SPACE AND TIME CURVATURE IN INFORMATION PROPAGATION IN MASSIVELY DENSE AD HOC NETWORKS

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Abstract Gupta and Kumar have shown that effective wireless range decreases in inverse function of local traffic density. We show that a variable traffic density impacts the curvature of paths in a dense wireless ad hoc network the same way a variable optical density bends light paths. We set up the general laws that paths must satisfy in presence of traffic flow density. Introducing Time constraint in packet delivery, we generalize this curvature problem to a space-time problem with mobile networks.

1. Introduction

Mobile ad hoc networks involve nodes that are moving on a network domain and communicate via radio means. The domain of a network can be indifferently a battlefield, a urban quarter, a building floor, etc. Most papers in the litterature take as an assumption models where the distribution of traffic and nodes are uniform over their network domain. This basic model leads to fundamental results, as the illuminating result of Gupta and Kumar **?** which states that the maximum capacity per node in a flat domain is in $O(\sqrt{\frac{1}{N \log N}})$. In this paper we will depart from the uniform model and assume that the traffic density varies with node location. We will provide results on how shortest path are affected by traffic density gradients. In particular we will show that in asymptotic conditions the routing paths obey to similar laws as in non linear optic.

We also generalize this result to the case where nodes are mobile and can hold the packet for some time before retransmitting it. Relaxing time constraint in the packet delivery and introducing mobility pattern depending on node position, we generalize the equation we obtained for a stationary network and prove that mobility can actually increase the network capacity.

In their reference paper on the capacity of mobile ad hoc networks, Gupta and Kumar **?** showed that in presence of traffic density of λ bit per time unit per square unit area, the typical radius of correct reception decays in $O(\frac{1}{\sqrt{\lambda}})$. This result assumes an uniform density model and quantity λ is the density of traffic *including* the load generated by packet that are retransmitted on their way to their destination on multihop paths. To our view, this estimate is the most fundamental result. As a direct consequence the average number of hops needed to connect two arbitrary points in a bounded domain is therefore $O(\sqrt{\lambda})$ since the distance must be divided by the radio ranges. As pointed out by Gupta and Kumar, this property has a strong implication in the evaluation of the maximum capacity attainable by a random node when the node density increases. If C is the capacity generated by each node and N is the number of nodes in the network, Gupta and Kumar found that the maximum bandwidth attainable is $C_N = \frac{\alpha}{\sqrt{N \log N}}$, quantity α depending on propagation models. However the order of magnitude is easy to get: the density of traffic generated per unit square are is $O(CN)$. Let $r(C,N)$ be the typical radio range, thus the number of retransmissions needed to route a packet from its source to its destination is $O(\frac{1}{r(C,N)})$. The latter estimate, in turn, yields a traffic density (including retransmissions) of $O(\frac{CN}{r(C,N)})$. Therefore $r(C,N) = O(\sqrt{\frac{r(C,N)}{CN}})$, namely $r(C,N) = O((CN)^{-1})$. The average number of neighbor per node is $O(\pi r(C,N)^2 N)$; it should be larger than $\log N$ in order to guarantee connectivity, which leads to the estimate $C = O((N \log N)^{-1/2})$.

If instead we consider that the network spans on a domain in dimension D, then the radius of correct reception decays in the inverse of the Dth root of emitter density, which impacts the maximum capacity (replacing the exponent $-1/2$ by $-1/D$).

This paper addresses the case where the traffic pattern is not uniform but varies as a continuous function of node location. We investigate the case where the traffic and node densities are large enough to make the efficient radio ranges infinitely small (compared to traffic density gradient and domain size). In this perspective, shortest paths (in number of hops) from sources to destinations look like continuous lines. In the sequel we call lines generated by shortest path routes, *propagation lines*. In the uniform model propagation lines are expected to be straight lines. In this paper we limit our investigation to the propagation lines and we analyze how the latter are affected by the variation of traffic density. We summarize our findings into macroscopic differential equations involving propagation lines curvatures.

