A Model for a Multi-level Disassembly System Under Random Disassembly Lead Times

Ilhem Slama, Oussama Ben-Ammar, Alexandre Dolgui

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Ilhem Slama[1], Oussama Ben-Ammar[2][0000−0002−1428−6199], and Alexandre Dolgui[1][0000−0003−0527−4716]

[1] IMT Atlantique, LS2N-CNRS, La Chantrerie, 4 rue Alfred Kastler - B.P. 20722, 44307 Nantes, France
{ilhem.slama,alexandre.dolgui}@imt-atlantique.fr

[2] Department of Manufacturing Sciences and Logistics, Mines Saint-Etienne, University Clermont Auvergne, CNRS, UMR 6158 LIMOS CMP, Gardanne, France
oussama.ben-ammar@emse.fr

Abstract. This paper deals with the problem of planned disassembly lead time calculation in a Reverse Material Requirement Planning (RMRP) environment under uncertainty. A multi-level disassembly system with one type of end-of-life product and several types of components at each level is considered for the first time in the disassembly planning problem under uncertainty. The paper presents a mathematical model with corresponding proofs. The objective of the proposed model is to minimise the total expected cost which is composed of holding and backlogging costs. Some advantages of the proposed model and perspectives of this research are discussed.

Keywords: Reverse supply chain · Disassemble-to-order · Stochastic lead times.

1 Introduction and related publications

With accumulated environmental pressure on industrial activities, the reverse flow has become indispensable. The remanufacturing process is a crucial step in the reverse logistics network, and it concerns all tasks associated with the collection, disassembly, refurbishing, repair, recycling, disposal, etc. of end-of-life (EoL) products [1]. In the scientific literature, the disassembly process has recently received a lot of attention due to its importance in the recovery of products. It allows the selective separation of parts in order to recover materials, isolate hazardous substances and separate reusable items [2].

The current paper proposes to model the disassembly planning problem, which is one of the main problems associated with product recovery. The motivations for initiating tactical disassembly planning are very varied and multiple. On a large scale, optimal planning of disassembly operations is necessary to efficiently process a large volume of products to be upgraded. It also aims to organise a succession of operations over time in order to meet the demands for components on predefined delivery dates.

Making decisions in an uncertain environment is a difficult task for many industrial sectors. In fact, some parameters are often considered deterministic, when in reality they are inherently uncertain. There are many sources of uncertainty related to demand and the quantity and quality of parts that can disrupt the disassembly process. For example, a machine failure can interrupt the disassembly process and subsequently, component disassembly times become longer than expected. Consequently, these uncertainties disrupt the level of stock, and thus, create disruption and unnecessary storage. Several methods have been developed in the literature to take into account the effect of uncertainty in the disassembly planning problem. Without trying to do an exhaustive review of the literature, we discuss the previous works on disassembly planning problem under uncertainty. For a more exhaustive literature review, readers can refer to [3,4].

The models proposed in the literature can be classified as a single or multi-period planning problem, two or multi echelon bill of materials (BOM) (see Table 1). Also, they can be divided into four different categories depending on the type of uncertainty: (1) demand (see [5,6]), (2) disassembly yield (see for example the works of [7,8,9]), (3) yield and demand [10] and (4) disassembly lead time (DLT). The latter is defined as the time difference between placing a disassembly order and receiving the disassembled item at each level of the BOM.

Table 1: Summary of relevant literature under uncertainly

Authors	**Resolution**	**Uncertainty**				
		Yield	**Demand**	**DLT**	**#Periods**	**#Levels**
[5]	MILP, Lagrangian heuristics		√		m	2
[6]	MILP, Lagrangian heuristics		√		m	2
[7]	Lagrangian heuristics	√			1	2
[8]	Heuristics	√			1	2
[9]	Fuzzy goal programming	√			1	2
[10]	MILP, Outer-approximation	√	√		m	2
[11]	Analytical model, Heuristic			√	1	2
[12]	S-LP			√	m	2
[13]	Analytical model, Newsboy formulae			√	1	2
[14]	Analytical model, Newsboy formulae			√	1	2
[15]	2S-MILP, SAA			√	m	2
[16]	2S-MILP, GA			√	m	2
Current Paper	Analytical model			√	**1**	**m**

