Pilot Assisted Channel Estimation in MIMO-STBC Systems Over Time-Varying Fading Channels

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Abstract—In this paper, challenges regarding the provision of channel state information (CSI) in multiple-input multiple-output (MIMO) systems based on space time block codes (STBC) over slow time-varying Rayleigh fading channels are addressed. We develop a novel MIMO channel estimation algorithm that adopts a pilot symbol assisted modulation (PSAM) which has been proven to be effective for fading channels. In this approach, pilot symbols are periodically inserted into the data stream that is sent through the orthogonal STBC encoder. At the receiver, we propose a straightforward MIMO channel estimation method before being used by STBC decoder. Simulation results indicate that the proposed pilot-assisted MIMO concept provides accurate channel estimates. The impact of Doppler frequency on performance scheme is also investigated by simulation.

Keywords— pilot symbol assisted modulation; MIMO system; STBC codes.

I. INTRODUCTION

Over the past several years, there has been a great deal of research to improve performance of multiple input multiple output wireless communications in fading environments by using space-time block coding [1]-[2]. These linear codes exploiting both the space and time diversities have been widely studied for combating the fading channels due to its simple implementation. The pioneering work by Alamouti [1] operates with two transmit antennas and one and two receive antennas. Later it has been generalized by Tarokh for an arbitrary number of antennas [2]. This coding scheme is able to achieve the full diversity promised by both transmitting and receiving antennas. Moreover, the STBC coding improves the reliability of the transmission. For decoding purpose, the MIMO-STBC systems require the channel state information which is basically obtained through channel estimation techniques at the receiver side. One of the most popular approaches to the MIMO channel estimation is to employ training symbols that are inserted into the transmitted information data and investigated at the receiver to render accurate CSI. The design of MIMO channel estimator is closely related to the channel characteristics. For quasi static MIMO fading channels, the obvious choice for channel estimation is through training sequences known by the receiver and inserted in the preamble at the beginning of each transmitted data burst at the expense of spectral efficiency. Several MIMO estimation methods have been introduced in the literature such as the least squares (LS) method, the minimum mean square error (MMSE) and the recursive MMSE estimator for quasi static fading channel [3]. However, mobile communication systems are characterized by channel responses with time-varying magnitude and phase. The performance of these channel estimators based on preamble training symbols is significantly degraded over time-varying channels. In order to efficiently combat variations on a fading channel, the channel may be estimated using the well-known pilot symbol assisted modulation (PSAM) scheme [4]. PSAM, originally established in [4] for single-input single-output (SISO) systems, has attracted intense interest by its ability of estimating the time varying fading channel [5]-[6]. The basic idea of PSAM is to periodically insert a known pilot symbol into data blocks, instead of sending the training sequences at the beginning of transmission. The received signals at pilot positions are used as a reference to estimate the amplitude and phase of the fading channel.

There have been a large number of previous works on PSAM. It is widely studied over selective and nonselective fading channels [7]. The optimum number of pilot symbols was derived in [8] to yield minimum bit error rate of a single antenna wireless system for slow fading channel. PSAM has been used in [9] to compensate the fading SISO channel and carrier frequency offset. In [10]-[11], the achievable rate PSAM of single and multiple antenna systems was analyzed. Recently, channel estimation based on PSAM for Alamouti coded transmission has been studied in [12]-[13]. Moreover, the authors in [14] investigate the relationship between the feedback transmission rate of pilot-assisted MIMO system and the capacity gain.

According to the literature, the pilot symbols based on PSAM estimator has recently emerged as a promising MIMO estimator for time-varying wireless communication systems especially due to its implementation ease. It also offers acceptable performance with reasonable computing.
complexity [12]. Therefore, the use of PSAM approach to perform channel estimation is suggested here for practical setting. In this work, a performance analysis of the novel pilot symbol assisted modulation system operating on MIMO channels and orthogonal STBC codes is explored. Pilot symbols on PSAM estimation are inserted in a periodic manner into the data symbols at the emitter side. Our investigation will focus on two types of STBC codes: the Alamouti code and the STBC operating with four transmit antenna and half rate. The channel characteristics are assumed to be slow fading and constant over the STBC codeword period. The received pilot signals provide information about the MIMO channel as estimated by the novel channel estimation before being interpolated. Simulation results prove the efficiency of the proposed estimation method which shows good bit error rate (BER) performance compared to the coherent system without any excessive complexity growth.

