Analyzing Dynamics of MVNO Market Using Evolutionary Game

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***Abstract*—In many countries, mobile virtual network operators (MVNOs) provide mobile network services to users by leasing the wireless bandwidth from mobile network operators (MNOs). To attract many users and increase the number of subscribers, some MVNOs introduce the strategy called zero rating (ZR) which exempts traffic of specific content providers (CPs) from usage-based charging. The ZR differentiates traffic of specific CPs from that of other CPs, so the ZR violates the principle of network neutrality, and the ZR is prohibited in some countries. However, to clarify the desirable rules against the ZR, we need to analyze its impact on end users. In this paper, we investigate the charging strategy of ZR MVNOs by analyzing the price plans of major MVNOs in Japan. Moreover, we model the dynamics of the MVNO market consisting of low-price (LP) MVNOs and ZR MVNOs by the evolutionary game which can model the dynamics of social environment described by strategic distribution. We show that the MVNO market will be monopolized by MVNOs using either strategy, and the monthly fee of users will increase at the steady state. Therefore, we conclude that ZR MVNOs are required to introduce a service plan for users who do not benefit from the ZR.**

I. Introduction

Mobile network services have been widely provided in many countries in the world. In addition to MNOs (mobile network operators) which own network equipment, e.g., base stations and backhaul networks, to provide mobile network services to users, MVNOs (mobile virtual network operators) which provide network services to users by leasing the wireless bandwidth from MNOs are also becoming popular in many countries [6]. For example, in Japan, there are 58 MVNOs with 3 million or more subscribers, and MVNOs have about 10% subscriber share in the mobile network market in June 2018 [8].

Although MNOs are major and large scale companies, many MVNOs are small to medium scale companies, so MVNOs use various strategies to increase the number of subscribers. Typical strategies used by MVNOs are *cost leadership* providing services at low price, *differentiation* giving additional value, e.g., original content, and *segmentation* proposing plans which give high customer appeal to focused user segment [6]. Moreover, as a new strategy to attract subscribers, some MNOs and MVNOs introduce *zero rating* (ZR) exempting traffic caused by delivering content of specific providers, e.g., YouTube and Google Play Music, from the monthly charge [12]. As the charging model applied to users, many MNOs and MVNOs use the monthly cap which charges a fixed price to users until the amount of traffic transmitted by users reaches the predefined threshold and charges additional fee proportional to the traffic amount exceeding the threshold [13]. When subscribing to MVNOs with the ZR, the amount of traffic caused by delivering ZR content is exempted from the charging target, so the ZR is a strong strategy to attract users who consume many content items of the ZR target.

The idea of providing services for free has been widely used in various services, and the tall-free 800 numbers is a well-known example which was applied to telecommunication services. In addition to the purpose of increasing the subscribers by attracting users, there are two kinds of purposes of introducing the ZR by MVNOs: (i) resolving the digital divide in developing countries and (ii) creating a new earnings model from content providers (CPs) [1]. As the ZR aiming to resolving the digital divide, for example, Wikipedia zero and Facebook zero which provide data communication services for using Wikipedia or Facebook without fee in various developing countries are widely known. The ZR aiming to obtain revenue from CPs is also called *sponsored delivery*, and CPs pay the fee of data communication instead of users. For example, Sponsored Data of AT&T and FreeBee Data of Verizon are the ZR of this type. In return for paying fee of users, CPs can expect to increase the view count of users for their content items and increase the revenue obtained by advertisements. Finally, the ZR as the marketing strategy is introduced by MVNOs to increase the number of subscribers by differentiating their service with other competitive MVNOs and MNOs, and Binge On of T-Mobile is an example of the ZR of this type. In the Binge On, traffic caused by delivering content of major CPs, e.g., YouTube, Netflix, Hulu, and Amazon prime video, are exempted from the monthly fee, and these CPs are not required to pay fee for this traffic.