The paper is organized as follows. The second section investigates more thoroughly a model of mobile ad hoc network in order to provide more accurate estimates about Gupta and Kumar results. The model assumes a 2D domain under slotted time with a density of λ emitters per slot and square area unit. Under this simple model we will provide quantitative confirmation of Gupta and Kumar results. The third section introduces the concept of massively dense networks and how propagation lines are affected by variable traffic density. At a macroscopic level the variable traffic density acts like a variable optical index and curves the propagation lines from sources to destinations. The fourth section introduces the time component and node mobility in the problem and shows that larger packet delivery delay can reduce the number of hops.

2. Quantitative results on time slotted networks

Quantification of the problem

Kumar and Gupta estimates were originally derived from information theory considerations and are not related to any particular network implementation. If we assume a specific implementation, then there will be a quantity β such that the typical radius of correct reception of a packet is equal to $\frac{\beta}{\sqrt{\lambda}}$. By typical radius we mean the radius below which probability of correct reception of a packet is above a given threshold. The quantity β will depend on many parameters such as the probability threshold, the attenuation coefficient of wave propagation and the minimum signal-over-noise ratio required for correct reception. Notice that the typical disk of correct reception contains in average a finite number of transmitters per slot, since the area is proportional to $\frac{1}{\lambda}$.

If we consider a network dispatched in a domain of dimension D then the estimate of the radius will be $\frac{\beta}{\lambda^{1/D}}$. In the sequel we will look at 2D domains generalizing occasionnaly the results on other dimensions.

When the density λ increases in a fixed domain, then the minimum number of hops connecting two points A, B tends to be equivalent to $d(A,B)\frac{\sqrt{\lambda}}{\beta}$ where $d(A,B)$ denotes the euclidian distance between mobile node A and mobile node B. Meanwhile, the increase of the number of relays naturally increases the traffic density. If ν is the actual traffic density generation per unit area, *i.e.* the traffic locally generated on mobile nodes, not the traffic relayed by the mobile nodes, then the average density traffic will satisfy the identity: $\lambda = \nu\bar{d}\frac{\sqrt{\lambda}}{\beta}$ where $\bar{d}$ is the average euclidian distance between two end points in a connection.

This previous identity assumes that the pattern of path between points covers the domain in an uniform manner so that the traffic density, generated and relayed, is constant on the whole domain. In this case the path that connect two points with the minimum number of hops is very close to the straight line. But the question arises about the shape of the shortest path when the traffic density is not uniform. We will show that when the density increases while keeping proportional to a given continuous function, then the propagation paths tend to conform to continuous line, that we call propagation lines. Under these assumptions we will provide the general equations that the propagation lines must satisfy. We will show that variable traffic densities affect shortest path the same way as variable optical indices affect light path in a physical medium.

Propagation model

We consider the following model: time is slotted, all mobile nodes are synchronized, transmissions on begining of slots. We consider an area of arbitrary size $\mathcal{A}$ (we will ignore border effect). N transmitters are distributed according to a Poisson process. We call λ the density of transmitter per slot and per square area unit. We have $\lambda = fN/\mathcal{A}$ where f is the rate of packets transmission per slot and per node.

Let a node X at a random position (we ignore border effects). We assume that all nodes transmit at the same nominal power. The reception signal at distance r is $P(r) = r^{-\alpha}$ with $\alpha > 2$. Typically $\alpha = 2.5$. Let W the signal intensity received by node X at a random slot. Quantity W is a random variable: let $w(x)$ its density function. In **?** it is shown that the Laplace transform of $w(x)$, $\tilde{w}(\theta) = \int w(x)e^{-x\theta}dx$ satisfies the identity:

$$\tilde{w}(\theta) = \exp(2\pi\lambda \int_0^\infty (e^{-\theta r^{-\alpha}} - 1)rdr) . \qquad (1)$$

Using standard algebra we get

$$\tilde{w}(\theta) = \exp(-\lambda\pi\Gamma(1 - \frac{2}{\alpha})\theta^{2/\alpha}) \qquad (2)$$

If the node location domain was a line instead of an area (consider a sequence of mobiles nodes on a road) then we would have

$$\tilde{w}(\theta) = \exp(-\lambda\Gamma(1 - \frac{1}{\alpha})\theta^{1/\alpha}) \qquad (3)$$

If, instead the location domain was a volume (consider aircrafts network), then

$$\tilde{w}(\theta) = \exp(-\frac{4}{3}\lambda\pi\Gamma(1 - \frac{3}{\alpha})\theta^{3/\alpha}) \qquad (4)$$

In the following we restrict ourselves on a 2D domain.