The rest of this paper is organized as follows. The pilot-assisted MIMO system model is given in the next section in which pilot insertion into the data blocks is also discussed. Section 3 presents the proposed channel estimation algorithm for slow time-varying fading channels, and the pilot spacing optimization and the interpolation method are presented in this section. Simulations are carried out to evaluate the performance of the proposed channel estimation method based on PSAM technique and the results are presented in Section 4. Finally, Section 5 is devoted to the conclusions drawn from this work.

II. SYSTEM MODEL

Consider a pilot symbol assisted MIMO system based on STBC orthogonal codes equipped by $N_T$ transmit antennas and one receive antenna as shown in Fig. 1. With the aim of enabling MIMO channel estimation, pilot symbols are periodically inserted into the data sequence at the transmitter so that the MIMO channel can be extracted and interpolated at the receiver. More precisely, modulated symbols are transmitted in frames of length $F$. We assume that, in each frame, the $N_T$ first symbols are pilot symbols and the following $(F-N_T)$ symbols are data symbols. Note that single pilot symbol is used in conventional PSAM, into the stream before transmission.

![Image](image-url)

Fig1: Block diagram of pilot-assisted MIMO system.

Linear M-ary phase-shift keying (M-PSK) modulation is considered in this paper. At the transmitter side, the information bits are mapped using gray mapping. The signal amplitude is divided by $\sqrt{N_T}$ such that the total transmitted power of the baseband signals in the $N_T$ transmitting antennas system is unitary. The known pilot symbols come from the same signaling set. Typically, each pilot symbol is selected from the constellation point with real value, i.e., $s_p = 1/\sqrt{N_T}$ for 8-PSK modulation and $N_T$ transmit antennas. Let $E_p$ and $E_s$ denote the required pilot and symbol energy, respectively. In the proposed estimation approach, pilot and data symbols have the same transmit power. Each transmit vector $S$ composed by $N_T$ modulated symbols is mapped according to the code matrix $C$ corresponding to the orthogonal STBC. These frames are transmitted over a slow flat fading channel with additive white Gaussian noise (AWGN) denoted by $N$. At the receive antenna, the symbol corresponding to $T$ STBC codeword period, $Y_{1 \times T}$, can be written as:

$$ Y = HC + N $$

The MIMO channel vector $H_{1 \times N_T}$ contains the elements $h_{k}, k = 1, \ldots, N_T$ corresponding to the channel fading coefficients from transmit antenna $j$ to the receive antenna. These complex path gains are generated according to slow fading channel. Among different models, the well-known Jakes model was shown to be sufficient for generating slow channel variations and will be adopted herein. The real and imaginary parts of $\{h_k\}$ are independent with autocorrelation function $R_e$ formulated by $R_e(\tau) = 0.5f_f(2\pi f_f T_d \tau)$, where $f_f(\tau)$ is the zeroth order Bessel function of the first kind and $\tau$ represents the correlation lag. $f_d$ denotes the Doppler frequency shift due to the terminal mobility, and $T_d$ is the transmitted symbol period, then the normalized fading rate is given by $\tau_d = f_d T_d$. The complex noise is represented by the vector $N_{1 \times T} \sim \mathcal{CN}(0, \sigma_n^2)$, where $\sigma_n^2$ denoted the noise variance.

Since each $N_T$ modulated symbols $S = (s_1, \ldots, s_{N_T})$ are mapped according to the orthogonal linear space-time block coding, the transmission matrix $C$ aforementioned also verifies the orthogonality property formulated by $CC^{H} = tr(C^{H}C)I_{N_T}$, where $tr(\cdot)$ is the trace function and $(\cdot)^H$ is the Hermitian operator. Without loss of generality, we consider in this paper only the Alamouti code and the orthogonal STBC code operating with four transmit antennas. The corresponding orthogonal matrix code $C_2$ and $C_4$ respectively to these STBC codes are given as follows:

$$ C_2 = \begin{bmatrix} s_1 & -s_1^* \\ s_2 & s_2^* \end{bmatrix} $$

$$ C_4 = \begin{bmatrix} s_1 & -s_2 & -s_3 & s_4 & s_1^* & -s_2^* & -s_3^* & -s_4^* \\ s_2 & s_1 & -s_3 & -s_4 & s_2^* & s_1^* & -s_3^* & -s_4^* \\ s_3 & -s_4 & s_1 & s_2 & s_3^* & -s_4^* & s_1^* & s_2^* \\ s_4 & s_3 & -s_2 & s_1 & -s_4 & s_3^* & -s_2^* & s_1^* \end{bmatrix} $$
The Alamouti coded transmission scheme achieves full diversity with full rate. The encoder outputs are transmitted throughout consecutive transmission periods using the two transmit antennas. During the first period, $s_1$ and $s_2$ are transmitted simultaneously from the first antenna and the second antenna, respectively. In the second transmission period, the encoder transmits $-s_2^*$ and $s_1^*$ respectively from the first and the second transmit antenna, where $(\cdot)^*$ denotes the complex operator. For the pilot-assisted MIMO system operating with four transmit antennas, the modulated symbols are mapped by the transmission matrix $C_4$. In this case, eight columns are transmitted in successive time intervals with four elements sent through four transmit antennas; respectively. Thus, this STBC coding achieves full diversity with a rate equal to 0.5.

It should be noted that because of power consumed by the $N_F$ pilot symbols periodically inserted in the blocks of transmitted symbols, the proposed scheme suffers from a power loss equal to:

$$D = 10 \log \left( \frac{F}{F-N_F} \right) \text{dB} \quad (4)$$

Therefore, the transmitted signal-to-noise ratio (SNR) per symbol is greater than the effective SNR per symbol; it is given by (5). It implies that the proposed pilot-assisted scheme provides a channel estimation method at cost of a low degradation compared to the coherent counterpart due to the insertion of few pilot symbols.

$$\text{SNR}_{\text{eff}} = \frac{F-N_F}{F} \cdot \text{SNR} \quad (5)$$

III. CHANNEL ESTIMATION BASED ON PSAM

Before being transmitted, pilot symbols $s_p$ are inserted into the data signal block $s_d$ at optimized period $F$. At the receiver side, the received composite signals are first demultiplexed. The received pilot symbols $Y_p$ are separated from the data received symbols $Y_d$. The signals $Y_p$ are used in order to estimate and to interpolate the slow time-varying channel MIMO at pilot positions. Finally, the linear maximum-likelihood (ML) detector is applied for decoding the received signals $Y_d$.

A. Optimization of pilot spacing

The pilot symbols are equally spaced with a pilot spacing $F$. The pilot symbol transmission and detection can be viewed as sampling of a band-limited process. Therefore, the pilot spacing in a fading channel with Jakes’ spectrum must satisfy the Nyquist criterion. We note $T_s$ the sampling time. According the sampling theorem, the pilot symbols should be transmitted at a minimum rate of $2f_D$. Consequently, the pilot spacing $F$ should satisfy the following expression:

$$F \leq \frac{1}{2f_D T_s} \quad (6)$$

Moreover, it should be carefully chosen to balance between the channel estimation error and the pilot sequence length. The optimum pilot spacing is given as [15]:

$$F_{opt} = 2 \left[ 1 + \frac{1}{\sqrt{1 + \frac{1}{4f_D}}} \right] \quad (7)$$

As clearly seen in (7), $F_{opt}$ does not depend on SNR. The unique dependence of the optimal pilot spacing is on the normalized Doppler spread $f_D$.

B. Pilot extraction

In the following discussions, we will focus on the pilot extraction method in two cases: the Alamouti code and the orthogonal STBC for four transmit antenna as given in (3).

1) Alamouti case: Consider a MIMO system equipped by two transmitting antennas and one receiving antenna over slow flat fading. The corresponding matrix code $C_2$ is given by (2). The transmitted symbols are formatted into frames of length $F$ in which the pilot symbols are inserted at times $t = (iF)$ ($i$ is an integer). Assuming all pilot symbols $s_p$ equal to $1\sqrt{2}$, the noisy and faded pilot signals at these times can be written as:

$$\begin{align*}
y_1(iF) &= (h_1(iF) + h_2(iF))s_p + n_1(iF) \\
y_2(iF) &= (-h_1(iF) + h_2(iF))s_p + n_2(iF)
\end{align*} \quad (8)$$

Where $y_1(iF)$ is the received pilot signal at time $t$, and $y_2(iF) = y_3(iF) + 1$. We propose an estimation method of the coefficients channel at pilot locations as follows:

$$\begin{align*}
h_1(iF) &= \frac{y_1(iF) - y_2(iF)}{2s_p} \\
h_2(iF) &= \frac{y_1(iF) + y_2(iF)}{2s_p}
\end{align*} \quad (9)$$

Substituting (1) into (9), the expression of the estimated complex path gains can be expanded as:

$$\begin{align*}
h_1(iF) &= h_1(iF) + \frac{(n_1(iF) - n_2(iF))}{\sqrt{2}} \\
h_2(iF) &= h_2(iF) + \frac{(n_1(iF) + n_2(iF))}{\sqrt{2}}
\end{align*} \quad (10)$$

2) STBC with four transmit antennas case: By considering four transmit antennas, the transmitted symbols are mapped into the $C_4$ matrix code. For pilot symbols $s_p = 0.5$, the first four received signals at time $t = iF$ are given by:
Where \( y_k(IF) = y_k(IF + k - 1); k = 2, 3, 4 \), and \( (\cdot)^T \) is the transpose operator. The noise vector \( \mathbf{N} \) is equal here to \( [n_1(IF) \ n_2(IF) \ n_3(IF) \ n_4(IF)]^T \).