However, because the ZR exempts traffic of content of just a specific set of CPs from monthly fee, it is pointed out that the ZR violates the network neutrality which prohibits differentiating the treatment of traffic in the Internet [11], and the ZR of MNOs and MVNOs is actually prohibited by governments in Chili and India [1]. Main reasons for prohibiting the ZR are that content items of CPs which are not ZR target will not be viewed frequently as a result of free delivery of ZR content of major CPs, so small and medium-sized CPs as well as newcomer CPs are difficult to obtain revenue [1]. However, governments of many countries support or wait and see the ZR. The FCC in USA claims that the ZR should be allowed if competitive environment among MNOs and MVNOs is fully maintained, and only problematic ZR, e.g., blocking content of non-ZR CPs or locking users in a specific application, should be restricted [1].

It is desirable to judge the treatment for the ZR based on the

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benefit and loss of users by investigating the influence of the ZR on the competitive market structure of the mobile network service in which many MVNOs exist. However, no existing works investigated the influence of the ZR on users considering a large number of MVNOs. In this paper, we analyze the competitive market of mobile network service consisting of a large number of MVNOs with multiple strategies by using an evolutionary game to clarify the impact of the ZR on the benefit and loss of users. The contributions of this manuscript are summarized as follows.

- We model the environment in which a large number of MVNOs using the LP or the ZR strategies exist by the evolutionary game to investigate the dynamics of the number of MVNOs using each strategy.
- Through the numerical evaluations, we clarify that the MVNO market will be monopolized by MVNOs using either strategy after long time elapses.
- We also show that a part of users suffer disadvantage when the mobile network market is monopolized by MVNOs using the ZR, and we suggest an appropriate treatment for the ZR which benefits all users.

After summarizing the related works in Section II, we analyze the dynamics of the MVNO market by the revolutionary game in Section III. We show the numerical results in Section IV and conclude this manuscript in Section V.

II. Related Works

Debbah et al. analyzed the influence of the brand power of MNOs and the additional values of MVNOs on their user shares when one MNO and one MVNO exist [3]. However, they assumed that the sensitivity of users against the price, brand, and additional value of MNO and MVNO were fixedly given, and they did not analyze the interaction among MNO, MVNO, and users. Moreover, when multiple ISPs and a single cross-carrier (cc) MVNO which virtually provided the mobile network service by leasing the network resources from multiple ISPs existed, Zheng et al. derived the optimum charging parameters of the cc MVNO against users and those of the ISPs against users as well as cc MVNO [14]. However, the probability that each user used the cc MVNO was given by a fixed value, and the authors did not analyze the interaction between the cc MVNO and the ISPs.

Cadre et al., and Guijarro et al., and Meidanis et al. analyzed the interaction between the MNO and the MVNO [2][4][7]. Cadre et al. proposed methods of optimally allocating the wireless-network resources to the MVNO and optimally setting the wholesale price of MNO to MVNO as well as the retail prices of MNO and MVNO to users when just a single MNO and MVNO existed [2]. They investigated the interaction between the MNO and the MVNO by modeling the market using a three-stage Stackelberg game, i.e., allocating wireless resources by MNO, setting the price to users by MVNO, and subscribing the MNO or MVNO by users. Guijarro et al. also modeled the competition between one MNO and one MVNO by a three-stage Stackelberg game consisting of setting the wholesale price to MVNO by MNO, setting the retail prices of MNO and MVNO against users, and subscribing MNO or MVNO by users when just a single MNO and MVNO existed [4]. Moreover, Meidanis et al. modeled the mobile-network market by a two-stage Stackelberg game, i.e., setting the price by MNO and setting the price by MVNO, when a single MNO and MVNO existed, and they showed that the profits of both the operators increased by utilizing the information of willingness to pay of users [7].

All these three existing works modeled the interaction among players by a static model and used the traditional non-cooperative game which derived the static Nash equilibrium. However, it was difficult to derive the Nash equilibrium in the non-cooperative game when a large number of players existed, so they considered just a single MNO and MVNO. It is difficult to investigate the market consisting of a large number of players by the non-cooperative game. Moreover, although non-cooperative game assumes the complete information in which each player knows the strategy sets of itself and all other players as well as the utility of all players against each strategy set, the assumption of complete information is difficult in the environment consisting of a large number of MVNOs.

