

Interplay of Contact Times, Fragmentation and Coding in DTNs

Francesco De Pellegrini
CREATE-NET
via Alla Cascata 56 c
38100 Trento, Italy
fdpellegrini@create-net.org

Rachid El-Azouzi,
LIA, Université d'Avignon
chemin des Meinajaries 339
84911 Avignon, France
rachid.elazouzi@univ-avignon.fr

Fabio Albini
CPGEI - UTFPR Curitiba,
Av. Sete de setembro 3165
80230-901 Curitiba, PR, Brazil
fabio.albini@univ-avignon.fr

Abstract—Models describing DTNs proposed in the last few years have been focusing on message replication policies able to achieve high delivery probability at the cost of network resources, e.g., message copies. To this respect, the duration of contact events is the physical constraint that dictates how fine a large message should be fragmented into packets in order to match finite contacts duration. The price to pay, indeed, is that the source has to deliver a larger number of packets per message.

In this paper we model the combined effect of message fragmentation and buffering and describe the structure of the forwarding process in closed form when the message is split into K packets and delivered to the destination.

We consider the specific case of sequential forwarding, where the source delivers fragmented message packets in order to relays. In this case the interplay of forwarding and message fragmentation can be expressed in closed form by coupling the combinatorial structure for packet forwarding and fluid models for the replication of packets in the network. By deriving the closed form expression for the delivery probability, we are able to derive the optimal fragmentation K^* as a function of the contact time distribution.

Finally, results on sequential forwarding are applied to derive performance figures for the case when fountain coding is applied: redundancy is added to the original information with the aim to increase the message delivery probability. The paper is completed by numerical results.

Index Terms—Delay Tolerant Networks, Contact Times, Fragmentation, Buffering, Fluid Approximations

I. INTRODUCTION

In Delay Tolerant Networks lack of persistent connectivity suggests to use redundancy in order to reach destination nodes. One technique is to store information on intermediate carries, namely the relay nodes, in the form of additional message copies. The typical envisioned scenario is that of highly mobile wireless networks: in such systems the end-to-end paths do not last long enough to sustain the operations of standard protocols. The TCP protocol, for instance, works under the assumption that timeouts are dimensioned for a certain bounded round trip delay. In particular, the exchange of messages between two nodes in a DTN is possible when they come into radio range: when this happens it is said that an intermeeting contact event occurs between the two nodes. For such a reason, in order to characterize the exchange of messages that is sustained among nodes in the network, modeling of DTNs poses major focus on the intermeeting process and on how it impacts the performance of such communication systems.

Clearly, given a finite time horizon, not always a message originated at a certain source node will be able to reach the intended destination. In fact, two concurrent effects can contribute to erasures of message copies. The first one is lack of a set of intermeetings through which message copies can reach the destination. The second one is due to the duration of contact times: for some contact events, in fact, the transmission time will not be sufficient to complete the message exchange successfully. In the extreme case of a very large file, it is possible that the size of the file hinders the possibility to deliver to the destination irrespective of the intermeeting pattern.

In order to overcome the latter issue, one can resort to fragmentation, a customary technique in telecommunication networks to cope with large sized messages. In the context of DTNs, the effect is to produce a set of K packets that are independently delivered to the destination. The destination, in turn, needs to receive all such K packets to decode the original message with success.

In order to make use of many intermeeting events, similarly to what done with repetition codes, several identical copies of each packet can be released. However, as known from coding theory, this neither the unique nor the optimal way to achieve robustness to messages erasures by adding redundancy.

In this paper, in particular, we will investigate the performance of coding techniques in combination with the presence of finite contact times durations. The main problem that we aim to model is the performance of a DTN where we make use of either coded or uncoded storage of packets at intermediate relays. For the sake of simplicity, we assume that no timeouts are used and two hop routing is employed. The former assumption describes the case when the buffer of relays is sufficient to store all packets; in the scope of this paper we restrict to the analysis of a single message per source-destination pair so that this assumption is not restrictive. We observe that two hop is the simplest known protocol whose per source-destination capacity does not vanish with increasing number of nodes [1].

Our analysis starts from the case of sequential packet delivery. This is a suboptimal scheme according to which the source populates the nodes' buffer with packets in sequential order. For instance, the TCP protocol adheres to this model since it delivers all the packets sequentially within a predefined congestion window. Indeed, as it was showed in [2], [3], in the

homogeneous case, it is optimal to distribute packets as evenly as possible among the relays in order to maximize the success probability. As a consequence, the sequential packet delivery strategy described before cannot be optimal. However, as showed in the following, it provides a tool that let us perform a companion performance analysis of the second scheme that we analyze in this paper, i.e., the case when fountain codes are employed.