Neighbor model

A node is considered neighbor of another node if the probability of receiving packets from each other is greater than a certain threshold p_0. For example $p_0 = 1/3$. Under this model, we can affect to p_0 the value which optimizes the distance traveled by a packet per transmission as in **?**. We assume that the slotted system contains an acknowledgment mechanism so that each succesful transmission does not triger any new retransmission for the same hop. In this case the distance travelled by the packet is equal to the distance from the transmitter to the receiver. When the transmission fails then the distance is zero and the node reschedule a new retransmission at a random time (we assume that λ involve the load due to retransmissions).

We assume that a packet can be decoded if its signal-over-noise is greater than a given threshold K. Typically $K = 10$. Therefore another node is neighbor if its distance r is such that $P(W < r^{-\alpha}/K) > p_0$, i.e. when $r < r(\lambda)$ where $r(\lambda)$ is the critical radius such that $\int_0^{r(\lambda)} w(x)dx = p_0$. By simple algebra it comes that $r(\lambda) = \lambda^{-1/2}r(1)$. This result confirms the result of Gupta and Kumar in this very specific network model. We find $\beta = r(1)$. The surface covered by the radius $r(\lambda)$ is the neighborhood area $\sigma(\lambda) = \frac{\sigma(1)}{\lambda}$ and $\sigma(1) = \pi r(1)^2$.

Computation of β, $r(1)$ and $\sigma(1)$

In order to simplify the presentation we set $C = \pi\Gamma(1 - \frac{2}{\alpha})$ and $\gamma = \frac{2}{\alpha}$. By application of reverse Laplace we have:

$$P(W < x) = \frac{1}{2i\pi}\int_{-i\infty}^{+i\infty} \frac{\tilde{w}(\theta)}{\theta} e^{\theta x} d\theta \tag{5}$$

Expanding $\tilde{w}(\theta) = \sum_n \frac{(-C)^n}{n!}\theta^{n\gamma}$, we get

$$P(W < x) = \frac{1}{2i\pi}\sum_n \frac{(-C)^n}{n!}\int_{-i\infty}^{+i\infty} \theta^{n\gamma-1}e^{\theta x}d\theta \tag{6}$$

By bending the integration path toward the negative axis we get

$$\begin{aligned}\frac{1}{2i\pi}\int_{-i\infty}^{+i\infty} \theta^{n\gamma-1}e^{\theta x}d\theta &= \frac{\sin(\pi n\gamma)}{\pi}\int_0^{\infty} \theta^{n\gamma-1}e^{-\theta x}d\theta \\ &= \frac{\sin(\pi n\gamma)}{\pi}\Gamma(n\gamma)x^{-n\gamma}\end{aligned}$$

Figure 1 shows the plot of $P(W < x)$ versus x for $\alpha = 2.5$ and $\lambda = 1$. Notice that $P(W < x)$ reaches $p_0 = 1/3$ close to $x = x_0 = 20$. Therefore $\beta = r(1) = (x_0K)^{-1/\alpha} \approx 0.12$. Therefore $\sigma(1) \approx 0.045$.

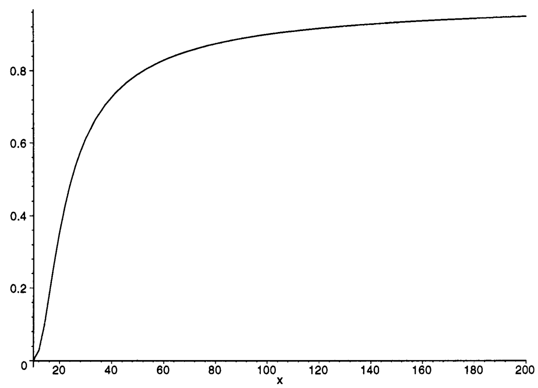

Figure 1. Quantity $P(W < x)$ versus x for $\alpha = 2.5$, no fading.