III. Modeling MVNO Market Using Evolutionary Game

A. Evolutionary Game

The evolutionary game is a new game theory which models social and economic phenomena without assuming the complete information of players. The classical game theory, i.e., non-cooperative game, models the target phenomenon by a static way, whereas the evolutionary game models the target phenomenon by a dynamic way. The evolutionary game describes the target phenomenon by simultaneous differential equations which represent the changes of strategy distribution of each player set through the learnings of players [10]. In the classical non-cooperative game, multiple Nash equilibrium exist in many cases, and which Nash equilibrium is actually realized is not obvious. On the other hand, by using the evolutionary game, we can derive the Nash equilibrium which is realized from the given initial state, and we can investigate the initial state and conditions which are required to achieve any Nash equilibrium.

There are various types of evolutionary games. First, according to the sets of players, there are two types: (i) the non-group model in which just two players repeatedly play the game and (ii) the group model consisting of a large number of players which play the game with randomly selected players. Second, according to the homogeneity of players, there are two types: (i) symmetry game in which all players have the identical strategy set and utility matrix and (ii) asymmetry game consisting of multiple player groups in which pairs of randomly selected from each player group play the game. Third, according to the way of describing the dynamics of game, there are also two types: (i) replicator dynamics which investigates the change of player count in each player group through a birth and death process and (ii) learning dynamics which investigates the process of learning of players through repeated plays of game.

B. Replicator Dynamics

The replicator dynamics assumes that each player of each strategy group plays a game with a randomly selected player and dies after leaving a number of offspring which use the same strategy. The strategy used by a child player is the same with its parent player. The average number of offspring left by each player after the game is determined by the average fitness of each strategy against the environment, and the average fitness of each strategy is given by the utility matrix. We can derive the convergent point depending on the initial state by solving the simultaneous differential equations of the number of players of each strategy [10].

The most primitive replicator dynamics consists of the two player sets using strategy A or strategy B. We define player i as players using strategy i, and let us consider a game between two players randomly selected: one is player i and the other is player j. We define g_{ij} as the utility of player i which is the increase or decrease ratio of player count belonging to player group i when it plays the game with player j. Let x denote the ratio of players using strategy A, and the probability that any player using strategy A plays the game with a player using strategy A or B is x or $1-x$, respectively. Therefore, u_a, the average utility of any player a obtained by one game, is given by $u_a = xg_{aa} + (1-x)g_{ab}$. Similarly, u_b, the average utility of any player b obtained by one game, is given by $u_b = xg_{ba} + (1-x)g_{bb}$. Therefore, the average utility of any player obtained by one game, u, is $u = xu_a + (1-x)u_b$, and we obtain the following differential equation for x,

$$\frac{dx}{dt} = (u_a - u)x. \tag{1}$$

By solving this differential equation for x from any given initial value x_0, we can obtain x at any t.

C. Modeling Approach of MVNO Market

In this paper, we assume the competitive market of MVNOs without MNOs, and we assume the LP and the ZR as the strategies of MVNOs to attract users. Let $\boldsymbol{M}_1$ and $\boldsymbol{M}_2$ denote the set of MVNOs using the LP and the ZR, respectively, and we define x_s as the number of MVNOs in $\boldsymbol{M}_s$. When a large number of MVNOs exist, we derive the convergence value of x_s after repeating the games acquiring users between two MVNOs randomly selected. Because there are many MVNOs which are classified into either of the two groups, our model corresponds to the asymmetry game of the group model. Moreover, we use the replicator-dynamics model because we analyze the time evolution of the MVNO count using each strategy. However, although the replicator dynamics assumes that players playing the game die after leaving some offspring, we regard the continuation, withdrawal, and new entry of MVNOs in the mobile network services as the dynamics of players in the replicator dynamics.

In the replicator dynamics, we need to reflect the result of game between players in the change of player counts. However, in our case, the game between players, i.e., MVNOs, results in the difference of the number of users acquired by each MVNO, so the result of game is not reflected in the change of MVNO counts. Therefore, we need to introduce a mechanism which reflects the number of users acquired on each game in the increase and decrease of the number of MVNOs of each strategy group. To achieve this goal, we define $\phi(\pi)$ as the utility function which is the change ratio of the MVNO count using each strategy and is a function of π, the profit of MVNO determined by the average number of users acquired in one game. When $\phi(\pi)$ of MVNOs using strategy s is greater than unity, MVNOs using strategy s continues to use the same strategy with high probability, and there exists many MVNOs with strategy s entering the mobile network market. On the other hand, when $\phi(\pi)$ MVNOs using strategy s is smaller than unity, MVNOs using strategy s leave the market with high probability, and the number of new MVNOs using strategy s is small.