m: multi-

For different reasons (machine breakdowns, absenteeism, quality problems, etc.), the disassembly lead times are often stochastic. To minimise the effects of these random factors, companies can implement safety lead times (or safety stock), but theses stocks are very expensive. On the other hand, if there are not enough stocks, we can observe stockout and the corresponding backlog cost. So, the main goal is to minimise the total cost which is composed of holding and backlog costs.

As far as can be determined from the literature, for the uncertainty of timing, the number of publications that study disassembly systems is very modest compared to those that deal with assembly systems [17,18]. In the paper of [12], only the uncertainty of DLT of the EoL product is studied. The case of multi-period, single product type and two-level disassembly system is investigated. The disassemble capacity is supposed infinite. The Scenario-based stochastic Linear Programming model (S-LP) is developed. The proposed approach is used to determine the optimal EoL products to be disassembled in order to minimise the Average Total Cost (ATC) over the planning horizon. In the work of [11], the authors developed a mathematical model to found the optimal plan of disassembly/assembly system under breakdowns machine and stochastic lead times. A heuristic is developed to determine the optimal ordered date of the EoL product as well as the optimum release dates of new external components. Later, [14] proposed a generalisation of the discrete Newsboy formulae to find the optimal release date when the time of disassembling the EoL product is random variable. The case of one-period, single product type and two-level disassembly system is treated. In the same year, [13] extend the work of [14] by considering the uncertainty of disassembly lead times of each component. In the same context, [15] extend the work proposed by [12] by considering the time limit of disassembly capacity in each period of the planning horizon. The problem is formulated as a two-stage stochastic Mixed Integer Linear Programming (2S-MILP) model. The Sample Average Approximation (SAA) approach is developed in order to minimise the ATC. Recently, [16] addressed a multi-period disassembly lot-sizing problem. The case of single product type and two-level disassembly system is studied. The problem is formulated as 2S-MILP model through all possible scenarios. To solve large scale problems, the authors proposed a basic genetic algorithm (GA).

As presented above, all the previous works under uncertainty of DLT are confined to the two-level disassembly system. Solving the studied problem with a multi-level BOM is more complex because of dependencies between the different sub-assemblies and components at each level. For this reason, this paper extends the work of [14] and [13] by considering a multi-level disassembly system to study a one-period planning for the disassemble-to-order (DTO) problem. A mathematical model is developed to calculate the total expected costs composed of the component backlog and holding costs.

The rest of this paper is organised as follows. Section 2 describes the problem. Section 3 presents the stochastic model of the multi-level disassembly to-order problem. Section 4 presents the conclusions with avenues for future studies.

2 Problem description

We consider the problem with the multi-level disassembly system (with several types of components at each level) as shown in Figure 1. The non-leaf item i (can be root or sub-assembly $(i \in \mathcal{A}^1)$) represents the item to disassemble. It has more than one child and a child element denotes non-root item (can be part or sub-assembly) that has only one parent. Customers have a fixed demand for all disassembled leaf item $(i \in \mathcal{A}^0)$. Theses demands must be delivered with certain delivery day. Once the disassembly process is started, the items recovery pro-

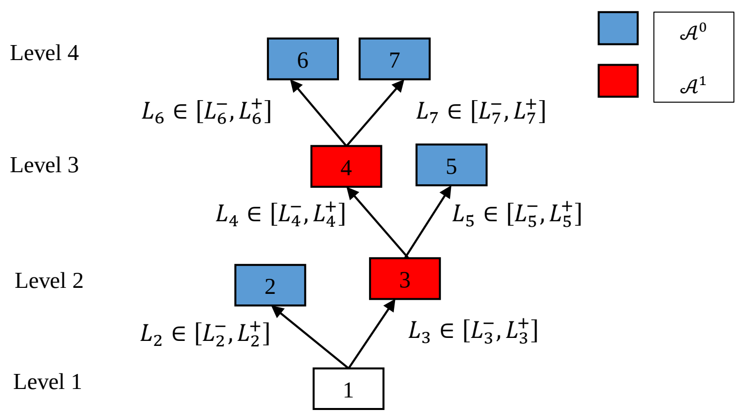

Fig. 1: A multi-level disassembly system.