We construct the four estimated complex path gains \( \{h_k(IP)\} \) from the pilot received signals as below:

\[
\begin{align*}
\hat{h}_1(IF) &= \frac{y_1(IF) - y_2(IF) - y_3(IF) - y_4(IF)}{4s_p} \\
\hat{h}_2(IF) &= \frac{y_1(IF) + y_2(IF) + y_3(IF) - y_4(IF)}{4s_p} \\
\hat{h}_3(IF) &= \frac{y_1(IF) - y_2(IF) + y_3(IF) + y_4(IF)}{4s_p} \\
\hat{h}_4(IF) &= \frac{y_1(IF) + y_2(IF) - y_3(IF) + y_4(IF)}{4s_p}
\end{align*}
\]  
(11)

Similar to Alamouti case, for a pilot-assisted MIMO system based on STBC code \( C_4 \), each estimated path gain is given by:

\[
\hat{h}_k(IF) = h_k(IF) + \Delta h_k, k = \{1, \ldots, 4\}
\]  
(13)

Where \( \Delta h_k \) represents the estimation error.

**C. FFT Interpolation**

We express the estimated channel response at pilot positions as \( \hat{H}(IF) \) as follows:

\[
\hat{H}(IF) = \begin{bmatrix}
\hat{h}_1(IF) \\
\vdots \\
\hat{h}_{NF}(IF)
\end{bmatrix}
\]  
(14)

Each element of the complex estimated channel at pilot symbol location is written by:

\[
\hat{h}(IF) = \hat{a}(IF)e^{j\hat{\theta}(IF)}
\]  
(15)

Where \( \hat{a}(IF) \) is the estimated fading channel envelope and \( \hat{\theta}(IF) \) is the estimated phase offset. The fading distortion at the information symbols can be estimated by interpolation the sequence of \( \hat{H}(IF) \) using the Fourier Transform (FFT) interpolation. This interpolation method is done in three steps. Firstly, the estimated channel vector corresponding to pilot positions are transformed from the time domain into the frequency domain by taking the fast version of the Discrete Fourier Transform (DFT). The resulting vector is denoted by \( H_{FFT} \). Next, zeros are stuffed in the middle of \( H_{FFT} \) to yield the frequency samples with length \( L \) equal to that of the channel coefficients vector corresponding to data information, where \( L \) is an integer power of two. Then, the inverse fast discrete Fourier transform (IDFT) is applied to the obtained signal. The signal at the output of the IDFT represents the interpolated samples of the estimated MIMO channel coefficients. We note the interpolated channel vector for each data symbols coded by STBC by \( \hat{H} \):

\[
\hat{H} = [\hat{h}_1, \ldots, \hat{h}_{NF}]
\]  
(16)

It is necessary to point out that this zero padding in the frequency domain is similar to the interpolation between the pilot symbols in the time domain. This interpolation method is simple because only the zero padding and the discrete FFT operation are required. In addition to the performance loss involved by the power consumed by pilot symbols as discussed above, we can notice additive fading estimation error. This occurs due to the received pilot symbols distortion and also to the interpolation error.

**D. Decoding**

By considering an orthogonal space time block coding and a slow time-varying fading channel \( (f_0T_s \ll 1) \), the MIMO channel coefficients are assumed be constant over STBC codeword period, therefore the well-known and straightforward ML decoder is applied at the receiver side. However, in the case of fast fading channel such as in [12], when the channels vary on a symbol-by-symbol basis, the ML detection has high complexity.

For a MIMO system equipped by two transmit antenna, the ML receiver is employed for computing the decision metric and deciding the codeword that minimizes this decision metric:

\[
\begin{align*}
\hat{s}_1 &= \arg\min_{s_1 \in A} \left( (\hat{h}_1 y_1 + \hat{h}_2 y_2) - s_1 \right)^2 + \beta |s_1|^2 \\
\hat{s}_2 &= \arg\min_{s_2 \in A} \left( (\hat{h}_2 y_1 + \hat{h}_1 y_2) - s_2 \right)^2 + \beta |s_2|^2
\end{align*}
\]  
(17)

Where \( \beta = -1 + \sum_{k=1}^{2} |h_k|^2 \).