Let π_0 denote the lower bound of monthly profit required for MVNOs to continue the mobile network service, and the utility function $\phi(\pi)$ satisfies $\phi(\pi_0) = 0$. Moreover, $\phi(\pi)$ monotonically increases as π increases, and the increase ratio of $\phi(\pi)$ is large at around π_0 whereas the increase ratio of $\phi(\pi)$ decreases as π departs from π_0. We also assume that $\phi(\pi)$ approaches to L or $-L$, i.e., $\phi(\pi) \to L$ or $-L$, when $\pi \to \infty$ or $-\infty$, where L is a given parameter taking a real positive number. As the function satisfying these requirements, we assume the Hyperbolic tangent function, tanh $x = (e^{2x}-1)/(e^{2x}+1)$, and we give $\phi(\pi)$ as

$$\phi(\pi) = L \cdot \tanh\Big(\frac{\pi - \pi_0}{z}\Big), \tag{2}$$

where z is a given parameter to scale π according to the order of π_0, and we set $z = 1,000$ in the numerical evaluation.

D. Assumption of MVNO

MVNOs provide mobile network services by leasing wireless bandwidth from MNOs, and we assume that the cost of leasing wireless bandwidth from MNOs per each user is c in all the MVNOs. As the charging model of MVNOs to users, we assume the monthly cap which charges a fixed price to users until the amount of traffic transmitted reaches the predefined threshold and charges additional fee proportional to the traffic amount exceeding the threshold. Moreover, we assume that all the MVNOs using each strategy s, i.e., $\boldsymbol{M}_s$, use the identical price plan. MVNOs of $\boldsymbol{M}_s$ set n_s charge categories, and we define $F_{s,k}$ and $f_{s,k}$ as the upper limit of monthly traffic amount and fixed monthly fee in the k-th charge category. When the monthly traffic amount exceeds $F_{s,k}$ in the contracted charge category k, q_s is charged to users per 1 GB for the excess traffic.

For example, the AEON mobile which is a major LP MVNO in Japan sets 10 charge categories from $F_{1,1} = 1$ GB, $f_{1,1} = 4.25$ USD[1] to $F_{1,10} = 50$ GB, $f_{1,10} = 91.15$ USD, and the DMM mobile which is also a major LP MVNO in Japan sets 9 charge categories from $F_{1,1} = 1$ GB, $f_{1,1} = 4.25$ USD to $F_{1,9} = 20$ GB, $f_{1,9} = 35.30$ USD. Moreover, the BIGLOBE mobile which is a major ZR MVNO in Japan provides the ZR service for video and music of 10 CPs including YouTube, Google Play Music, Spotify, and Amazon Music, and the BIGLOBE mobile sets 5 charge categories from $F_{2,1} = 3$ GB, $f_{2,1} = 12.21$ USD to $F_{2,5} = 30$ GB, $f_{2,5} = 63.98$ USD. Figure 1 plots $f_{s,k}$ against $F_{s,k}$ in these three MVNOs[2]. We confirmed that the price plans of the two

[1] In this paper, we assume the exchange rate of 1 USD = 113 JPN.
[2] $f_{s,k}$ are based on the price in Mar. 2019.

LP MVNOs were almost identical, and $f_{s,k}$ of the ZR MVNO were higher than $f_{s,k}$ of the LP MVNOs for the same $F_{s,k}$. Moreover, we also confirmed that the granularity of charge categories of the ZR MVNO was coarser than those of the LP MVNOs.