The use of fountain codes has two main advantages: first of all no resequencing is needed at the destination; from the modeling standpoint this is also the basic reason why the sequential delivery protocol analysis can be adapted to this case. Indeed, the best advantage that fountain codes give compared to customary block coding, e.g., Reed Solomon codes, is that the amount of redundancy introduced by fountain codes is not fixed and information can be retrieved with any combination of encoded packets, provided that the set of received packets form a full rank matrix [4].

A. Related works

The problem of information fragmentation and reassembly is a traditional topic in communication networks. However, the idea of encoding fragments to ease the reconstruction when some fragments may be lost appeared first in satellite communications to cope with *ACK implosion*. Seminal work [5] proposed first a solution using packet-level forward error correction (FEC), namely, Reed–Solomon coding. A large block of data can be split into several packets and coding is a mean to avoid retransmission due to channel errors. This becomes mandatory for collisions’ sake in order to reduce the number of sites requesting concurrently the satellite to retransmit. If one transmits additional H redundant packets, so that $K + H$ total packets are broadcasted, all stations receiving K packets are able to decode. Several works combining FEC and acknowledgment-based retransmission protocols [6], [7], [8] appeared later on. In that context coding was to reduce the potential delay due to retransmissions’ for multicasting multimedia streams.

In DTNs, works [9] and [10] suggested to encode a file using erasure codes and then distribute all the resulting code-blocks to the relays. Under random mobility patterns, it is clear that reconciling erasures can increase the delivery probability; a connection with standard replication-based routing (e.g., spray-and-wait, epidemic routing or two hop routing) would relate those techniques to repetition codes. Those are known to be the codes with highest overhead for a given codeword length. [10] assessed first the performance gain of erasure coding by means of extensive simulations and for different routing protocols. [9] specialized to non-uniform encounter patterns where the optimal successful delivery probability is dependent on the path taken by each packet; also, the problem is NP-hard. Routing protocols based on network coding are proposed in [11] and tested when delivering multiple packets. In [12] ODE based models are employed under epidemic routing; in that work, semi-analytical numerical results are reported describing the effect of finite buffers and contact

times; the authors also propose a prioritization algorithm. The same authors in [13], [14] investigate the use of network coding using the Spray-and-Wait algorithm and analyze the performance in terms of the bandwidth of contacts, the energy constraint and the buffer size. In [15] the integration of the fountain codes and the Optimal Probabilistic Forwarding (OPF) protocol is proposed. Data are encrypted and a forwarding rule is set to decide where a packet will be sent to.

In [16] the authors derive the performance analysis for a unicast session, under spray-and-wait routing and inter-session network coding, in the presence of background traffic in DTNs with homogeneous mobility.

Novel contribution Compared to existing works, in this paper we are able to provide a model that can jointly describe the effect of buffering at relays and the effect of finite contact times durations. This provides a connection between the distribution of the duration of contacts and the message delivery probability. The model is applied to fountain coding providing a closed form solution for the message delivery probability. To the best of the authors’ knowledge, the joint impact of buffering and finite contact times durations has not been investigated so far in DTNs.

II. NETWORK MODEL

We consider a DTN composed of a source node, $N - 1$ relay nodes and a destination node. A message is generated at the source node at time $t = 0$ and needs to be forwarded to the destination node. In this context a simple way to model the intermeeting process is by means of a point process, where arrivals model the intermeeting events that rule the transmission opportunities between DTNs nodes. In our model we adopt a Poisson process so that the time between contacts of any two nodes is assumed to be exponentially distributed with intensity $\lambda > 0$.

In modeling of DTN forwarding one may focus on contact events and describe the forwarding process accordingly. In the case of two hop routing, at each contact between the source and a mobile that does not have the message, the message is relayed to that mobile. If a mobile that is not the source has the message and it is in contact with another mobile then it transfers the message if and only if the other mobile is the destination node. However, such reasoning can be refined by including in the model the probability of transferring a message during a contact event, which indeed relates to the distribution of *contact times*.

The contact time between two mobile nodes is the period of time during which two nodes have the opportunity to communicate since they stay in radio range. Throughout the variety of research works in this topic, most of research interest focused on the inter-contact time under the assumption that the contact is sufficient to exchange the message at each encounter.

A closer look reveals the distribution of contact times should have an impact the delay and capacity of the network. In particular, since the contact time between two nodes sometimes is not enough to transfer a copy of message to a relay, the source should divide the message into K packets. In this paper, we

describe in detail how fragmentation improves the efficiency of the DTN's operations by connecting the delivery probability of a certain message and the number of packets K that the source needs to split the message into.