cess provides no information about the state of components until the end of the disassembly process. Then, each item can be received after a stochastic disassembly lead time where the items undergo several renovation processes (such as repair and cleaning). The DLT of each item L_i is a random discrete variable with a known probability distribution and bounded over known intervals $(L_i \in [L_i^-, L_i^+])$. Here, we supposed that the probability is not identically distributed, that's to say, the DLT for each item don't following the same probability distribution.

The assumptions of the studied problem can be summarised as follows:

1. The one-period planning for the DTO environment and a single demand for each type of component is considered;
2. At each level, all parent items must proceed to disassembly once they are ready;
3. If the date of obtaining an item from its parent is less than its scheduled disassembly start date, this item will be kept until the scheduled release time, which incurs a holding cost;
4. If, at a certain level in the BOM, a component is not received as expected, a backlog cost is incurred.

3 Problem formulation

The aim of this research is to develop a mathematical model for multi-level disassembly systems under a fixed part's demand and uncertainty of parts disassembly lead times.

Let $i = 1$ be the index of the EoL product, n the index of the last disassembled item and the following three sets: (i) $\mathcal{A}$ the set of all elements $i = 1, \ldots, n$, (ii) $\mathcal{A}^0$ the set of all obtained items that we can not disassemble and to be delivered to clients and (iii) $\mathcal{A}^1$ the set of all obtained items that we can disassemble. Therefore $\mathcal{A} = \{1\} \cup \mathcal{A}^0 \cup \mathcal{A}^1$ Here, we have only demands for items i from the last level. i.e. $i \in \mathcal{A}^0$. The notation used in the paper are presented in Table 2.

Table 2: Model notation and definition

Parameters	
i	Index of items, $i = 1, ..., n$
$p(i)$	Index of the parent of item i, $\forall i \in \mathcal{A}^0 \cup \mathcal{A}^1$
T_i	Delivery date for item i, $\forall i \in \mathcal{A}^0$
h_i	Unit inventory holding cost for item i, $\forall i \in \mathcal{A}^0 \cup \mathcal{A}^1$
b_i	Unit backlogging cost for item i, $\forall i \in \mathcal{A}^0 \cup \mathcal{A}^1$
L_i	Actual disassembly lead time of item i, $\forall i \in \mathcal{A}^0 \cup \mathcal{A}^1$
Variables	
X_i	**Decision variable**: Planned date of availability of item i, $\forall i \in \mathcal{A}^1$
Functions	
$\mathbb{E}[\![.]\!]$	Excepted value
$\mathbb{F}_i[\![.]\!]$	Distribution function of the random variable L_i, $\forall i \in \mathcal{A}^0 \cup \mathcal{A}^1$
T_i^+	$max(M_i; T_i)$, $\forall i \in \mathcal{A}^0$
T_i^-	$min(M_i; T_i)$, $\forall i \in \mathcal{A}^0$

The total cost is equal to the sum of backlogging and inventory holding costs for all components.