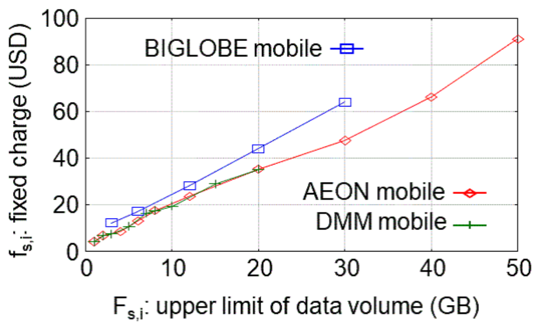


Fig. 1. Price plan of major three MVNOs in Japan

E. Assumption of Users

1) Selecting MVNOs: Let v denote the amount of data transmitted by a user within a month, and let ϵ denote the ratio of data which is the target of the ZR among v. Both v and ϵ are different among users, and the characteristics of each user is determined by two parameters, v and ϵ. We define e_s as a binary variable which takes unity when MVNOs of $\boldsymbol{M}_s$ use the ZR or takes zero otherwise, i.e., $e_1 = 0$ and $e_2 = 1$. We also define $\kappa_s(v, \epsilon)$ as the charge category which is selected by a user (v, ϵ) when subscribing to an MVNO of $\boldsymbol{M}_s$. We can assume that users will select the minimum charge category k in which $F_{s,k}$ is not less than the amount of charging-target traffic, $v(1-e_s\epsilon)$. Therefore, when $v(1-e_s\epsilon) \leq F_{s,n_s}$, $\kappa_s(v, \epsilon)$ is the minimum k in the range of $1 \leq k \leq n_s$ which satisfies $v(1-e_s\epsilon) \leq F_{s,k}$. When $v(1-e_s\epsilon) > F_{s,n_s}$, we assume $\kappa_s(v, \epsilon) = n_s$. We define $\psi_s(v, \epsilon)$ as the monthly fee paid by a user (v, ϵ) subscribing to an MVNO of $\boldsymbol{M}_s$, and it is obtained by

$$\psi_s(v, \epsilon) = f_{s,\kappa_s(v,\epsilon)} + q_s \max\{v - e_s\epsilon v - F_{s,\kappa_s(v,\epsilon)},\ 0\}. \quad (3)$$

Figure 2 plots $\psi_s(v, \epsilon)$ against v when using the price plan of AEON mobile as LP MVNO and BIGLOBE mobile as ZR MVNO, and setting ϵ to 0.2, 0.5, or 0.8. $\psi_1(v, \epsilon)$, i.e., the monthly fee when subscribing to LP MVNO, was independent of ϵ. Both LP MVNO and ZR MVNO used the price plan consisting of multiple stages, so $\psi_s(v, \epsilon)$ stepwisely increased as v increased. However, when v exceeded F_{s,n_s}, $\psi_s(v, \epsilon)$ linearly increased as v increased. Users can save the monthly fee by subscribing to LP MVNO when v was small, whereas users can save the monthly fee by subscribing to ZR MVNO when v was large. When subscribing to ZR MVNO, the monthly fee decreased as ϵ increased.

It is anticipated that users consider various factors, e.g., the brand image, the service quality, and the reputation of MVNOs, in addition to the monthly fee when selecting the MVNOs. However, to simplify the problem, we assume that users select the MVNOs with considering only the monthly fee. In other words, we assume the rational behavior of users selecting the MVNOs, i.e., contracting with LP MVNO when $\psi_1(v, \epsilon) < \psi_2(v, \epsilon)$, and contracting with ZR MVNO when $\psi_1(v, \epsilon) \geq \psi_2(v, \epsilon)$[3]

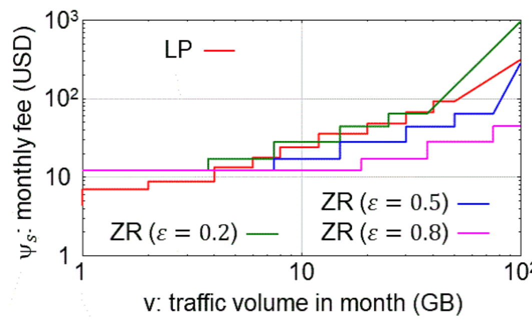


Fig. 2. Monthly fee against monthly traffic volume in major LP and ZR MVNOs in Japan