Modeling of fading

Signals propagating through random obstacles experience random fadings. An usual modeling of fading consists into introducing a random factor F to signal attenuation at distance r: $r^{-\alpha}$. For example $\log F$ is uniform on $[-v, v]$. In this case we have a new expression of $\tilde{w}(\theta)$:

$$\tilde{w}(\theta) = \exp(-\pi\lambda\Gamma(1 - \frac{2}{\alpha})\phi(-\frac{2}{\alpha})\theta^{2/\alpha}) \tag{7}$$

with $\phi(s) = E(F^{-s})$, the Dirichlet transform of the fading. When fading is uniform on $[-v, v]$ we have $\phi(s) = \frac{\sinh(sv)}{sv}$. For any given real number x we also have $P(W < xF)$ equaling

$$\sum_n \frac{(-CF(-\gamma))^n \sin(\pi n\gamma)}{\pi n!}\Gamma(n\gamma)\phi(n\gamma)x^{-n\gamma} \tag{8}$$

which helps the computation of $\sigma(1)$ with fading.

3. Massively dense networks

We now consider massively dense networks on a 2D domain. We denote by $\lambda(x, y)$ the traffic density at the point of coordinate (x, y) on the domain. We suppose that function $\lambda(x, y)$ is continuous in (x, y), or at least Lebesgue integrable. When $\lambda(x, y)$ are uniformly large, the results of Gupta Kumar together with the result of the previous section state that the radio ranges tend to be "microscopic" and routes can be considered as continuous lines between nodes. Packets travelling on a route $\mathcal{C}$ passing on the point of coordinate (x, y) will experience hops of length $\frac{\beta}{\sqrt{\lambda(x,y)}}$ passing in the vicinity of point (x, y).

Let $n(x,y) = \frac{\sqrt{\lambda(x,y)}}{\beta}$. The number of hops that a packet will experience on route $\mathcal{C}$ is something close to $\int_{\mathcal{C}} n(x(s), y(s))ds$ where s is a curvilign absciss on route $\mathcal{C}$.

In the sequel we are looking for route with the shortest hop number. Searching the path that minimize the hop number between two points A and B is therefore equivalent for looking for the path light between A and B in a medium with non-uniform optical index $\lambda(x,y)$. There is a known result about the optimal path that minimize a path integral $\int_{\mathcal{C}} nds$.

THEOREM 1 *The optimal path satisfies on each of its point* $\mathbf{z}(s) = (x(s), y(s))$ *such that s is acurvilign absciss (*$ds = \sqrt{(dx)^2 + (dy)^2} = |d\mathbf{z}|$*):*

$$\frac{d}{ds}(n(\mathbf{z}(s))\frac{d\mathbf{z}(s)}{ds}) = \nabla n(\mathbf{z}(s)) \tag{9}$$

where ∇ *is symbol of gradient vector.*

The proof is classical. If we consider a small perturbation $\mathcal{C}^*$ of optimal path $\mathcal{C}$ where $\mathbf{z}^*(s) = \mathbf{z}(s) + \delta\mathbf{z}(s)$ we should have $\int_{\mathcal{C}^*} nds^* - \int_{\mathcal{C}} nds = \delta[\int_{\mathcal{C}} nds] = 0$. We have $\delta[\int_{\mathcal{C}} nds] = \int_{\mathcal{C}} \delta[nds]$. Since $\delta[n] = \nabla n.\delta\mathbf{z}(s)$ and $\delta[ds] = \frac{d\mathbf{z}}{ds}.d\delta\mathbf{z}$ we get

$$\delta[\int_{\mathcal{C}} nds] = \int_{\mathcal{C}} \nabla n.\delta\mathbf{z}(s)ds + \int_{\mathcal{C}} n\frac{d\mathbf{z}}{ds}.\frac{d\delta\mathbf{z}}{ds}ds$$