For four transmit antennas, in order to estimate the four symbols, the ML minimizes the metrics given by (18). In which \( \gamma = -1 + 2 \sum_{k=1}^{4} |h_k|^2 \).

Then, the detected symbols can be obtained by:

\[
\hat{s}_k = \arg\min_{s_k \in A} (m_k); k = 1, \ldots, 4
\]  
(19)
\begin{align}
m_1 &= \left( y_1 \hat{h}_1 + y_2 \hat{h}_2 + y_3 \hat{h}_3 + y_4 \hat{h}_4 + y_5 \hat{h}_5 + y_6 \hat{h}_6 \right) - s_1^2 + y_1 |s_1|^2 \\
m_2 &= \left( (y_1 \hat{h}_1^2 - y_2 \hat{h}_2 + y_3 \hat{h}_3^2 + y_4 \hat{h}_4 - y_5 \hat{h}_5 - y_6 \hat{h}_6) - s_2^2 + y_2 |s_2|^2 \\
m_3 &= \left( (y_1 \hat{h}_1^3 + y_2 \hat{h}_2 + y_3 \hat{h}_3^2 - y_4 \hat{h}_4 + y_5 \hat{h}_5^2 + y_6 \hat{h}_6) - s_3^2 + y_3 |s_3|^2 \\
m_4 &= \left( (y_1 \hat{h}_1 - y_2 \hat{h}_2 + y_3 \hat{h}_3 + y_4 \hat{h}_4 - y_5 \hat{h}_5 - y_6 \hat{h}_6) - s_4^2 + y_4 |s_4|^2 \\
\end{align}

(18)

IV. RESULTS

In this section, we present numerical results that highlight the accuracy of the proposed pilot-assisted MIMO scheme in terms of bit error rate (BER). Simulations are conducted for MIMO wireless system with 8-PSK modulation and equally spaced pilot symbols as described in section 2. The training lengths are set to be the antenna number of the terminal \( N_T \). Two cases of STBC are considered: the Alamouti code \( C_2 \) and the orthogonal STBC for four transmit antennas \( C_4 \). The two proposed schemes are operated with one receive antenna. For evaluating the PSAM performance as function of SNR, slow fading is assumed. The considered slow fading channel corresponds to a typical digital cellular system operating at carrier frequency \( f_c = 900 \) MHz. The speed of the mobile user is \( v = 120 \) Km/h. We assume that the sampling frequency \( f_s \) is 100 KHz. The maximum Doppler frequency is equal to 100 Hz. Hence, the normalized fading rate is 0.001.

Considering the optimization of pilot symbol spacing given by (7), the optimal pilot period is \( F_{opt} = 32 \). In Fig. 2, we present the BER performance, as function of SNR, of the new pilot-assisted MIMO systems equipped by two and four transmit antennas; respectively. For comparison purposes, the BER performance of the coherent system is also plotted in this figure. The curves indicate that the actual estimation BER performance of the PSAM-STBC is within 2 dB apart from the coherent counterparts at typical SNR range in slow fading channel. This BER performance loss can be explained by the power loss due to the pilot insertion and the errors of the pilot estimation and the interpolation function. In Fig. 3, the BER of the proposed schemes are shown as function of the normalized frequency Doppler for \( SNR = 24 \) dB over slow fading channels. According to this figure, we can conclude that the BER degrades as function of the normalized fading rate.

V. CONCLUSION

In this paper, accurate and straightforward PSAM estimation method is proposed for MIMO based on orthogonal STBC codes. The transmitter just inserts known equally and optimally spaced pilot symbols in data information block. The combined signal is coded using orthogonal STBC code. The transmitted signal is corrupted by slow fading and additive noise. The slow fading channel is modeled by Jakes model; also it is chosen to be constant over the STBC codeword period. The receiver estimates and interpolates the channel measurements provided by the pilot symbols in order to obtain the amplitude and the phase reference for detection. Simulation results show that the channel estimation based on PSAM technique is accurate in terms BER for the two MIMO schemes. The benefit of this technique is its implementation ease in addition to its competitive performance. It is also shown that the estimation method is suitable for slow time-varying fading channel and it can be extended to fast time-varying fading channel.
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