2) Selection Probability of LP MVNO: We assume that W, the total number of users of MVNOs, is fixed, and let $D(v, \epsilon)$ denote the concurrent probability density function of user (v, ϵ) in W. We define p_1 as the probability that users select LP MVNO when they can freely select either LP MVNO or ZR MVNO, and p_1 is given by

$$p_1 = \int\int D(v, \epsilon) U\big(\psi_1(v, \epsilon) < \psi_2(v, \epsilon)\big) dv d\epsilon, \quad (4)$$

where $U(C)$ is a Boolean-valued function taking unity when condition C is satisfied or taking zero otherwise. The probability that users select ZR MVNO, p_2, is given by $p_2 = 1 - p_1$. Let r_s denote the average revenue of MVNOs of $\boldsymbol{M}_s$ obtained from each of W users within one month, and r_s is given by

$$r_s = \int\int D(v, \epsilon) \psi_s(v, \epsilon) dv d\epsilon. \quad (5)$$

We assume that v and ϵ obey the normal distribution. Delivery service of large-size video content is one of the main targets of the ZR, so it is anticipated that ϵ of users with large v is also large, and there is a positive correlation between v and ϵ. Therefore, as the joint probability distribution of v and ϵ, we assume a bivariate normal distribution with average μ_v and μ_ϵ, standard deviation σ_v and σ_ϵ, and correlation coefficient ρ.

F. Modeling MVNO Market

The average profit of an MVNO of $\boldsymbol{M}_s$ obtained from one user within a month is $r_s - c$. We define π_{ij} as the average profit of an MVNO of $\boldsymbol{M}_i$ when playing the user-acquisition game with an MVNO of $\boldsymbol{M}_j$. Assuming that each user selects the contracted MVNO by comparing two MVNOs randomly selected, any two MVNOs scramble for $2W/(x_1 + x_2)$ users on average in each game because the total number of MVNOs is $x_1 + x_2$. Therefore, when two MVNOs using different strategies play a game, π_{ij} is given by

$$\pi_{12} = \frac{2(r_1 - c)Wp_1}{x_1 + x_2}, \quad (6)$$

$$\pi_{21} = \frac{2(r_2 - c)W(1 - p_1)}{x_1 + x_2}. \quad (7)$$

[3]We assume that users select ZR MVNO when $\psi_1(v, \epsilon) = \psi_2(v, \epsilon)$.

On the other hand, when two MVNOs using the same strategy play a game, we have

$$\pi_{11} = \frac{(r_1 - c)W}{x_1 + x_2}, \quad (8)$$

$$\pi_{22} = \frac{(r_2 - c)W}{x_1 + x_2}, \quad (9)$$

assuming that each MVNO acquires each user with probability of 0.5. As mentioned in Section III-D, using ϕ defined by (2), the utility of an MVNO of $\boldsymbol{M}_i$ when playing a game with an MVNO of $\boldsymbol{M}_j$, g_{ij}, is given by

$$g_{ij} = \phi(\pi_{ij}) = L \cdot \tanh\left(\frac{\pi_{ij} - \pi_0}{z}\right). \quad (10)$$

Because each MVNO plays a game with an MVNO of $\boldsymbol{M}_s$ with the probability of $x_s/(x_1 + x_2)$, the average profit of an MVNO of $\boldsymbol{M}_s$ obtained in one game, G_s, is obtained by

$$G_s = \frac{x_1}{x_1 + x_2} g_{s1} + \frac{x_2}{x_1 + x_2} g_{s2}. \quad (11)$$

Using G_s, we have the following simultaneous differential equations [10],

$$\frac{dx_1}{dt} = G_1 x_1 = \frac{x_1^2}{x_1 + x_2} g_{11} + \frac{x_1 x_2}{x_1 + x_2} g_{12}, \quad (12)$$

$$\frac{dx_2}{dt} = G_2 x_2 = \frac{x_1 x_2}{x_1 + x_2} g_{21} + \frac{x_2^2}{x_1 + x_2} g_{22}. \quad (13)$$

As observed in (6)-(9), π_{ij} is a constant, so we can obtain x_1 and x_2 at any t using numerical solution, e.g., Runge-Kutta method, when giving the initial values of x_1 and x_2.

IV. Numerical Evaluation

We set W, the number of users, to 10 million, and we use the price plan of AEON mobile and BIGLOBE mobile mentioned in Section III-D as the charging parameters, $F_{s,k}$ and $f_{s,k}$. We also assume that v, the amount of data transmitted by each user, obeys the normal distribution with the average $\mu_v = 4.22$ GB [9].