Integrating by part the second right hand side integral of the above and assuming that both $\mathcal{C}$ and $\mathcal{C}^*$ share the same end points (*i.e.* $\delta\mathbf{z}(s) = 0$ at both ends), we get:

$$\delta[\int_{\mathcal{C}} nds] = \int_{\mathcal{C}} (\nabla n - \frac{d}{ds}(n\frac{d\mathbf{z}}{ds})).\delta\mathbf{z}(s)ds.$$

Since $\delta\mathbf{z}(s)$ can be arbitrary, and that in all case $\delta[\int_{\mathcal{C}} nds] = 0$, then $\nabla n - \frac{d}{ds}(n\frac{d\mathbf{z}}{ds}) = 0$ on the optimal path.

Therefore finding the optimal path is just an application of geometric optics. Notice that when $\nabla n = 0$ (uniform traffic density) propagation lines are straight lines (no curvature).

However we face a major problem in the fact that the distribution of path is actually impacting traffic density. This lead to an egg-and-chicken problem which may not that easy to solve. We call $\Phi(x,y)$ the flow density of information transiting in the vicinity of point (x,y). Quantity $\Phi(x,y)$ is expressed in bit per meter, since it expresses the flow of packet crossing a virtual unit of segment of length of 1 meter centered on point (x,y). This flow impact the traffic density by the fact that each packet must be relayed every $\beta/\sqrt{\lambda(x,y)}$ meter in the vicinity of point (x,y). Therefore locally:

$$\lambda(\mathbf{z}) = \Phi(\mathbf{z})\frac{\sqrt{\lambda(\mathbf{z})}}{\beta} \tag{10}$$

In other words $\lambda(\mathbf{z}) = (\frac{\Phi(\mathbf{z})}{\beta})^2$ and

$$n(\mathbf{z}) = \frac{\Phi(\mathbf{z})}{\beta^2} \tag{11}$$

When considering domain of dimension D we have $\lambda = \Phi\frac{\lambda^{1/D}}{\beta}$ and $n = (\frac{\Phi}{\beta^D})^{\frac{1}{D-1}}$. Notice that the equations are singular when $D = 1$.

As an example we can assume a planar domain massively and uniformly filled with mobile nodes and gateway nodes. We denote by μ_G the spatial density of gateways. We assume that the mobile nodes are much more dense than the gateways. We denote by ν the traffic density generated in any point. ν is expressed in bits per square meters per slot. The flow density Φ is constant in the domain and is equal to $\nu\bar{d}$. We suppose that mobile nodes sends and receives flows from their closest gateway. Therefore $\bar{d} = \int_0^\infty \exp(-\pi\mu_G r^2)dr = \frac{1}{2\sqrt{\mu_G}}$:

$$\Phi = \frac{\nu}{2\sqrt{\mu_G}} \tag{12}$$

But in this case $\nabla n = 0$ and propagation lines are straight lines. The document **?** provides non trivial examples where the propagation lines are curved and can be exactly calculed. This is the cases when traffic is generated toward a central gateway or when the traffic is generated toward gateways regularly spaced on a circle circonference.

Practical implementation of shortest path protocol

Implementing a routing protocol that follows the geodesic lines is not a difficult task. Indeed there is no need that nodes monitor their local traffic density n nor to advertize the gradient vector. In fact a shortest path algorithm, such as OLSR **?**, will suffice. Of course one will need to limit the neighborhood of the node to those whose hello success rate exceeds p_0. To this end we make use of Hysterisis Strategy in *advanced link sensing* option and set up HYST-THRESHOLD-HIGH parameter to value p_0 that provides the best success rate, hop distance compromize. Tuned that way OLSR will automatically provide the shortest path that fit the traffic density gradient curvature.

4. Introduction of time component

In the previous section we were assuming very strict timing constraints so that packets are forwarded like *hot potatoes* without any pause between retransmissions. Recently Grossglauser and Tse **?** showed that mobility increases the capacity of wireless ad hoc networks. This due to the fact when nodes are moving one just wait that nodes come closer instead of immediately starting relaying when nodes are far apart. In particular the increase in capacity can be dramatic (in $O(1)$ instead of $O(\lambda^{-1/2})$) if one consider ergodic mobility patterns. Unfortunately the delay for packet becomes unbounded when the density increases. The aim of this section is to quantify the gain in retransmissions number while we let the time constraint on delay delivery T vary.