Proposition 1. *For the system described in the previous section, the total cost, noted by $TC(X, \mathcal{L})$, is as follows:*

$$\begin{aligned} TC(X, \mathcal{L}) = &\sum_{i \in \mathcal{A}^0} (b_i + h_i) \times max(T_i; M_i) + \sum_{i \in \mathcal{A}^1} (b_i + h_i) \times max(X_i; M_i) \\ &- \sum_{i \in \mathcal{A}^0} b_i T_i - \sum_{i \in \mathcal{A}^1} b_i X_i - \sum_{i \in \mathcal{A}^0 \cup \mathcal{A}^1} h_i M_i \end{aligned} \tag{1}$$

Proof. In this prof, several costs will be detailed. Let M_i be the actual availability date of item i; $M_i = X_1 + L_i$ $\forall i \in \mathcal{A}^1$ and $p(i) = 1$. We first calculate $C_{h1}(X, L)$ the inventory holding cost of components disassembled from EoL product ($\forall i \in$

$\mathcal{A}^1$, $p(i)=1$):

$$C_{h1}(X,L) = \sum_{\substack{i\in\mathcal{A}^1\\ p(i)=1}} h_i(X_i - min(X_i; M_i)) \tag{2}$$

Let M_i be the actual availability date of item i; $M_i = max(X_{p(i)}; M_{p(i)}) + L_i$ $\forall i \in \mathcal{A}^1$ and $p(i) > 1$. Then, we can calculate $C_h(X,L)$ the inventory holding cost of components disassembled from items:

$$C_h(X,L) = \sum_{\substack{i\in\mathcal{A}^1\\ p(i)>1}} h_i(X_i - min(X_i; M_i)) \tag{3}$$

Let M_i be the actual availability date of item i; $M_i = max(T_i; M_{p(i)}) + L_i$ $\forall i \in \mathcal{A}^0$. We calculate $C_{h0}(X,L)$ the inventory holding cost of items that we have to deliver to clients:

$$C_{h0}(X,L) = \sum_{i\in\mathcal{A}^0} h_i(T_i - min(T_i; M_i)) \tag{4}$$

From Expressions (2)-(4), we can deduce the total holding cost:

$$C_H(X,L) = \sum_{i\in\mathcal{A}^0} h_i(T_i - min(T_i; M_i)) + \sum_{i\in\mathcal{A}^1} h_i(X_i - min(X_i; M_i)) \tag{5}$$

By the same way, we can easily deduce the total backlogging cost:

$$C_B(X,L) = \sum_{i\in\mathcal{A}^0} b_i(max(T_i; M_i) - T_i) + \sum_{i\in\mathcal{A}^1} b_i(max(X_i; M_i) - X_i) \tag{6}$$

Finally, the total cost can be formulated from Expressions (5)-(6).

Proposition 2. *The mathematical expectation of the total cost is given by the following expression:*

$$\begin{aligned}\mathbb{E}[\![C(X,L)]\!] = &\sum_{i\in\mathcal{A}^0}(b_i+h_i)\times\sum_{s\geq T_i}(1-\mathcal{F}_i(s)) + \sum_{i\in\mathcal{A}^1}(b_i+h_i)\times\sum_{s\geq X_i}(1-\mathcal{F}_i(s))\\ &+\sum_{i\in\mathcal{A}^0} h_iT_i - \sum_{i\in\mathcal{A}_{\setminus 1}} h_i(\mathbb{E}[\![L_i]\!] + \sum_{s\geq X_{p(i)}}(1-\mathcal{F}_{p(i)}(s)))\\ &+\sum_{i\in\mathcal{A}^1} h_iX_i - \sum_{i\in\mathcal{A}_{\setminus 1}} h_iX_{p(i)}\end{aligned}$$

where:

$$\mathcal{F}_i(s) = \begin{cases} 1 & \text{if } i=1\\ \mathbb{F}_i(-X_{p(i)}+s) & \text{if } p(i)=1\\ \mathbb{F}_i(-X_{p(i)}+s)\displaystyle\sum_{\substack{v_i+w_i=s\\ v_i+w_i\in\mathbb{N}}}\mathbb{P}[\![L_i=v_i]\!]\times\mathcal{F}_{p(i)}(w_i) & \text{otherwise}\end{cases}$$