It is difficult to directly derive p_1 from (4), so we obtain p_1 by calculating the ratio of user samples with $\psi_1(v, \epsilon) < \psi_2(v, \epsilon)$ when generating 100,000 user samples with any (v, ϵ) according to the following procedure.

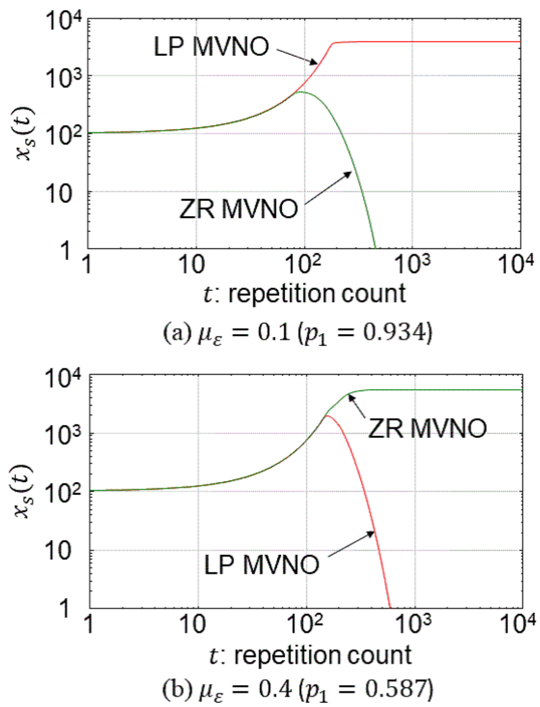
1) Randomly generates v from the normal distribution $N(\mu_v, \sigma_v)$
2) Randomly generates ϵ from the normal distribution $N(\mu_\epsilon + \rho(v - \mu_v)\sigma_\epsilon/\sigma_v,\ (1 - \rho^2)\sigma_\epsilon^2)$

Because v and ϵ must be $v \geq 0$ and $0 \leq \epsilon \leq 1$, we set σ_v and σ_ϵ to the maximum values with the condition that the probability of v and ϵ being outside this range is less than 0.001. We set $v = 0$ when $v < 0$, and we set $\epsilon = 0$ ($\epsilon = 1$) when $\epsilon < 0$ ($\epsilon > 1$) for the generated user samples.

A. Time Evolution of MVNO Count

The cost of leasing wireless bandwidth per user, c, depends on the degree of user aggregation to unit bandwidth, and MVNO needs to satisfy $c < r_s$ to guarantee the positive profit. r_s was greater than 11, so we set $c = 7$ USD in the following evaluation. Moreover, we assume that π_0, the lower limit of monthly profit required MVNOs to continue the mobile network service, was the average revenue of each LP MVNO obtained from B users within a month, and we set $\pi_0 = Br_1$ and $B = 1,000$. We set L, the limit value of the utility in each game between two MVNOs, to 2.

We obtain $x_s(t)$, the number of MVNOs of $\boldsymbol{M}_s$ at repetition count t, by numerically solving (12)(13) using the fourth-order Runge-Kutta method [5]. We set $x_s(0)$, the initial number of MVNOs of $\boldsymbol{M}_s$, to 100, and we set the step-width h of Runge-Kutta method to 0.01. Figure 3 plots $x_s(t)$ against t when setting $\mu_\epsilon = 0.1$ ($p_1 = 0.934$) and $\mu_\epsilon = 0.4$ ($p_1 = 0.587$). In the initial stage, the total number of MVNOs, $x_1(t) + x_2(t)$, was small, so $x_s(t)$ increased as t increased in both the two strategy groups. However, after time elapsed, the number of MVNOs using the strategy with higher competitive power continued to increase up to the converged value, whereas the number of MVNOs using the other strategy decreased to zero. The average monthly profit of MVNOs using the ZR, r_2, was larger than that of MVNOs using the LP, r_1, and the profitability of ZR MVNOs was larger than that of LP MVNOs, so the ZR MVNOs monopolized the mobile network market even when p_1 was greater than 0.5, i.e., $p_1 = 0.587$.