Although basic sensors networks or other smart dust are not expected to be mobile, However one can imagine more sophisticated sensors produced by via nanotechnology that can be mobile by themselves. The sensors can also travel because they are embedded in a mobile device or because the background medium is mobile (think about sensor in a river stream) Interestingly intermediate nodes can choose to store packet while moving instead of immediately retransmitting it. As long they move to the right direction this may considerably reduce the average number of retransmissions between source and destination. Of course the consequence is that packet delay delivery will considerably increase. This may be a solution for non urgent background traffic to take advantage of mobility and therefore have much less impact on network global load. Therefore we will model this very property by introducing space-time considerations in the framework presented in this paper.

Throughout this section we will assume that a node has a packet (or a sequence of packets) to transmit to a destination node with a time delivery constraint of T. In words, each packet should arrive to its destination no later than after a delay T. It basically means that we add the time dimension to our 2D problem. A path now contains the time dimension and will connect a source space-time point $(\mathbf{z}_0, t_0)$ to a destination space-time point $(\mathbf{z}_1, t_1)$, given that $t_1 = t_0 + T$.When $T = 0$, and neglecting propagation delays and processing in relay nodes, we get to our previous analysis restricted to space components. Our aim is to show that with some mobility models, when T tends to infinitely, the number of retransmissions needed to connect point $(\mathbf{z}_0, t_0)$ to point $(\mathbf{z}_1, t_1)$ tends to be negligible compared to the number of retransmissions needed to connect point $(\mathbf{z}_0, t_0)$ to point $(\mathbf{z}_1, t_0)$, (*i.e.* same spatial point but zero delay).

In order to set up notations and convention for this very general problem, we will first start with an unrealistic mobility model.

A simplistic mobility model

In this first example, we make the assumption that a node which has a packet to transmit or to relay can also trave with its packet at speed v. We also make the unrealistic hypothesis that the node that carries the packet can travel in any direction it wishes and that it makes the decision according to the destination of the packet. Therefore at any time the node that carries the packet has to make the decision of either transmitting it to the next hop or to travel with it on the propagation path. When the node chooses to hold the packet, we say that the packet is in hold state. We consider the optimal path $\mathcal{C}$ when $T = 0$ which connects $\mathbf{z}_0$ to $\mathbf{z}_1$, we assume that the space time path will be the path $\mathcal{C}$ plus a time component. In order to avoid too many notations we will still denote the space-time path by $\mathcal{C}$.

With these very hypotheses, the number of actual hops the packet will experience during its propagation on path $\mathcal{C}$ is equal to $\int_{\mathcal{C}} n|d\mathbf{z} - \mathbf{v}dt|$ where $\mathbf{v}$ is the vector speed at point $\mathbf{z}$. Since we assume that the speed can be made colinear with $d\mathbf{z}$ then the number of hops is equal to $\int_{\mathcal{C}} n|1 - v\frac{dt}{|d\mathbf{z}|}| \times |d\mathbf{z}|$. In the following we call $h = \frac{dt}{|d\mathbf{z}|}$ the average (local) packet holding time per distance unit, or we will denote by $\gamma = \frac{h}{v}$ the average fraction of distance traveled in hold state by a packet per distance unit. We therefore has $T = \int_{\mathcal{C}} h|d\mathbf{z}| = \int_{\mathcal{C}} \frac{\gamma}{v}|d\mathbf{z}|$.

Under this hypothesis it is clear that when $\gamma \to 1$, then the number of hops tends to be negligible compared to $\int_{\mathcal{C}} n|d\mathbf{z}|$ (zero delay case). In this case we don't need to have T unbounded, since in the most extreme case the node that generated the packet can simply drive his way to the destination and deliver the packet when in the neighborhood of the destination without transmitting it to intermediate nodes.