In order to proceed with the analysis, we resort to fluid approximations, where one models the evolution of the mean value of the the underlying process. For a discussion on the validity of such approximation model the reader can refer to [3].

III. SEQUENTIAL FORWARDING

In the sequential forwarding mechanism, the source delivers the packets in order to relays. I.e., packets are labeled $1, 2, \dots, K$, and upon meeting a relay which has $j-1$ packets, $0 \leq j \leq K$, the source will attempt to deliver packets from j onward.

Let $X_i(t)$ be the fraction of nodes excluding the source which have the i first packets at time t . If at time t it encounters a mobile which has i first packets, it gives it j packets $i+1, \dots, i+j$ whenever the contact time is $t_j \leq T_c < t_{j+1}$; here, t_j denotes the time needed to transmit j packets. The standard fluid approximation for the forwarding process can be written as

$$\begin{aligned} \dot{X}_0(t) &= -\lambda \bar{p}_1 X_0(t) \\ \dot{X}_i(t) &= \lambda \sum_{j=0}^{i-1} X_j(t) p_{i-j} - \lambda \bar{p}_1 X_i(t) \quad i = 1, \dots, K-1 \\ \dot{X}_K(t) &= -\sum_{j=0}^{K-1} \dot{X}_j(t) \end{aligned} \quad (1)$$

where $p_j = \mathbb{P}\{t_j \leq T_c < t_{j+1}\}$ and $\bar{p}_j = \mathbb{P}\{T_c \geq t_j\}$.

In particular, the first equation in (1) follows from a thinning argument: $\lambda \bar{p}_1$ is the frequency at which a relay which does not have any packet meets the source *and* the contact time lasts long enough to have at least one packet forwarded to the relay. The equations for $i = 1, \dots, K-1$ follow using a similar argument and the last one expresses the dynamics of absorbing state K .

In particular, the dynamics corresponding to (1) can be derived as follows. Denote $X_0(t)$ the fraction of nodes that do not have packets at time t . Indeed it holds

$$X_0(t) = e^{-\lambda \bar{p}_1 t} \mathbb{1}_{\{t \geq 0\}}(t)$$

where $\mathbb{1}_A(\cdot)$ is the standard indicating function of set A ; the system (1) can be solved iteratively using the Laplace transform $X_i(s) = \mathcal{L}[X_i(t)|s]$,

$$sX_i(s) = \lambda \sum_{j=0}^{i-1} X_j(s) p_{i-j} - \lambda \bar{p}_1 X_i(s) \quad i = 1, \dots, K-1$$

noticing that $X_0(s) = (s + \lambda \bar{p}_1)^{-1}$.

Using the Laplace transform, we obtain

$$\begin{aligned} X_1 &= \lambda p_1 X_0^2 \\ X_2 &= \lambda p_2 X_0^2 + \lambda^2 p_1^2 X_0^3 \\ X_3 &= \lambda p_3 X_0^2 + 2\lambda^2 p_1 p_2 X_0^3 + \lambda^3 p_1^3 X_0^4 \\ X_4 &= \lambda p_4 X_0^2 + \lambda^2 (2p_1 p_3 + p_2^2) X_0^3 \\ &\quad + \lambda^3 3p_1^2 p_2 X_0^4 + \lambda^4 p_1^4 X_0^5 \\ X_5 &= \lambda p_5 X_0^2 + 2\lambda^2 (p_1 p_4 + p_2 p_3) X_0^3 \\ &\quad + 3\lambda^3 (p_1^2 p_3 + p_1 p_2^2) X_0^4 + 4\lambda^4 p_1^3 p_2 X_0^5 + \lambda^5 p_1^5 X_0^6 \\ X_6 &= \lambda p_6 X_0^2 + \lambda^2 (2p_1 p_5 + 2p_2 p_4 + p_3^2) X_0^3 + \lambda^3 (3p_1^2 p_4 \\ &\quad + 6p_1 p_2 p_3 + p_2^3) X_0^4 + \lambda^4 (4p_1^3 p_3 + 6p_1^2 p_2^2) X_0^5 \\ &\quad + 5\lambda^5 p_1^4 p_2 X_0^6 + \lambda^6 p_1^6 X_0^7 \\ &\quad \dots \end{aligned}$$

which leads to the general expression

$$X_i(s) = \sum_{j=0}^{i-1} X_0(s)^{j+2} \lambda^{j+1} a_{i,i-j}(p_1, \dots, p_{i-j}) \quad (2)$$

In particular, we can derive from (2) the iterative calculation in the time domain by taking the inverse transform as

$$X_i(t) = \sum_{j=0}^{i-1} a_{i,i-j}(p_1, \dots, p_{i-j}) \lambda^{j+1} \Theta_{j+2}$$