(a) $\mu_\varepsilon = 0.1$ ($p_1 = 0.934$)

(b) $\mu_\varepsilon = 0.4$ ($p_1 = 0.587$)

Fig. 3. Time evolution of MVNO count of each strategy type

B. Abuses of Monopoly by MVNOs Using Single Strategy

The mobile network market will be monopolized by either LP MVNOs or ZR MVNOs depending on μ_ϵ. This means that the flexibility of users on selecting MVNOs decreases, and the monthly fee increases for some users. In other words, when LP MVNOs monopolizing the market, the monthly fee of users with $\psi_1 > \psi_2$ increases by $\psi_1 - \psi_2$ because they cannot select ZR MVNOs. On the other hand, when ZR MVNOs monopolizing the market, the monthly fee of users with $\psi_1 < \psi_2$ increases by $\psi_2 - \psi_1$ because they cannot select

LP MVNOs. The amount of increase of monthly fee depends on v and ϵ when MVNOs using a single strategy monopolizes the market.

To clarify the abuses of market-monopoly by MVNOs of single type against users, Fig. 4 shows the complementary cumulative distribution of the increase of monthly fee of users for five sets of μ_ϵ and ρ. As shown in Fig. 1, the prices of ZR MVNOs was higher than those of LP MVNOs for same v, so the increase of monthly fee when ZR MVNOs monopolized the market tended to be larger than that when LP MVNOs monopolized the market. Especially, when μ_ϵ was small, i.e., $\mu_\epsilon = 0.2$, the cost increase of users by market-monopoly of ZR MVNOs was large, and the monthly fee of about 60% users increased by 4 USD or more. In the other cases, the monthly fee of about 40 to 50% users increased by 3.5 USD or more. The influence of ρ on the cost increase of users was small.

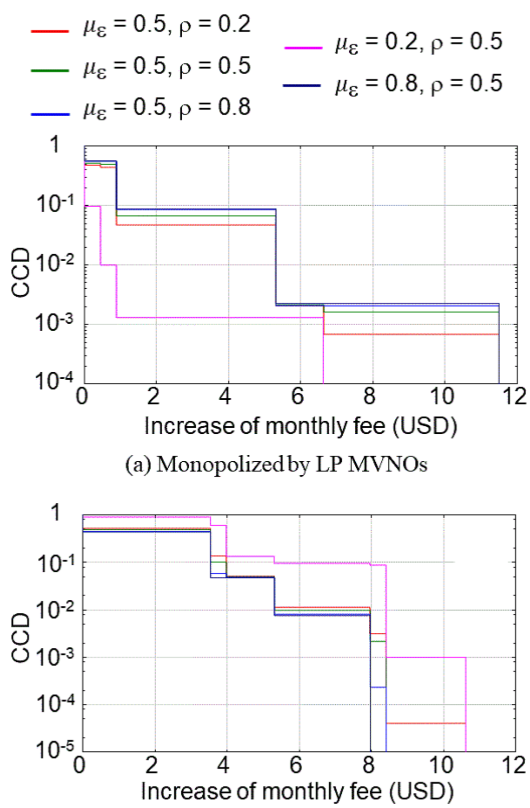


Fig. 4. Complementary cumulative distribution of increase of monthly charge of users when one type of MVNOs monopolize market

V. Conclusion

In this paper, we modeled the market consisting of many MVNOs which used either the low price (LP) or the zero ration (ZR) as the strategy by the evolutionary game, and we clarified that the mobile network market will be monopolized by MVNOs using either strategy. Moreover, we showed that the market will be monopolized by ZR MVNOs with high probability, and the monthly fee of many users will increase when assuming the price plan of typical MVNOs in Japan. Therefore, to protect the utility of users, for example, we suggest that governments should regulate ZR MVNOs to provide a price plan for users with small amount of ZR-target traffic to use the mobile network service with reasonable fee.

We assumed that users selected MVNOs contracted by considering just the monthly fee when modeling the dynamics of MVNO market using the evolutionary game. However, users will consider various factors, e.g., the brand image of MVNOs and the scale of MVNOs, in addition to the fee when selecting the MVNO. Therefore, we will extend the model to consider various factors including the brand image and the scale of MVNOs in the future. Moreover, we will also consider the cost of users to switch the MVNO contracted because some procedures and initial cost are required to change the MVNO contracted.

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