Proof. First, we formulate $\mathbb{E}[\![M_i]\!]$, $\mathbb{E}[\![max(X_i; M_i)]\!]$ and $\mathbb{E}[\![max(T_i; M_i)]\!]$. For all $i \in \mathcal{A}$ and $p(i) = 1$, M_i is the actual availability date of item i; $M_i = X_1 + L_i$. So $\mathbb{E}[\![M_i]\!] = \mathbb{E}[\![L_i]\!] + X_1$. For all $i \in \mathcal{A}^1$ and $p(p(i)) = 1$, M_i is the actual availability date of item i. So M_i is equal to $max(X_{p(i)}; M_{p(i)}) + L_i$ and:

$$\mathbb{E}[\![M_i]\!] = \mathbb{E}[\![L_i]\!] + \sum_{s \geq 0} (1 - \mathbb{P}[\![X_{p(i)} \leq s]\!]\mathbb{P}[\![M_{p(i)} \leq s)]\!]$$

Moreover, knowing that $Pr[\![X_{p(i)} \leq s]\!] = 0\ \forall s \in [0, X_{p(i)}[$ and $Pr[\![X_{p(i)} \leq s]\!] = 1\ \forall s \geq X_{p(i)}$, then:

$$\mathbb{E}[\![M_i]\!] = \mathbb{E}[\![L_i]\!] + X_{p(i)} + \sum_{s \geq X_{p(i)}} (1 - \mathbb{P}[\![M_{p(i)} \leq s)]\!]$$

Knowing that $M_{p(i)} = X_1 + L_{p(i)}$, so:

$$\mathbb{E}[\![M_i]\!] = \mathbb{E}[\![L_i]\!] + X_{p(i)} + \sum_{s \geq X_{p(i)}} (1 - \mathbb{F}_{p(i)}(-X_1 + s)) \tag{7}$$

For all $i \in \mathcal{A}$ and $p(p(p(i)))) = 1$, M_i is the actual availability date of item i. So M_i is equal to $max(X_{p(i)}; M_{p(i)}) + L_i$ and:

$$\mathbb{E}[\![M_i]\!] = \mathbb{E}[\![L_i]\!] + X_{p(i)}$$
$$+ \sum_{s \geq X_{p(i)}} (1 - \mathbb{F}_{p(i)}(-X_{p(p(i))} + s) \sum_{\substack{o_1+o_2=s \\ o_1+o_2 \in \mathbb{N}}} (\mathbb{P}[\![L_{p(i)} = o_1]\!]\mathbb{P}[\![M_{p(p(i))} \leq o_2)]\!])$$

By using the recursive function, we can easily formulate $\mathbb{E}[\![M_i]\!]$, $\mathbb{E}[\![max(X_i; M_i)]\!]$ and $\mathbb{E}[\![max(T_i; M_i)]\!]$ as follows:

$$\mathbb{E}[\![M_i]\!] = \mathbb{E}[\![L_i]\!] + X_{p(i)} + \sum_{s \geq X_{p(i)}} (1 - \mathcal{F}_{p(i)}(s)) \tag{8}$$

$$\mathbb{E}[\![max(X_i; M_i)]\!] = X_i + \sum_{s \geq X_i} (1 - \mathcal{F}_i(s)) \tag{9}$$

$$\mathbb{E}[\![max(T_i; M_i)]\!] = T_i + \sum_{s \geq T_i} (1 - \mathcal{F}_i(s)) \tag{10}$$

Then, the expression of the expected total cost can be deduced.

4 Conclusion and perspectives

In this preliminary work, we model a multi-level disassembly problem in a DTO environment. We consider a one type of EoL product and several types of components at each level. The disassembly lead times at each level are discrete independent random variables, and the items demand at the finished level is fixed. The disassembly of the components is carried out as soon as their parents are available. The demand of the leaf items should be satisfied on predefined delivery dates. Otherwise a backlogging cost is incurred. If a given component is available before the delivery date, we stored it until this period. The developed mathematical model calculates the expected value of the sum of backlogging and inventory costs.

The expected total cost is not linear. In the future, we will develop an optimisation approach to optimise the release dates for the non-leaf items. Research into solving this problem is in progress. The developed approach is based on genetic algorithm developed for assembly systems under uncertainty of lead times [19].

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