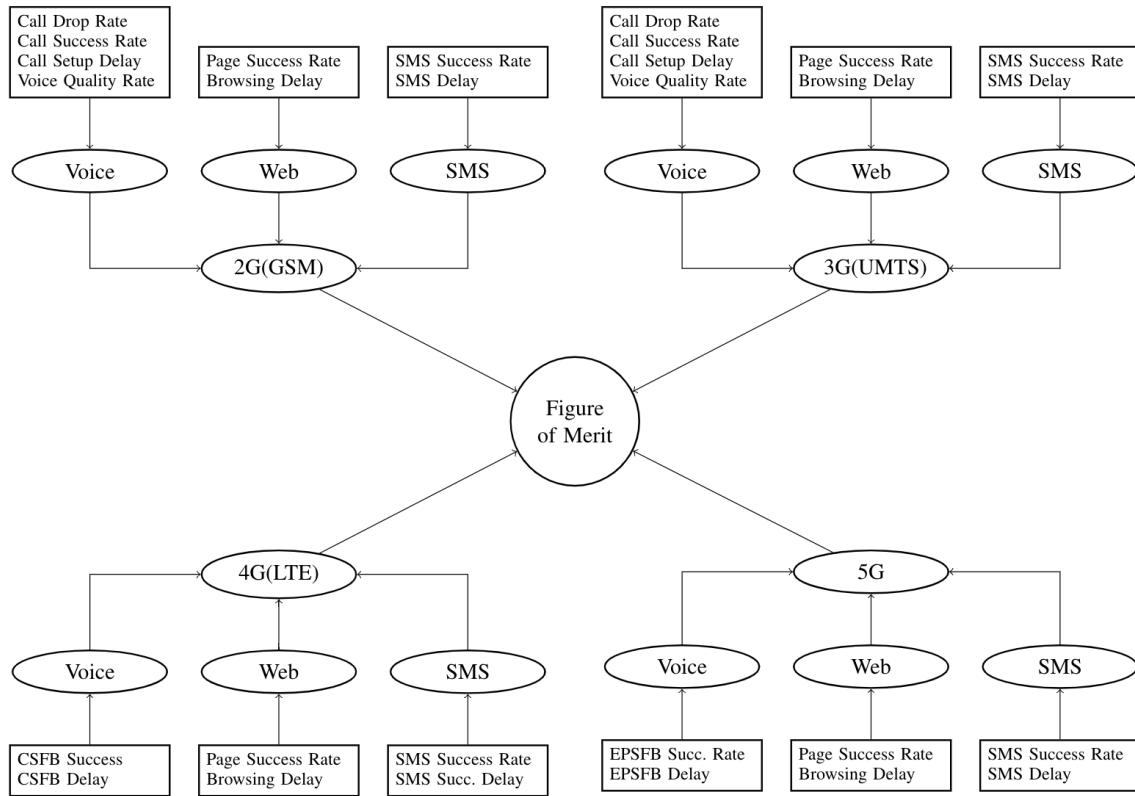


Fig. 2. Flow Chart of Performance Metric Collection and FoM Computation

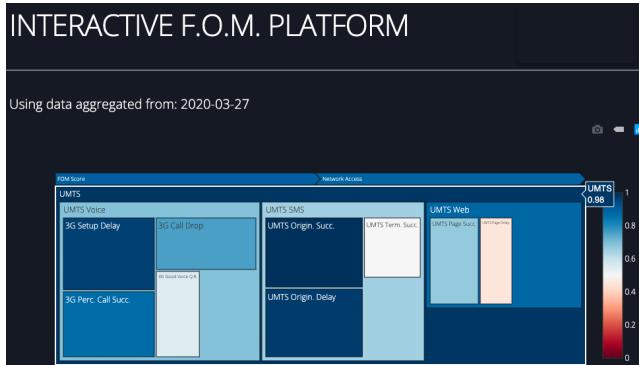


Fig. 3. GUI for FoM Platform

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