However this model is far from realistic. There is no reason that the mobility pattern of a node could depend that crucially on the destinations location of the packet it is holding. In fact a relay node can hold several packet to different destination at the same time and the node will have no way to split itself in several parts in order to move toward these different destinations at the same time. In the following subsection we consider a more realistic mobility model where the nodes are subject to random walks that are independent to data traffic conditions.

The random walk model

In this model we assume that at any time node travels at a random speed toward a random direction and keep its speed and heading during a time duration τ. After time τ it randomly change speed and heading. This like a particle in motion in a gas. Quantity τ refers to the free space motion delay during which the particle moves in straight line. At the end of period τ the particle experiences a collision that changes its motion vector in a random way. Notice that τ can be made random as well (we may assume that it is exponentially distributed). We assume that the expectation of speed vector $E[\mathbf{v}] = 0$. We also assume that the speed vectors have isotropic direction and $c_2\mathbf{I}$ is the covariance matrix, $\mathbf{I}$ being the identity matrix. We could accept some un-isotropic aspects so that covariance could depart from colinearity with identity matrix, but we will not do it for the sake of presentation.

Quantity $\sqrt{c_2}\tau$ is the standard deviation of node location after one free space travel. We assume that this quantity is of the same order as of hop distance r

(remaining that $r = \frac{1}{n}$. In other words the free space travel distance is of the same order as the hop distance. It is also instrumenetal in our proof that the speed is distributed on values that are not bounded by a finite number.

When a packet arrive in a mobile node, the router has to select whether it will transmit the packet to the next hop or keep it in hold state. We define a decision process which is based on the localisation of the next hop and the speed vector of the host node. If the node decides set the packet in hold state it will keep it as long its speed does not change. Therefore a hold state will last at least one τ period. The decision making automaton use a parameter x that is a positive real number and which depends on the delivery delay constraint T of the packet. Let θ be the angle made by the direction to the next hop. The node decide to immediately transmit its packet to the next hop iff the two following conditions hold:

1 its speed is larger than parameter x;

2 the speed direction angle is contained in interval $[-\theta - \frac{\pi}{1+x}, -\theta + \frac{\pi}{1+x}]$;

otherwise the packet stays in hold state. If the node keeps the packet in hold state it will keep it to its next motion vector change. At this moment it will proceed to a new packet state decision according to its new motion vector and to the localisation of the curent potential next hop. If the node has been decided to be transmitted immediately then the reciever will also proceed to a state selection. A packet may be transmitted over several hops before returning back in hold state. Basically when $x = 0$, then the packet is always immediately retransmitted to its next hops as with $T = 0$; and when $x \to \infty$, then the packet is less likely retransmitted and stay longer in hold state. The probability that a node chooses to immediately transmit the packet is $p(x) = \frac{1}{1+x} P(|\mathbf{v}| > x)$ and let $v(x) = E[|\mathbf{v}| \ , \ |\mathbf{v}| > x] \frac{1}{\pi p(x)} \sin(\frac{\pi}{1+x})$, it is clear that $v(x) > \frac{x}{\pi} \sin(\frac{\pi}{1+x})$. since the computation is done on speed greater than x with direction uniformly distributed in $[-\theta - \frac{\pi}{1+x}, -\theta + \frac{\pi}{1+x}]$. The average motion vector when the packet is in hold state is colinear with the direction to the next hop and has modulus equal to $p(x)v(x)$. According to hypothesis we have $\lim_{x\to\infty} p(x) = 0$ and $\lim_{x\to\infty} v(x) = \infty$.

The average distance the packet will travel before a new decision has to be taken (either in hold state or in immediate retransmission) is $p(x)r + (1 - p(x))p(x)v(x)\tau$ with variance $v_2(x)$. Notice that $\lim_{x\to\infty} v_2(x) = c_2\tau^2$. It comes that the average fraction of hold state travel per unit distance is

$$\gamma = \frac{(1 - p(x))v(x)\tau}{r + (1 - p(x))v(x)\tau} \tag{13}$$

We have clearly $\lim_{x\to\infty}\gamma = 1$ since $v(x)$ tends to infinity, in this case we also have

$$T = \int_{\mathcal{C}} \frac{(1-p(x))v(x)\tau^2}{(r+(1-p(x))v(x)\tau)^2 p(x)}|d\mathbf{z}| \tag{14}$$

which also tends to infinity since the product $p(x)v(x)$ tends to zero. However we cannot be sure that the packet actually reaches its destination. We know that the packet is *on average* on the path $\mathcal{C}$. In order to check how far from path it is actually we have to look at the variance of packet localization. After each decision step, the packet travels in average a distance $p(x)r+(1-p(x))p(x)v(x)\tau$ with a variance $v_2(x)$. Therefore in order to travel a distance of one unit the packet will have go through an average number of decision steps equal to $\frac{1}{p(x)r+(1-p(x))p(x)v(x)\tau}$. Therefore the variance of its position is close to $\frac{v_2(x)}{p(x)r+(1-p(x))p(x)v(x)\tau}$. In order to be safe we have to prove that this variance is small, so that the packet does not evade too far from the path and that when time limit will be critical the number of hops it will have to travel in emergency to reach its destination won't be too long. Since $v_2(x) \sim c_2\tau^2$ which is of order of r^2 the variance is at most equal to $\frac{c_2\tau^2}{p(x)r}$ which is order $r/p(x)$ which is small. In other words the number of hops is (using identity $r = \frac{1}{n}$):

$$\int_{\mathcal{C}} \frac{n}{1+n(1-p(x))v(x)\tau n}|d\mathbf{z}| + O(|d\mathbf{z}|\sqrt{\frac{n}{p(x)}}) \tag{15}$$

Similarily the delivery delay $T = \int_{\mathcal{C}} \frac{(1-p(x))v(x)\tau^2 n^2}{(1+(1-p(x))v(x)\tau n^2)^2 p(x)}|d\mathbf{z}|$. When the parameter $v(x)\tau$ is large compared to $1/n$ then the number of hops is equivalent to $\int_{\mathcal{C}} \frac{|d\mathbf{z}|}{v(x)\tau}$ and $T \sim \int_{\mathcal{C}} \frac{|d\mathbf{z}|}{p(x)v(x)}$. Notice that the optimal path may vary when T changes, for example if mobility model is uniform on the network domain then when T is large optimal path will be straight lines. In other word the curvature of optimal paths may also depend on the time component. Of course all the quantities x, $v(x)$ and $p(x)$ may also vary on the spatial domain leading to further optimization.

5. Conclusion and perspectives

It seems that propagation lines don't change when the route optimization criterium changes. For example if hop number is changed in packet total delay time, the route should basically remains the same. The reason for this conjecture is that the condition of traffic at any given point in the network location is basically the same *modulo* an homothetic factor $\lambda(\mathbf{z})$. The only aspect which changes is the distance travelled by the packet per hop, but the delay per hop will be the same in distribution. We have a similar point about bandwidth allocation criterium. The transmission at any point will take the same amount of bandwidth, only the per hop distance travelled by the packet will differ. Under this perspective simple shortest path algorithms such as OLSR **?** should be asymptotically close to optimal.

References

P. Jacquet, "Elément de théorie analytique de l'information, modélisation et évaluation de performances," INRIA Research Report RR-3505, http://www.inria.fr/rrrt/rr-3505.html, 1998.

P. Gupta and P.R. Kumar. Capacity of wireless networks. Technical report, University of Illinois, Urbana-Champaign, 1999. http://citeseer.nj.nec.com/gupta99capacity.html

F. Baccelli, B. Blaszczyszyn, P. Mühlethaler, "A spatial reuse Aloha MAC protocol for Multihop wireless mobile networks," INRIA Research Report RR-4955, 2003. http://www.inria.fr/rrrt/rr-4955.html

M. Grossglauser, D. Tse, "Mobility increases the capacity of ad hoc wireless networks," INFOCOM 2001, citeseer.nj.nec.com/grossglauser01mobility.html.

T. Clausen, P. Jacquet, "The Optimized Link State Routing protocol (OLSR)," IETF RFC 3626, 2003.

P. Jacquet, "Geometry of information propagation in massively dense ad hoc networks," INRIA Research Report RR-4992, 2003.