Here the combinatorial description of the sequential delivery of packets is encoded by the coefficients $a_{i,i-j}$: those are monomials of degree j of the p_i s, where

$$a_{i,i-j}(p_1, \dots, p_{i-j}) = \sum_{\{r: \sum r = i-j\}} \prod p_r \quad (3)$$

and $a_{i,i-j}$ is the probability of passing i packets in $i-j$ contacts. Also, Θ_i is the j -th self-convolution of $X_0(t)$, i.e., $\theta_0(t) = X_0(t)$ and $\theta_{j+1}(t) = \theta_j * X_0(t)$ and it also follows that $\theta_j(t) = \frac{t^{j-1}}{(j-1)!} X_0(t)$.

In order to understand the role of the coefficients $a_{i,i-j}$ we can resort to the concept of *partitions* and *ordered partitions* of an integer n into k parts. In general, the partition of n into k parts is the solution of $n = x_1 + \dots + x_k$, for $x_1 \geq x_2 \geq \dots \geq 1$. For example, $(3, 2, 1)$ is an ordered partition of $n = 6$ into 3 parts. From any such given solution we can determine the number of corresponding unordered partitions as $k! / \sum_{r=1}^n h_r!$, where h_r is the number of occurrences of integer r in the partition.

Back to (2), we need to express the combinatorial structure of the message delivery once it is fragmented into packets. In particular, the coefficients $a_{i,i-j}$ correspond to the *ordered partitions* of the integer i into $i-j$ parts, once we identify p_i with the integer i , i.e., the number of packets. For example, for $i = 6$, the partitions into 2 parts are 2 of the kind $\{1, 5\}$, 2 of the kind $\{2, 4\}$ and just one of the kind $\{3, 3\}$. In particular, the generic term $a_{i,i-j}$ is the probability of the events according to which it is possible to transfer i packets in exactly $i-j$ contacts: for example, the coefficient $a_{6,3} = (3p_1^2 p_4 + 6p_1 p_2 p_3 + p_2^3)$ describes the probability

of transferring two single packets in two contacts and four packets in one contact, plus the probability of transferring one, two and three packets at each contact respectively; the last addend accounts for the probability of transferring exactly two packets per contact.

Let $v(n, k)$ be the number of ordered partitions of n into k parts and $p(n, k)$ the number of partitions of n in k parts. In Table I we enumerated partitions and ordered partitions up to $n = 6$. In literature there exist several algorithms to determine the partitions of an integer in k parts [17].

The number of terms that appear in the polynomial $a_{i,i-j}$, in the general case grows according to the number of partitions; the coefficients in front of each monomial in the $p_i s$ is the number of ordered partitions. Notice that this observation provides an algorithm to construct the expression of (2) for example, $a_{5,3}$ accounts for the partitions of 5 into 3 parts, and corresponds to the following unordered partitions (3, 1, 1) and (2, 2, 1). Using the correspondence above, the two monomials $p_1^2 p_3$ and $p_1 p_2^2$ will appear in $X_5(s)$, both with coefficient $3 = 3!/2!$.

In order to carry out our calculations, we will leverage the following relation on the number of partitions

Lemma 3.1: Let $n \geq k \geq 1$, then $v(n, k) = \binom{n-1}{k-1}$

Proof: By induction on n . The fact it is indeed true for $n = 2$. Now, assume that it holds for $n - 1$, and for all $k \leq n - 1$, then

$$\begin{aligned} v(n, k) &= \sum_{h=1}^{n-k+1} v(n-h, k-1) = \sum_{h=1}^{n-k+1} \binom{(n-2)-(h-1)}{k-2} \\ &= \sum_{h=0}^{n-k} \binom{(n-2)-h}{k-2} = \sum_{r=k-2}^{n-2} \binom{(n-2)-r}{r} = \binom{n-1}{k-1} \end{aligned}$$

where the last step follows from the binomial identity $\sum_{r=M}^N \binom{r}{M} = \binom{N+1}{M+1}$. ■

Notice that from Lemma 3.1, the number of distinct monomials that are accounted in the right-hand terms of (2) is 2^{i-1} .

Finally, a simplified expression for (2) can be obtained in the case of a geometric distribution, i.e., when $p_i = (1-p)p^i$, $i = 1, 2, \dots$, for $0 \leq p \leq 1$. Notice that in this case, in order to write the dynamics, it is sufficient to use the number of ordered partitions in (2) and observe that the expression there writes

$$X_i(s) = p^i \sum_{j=0}^{i-1} v(i, i-j) [\lambda(1-p)]^{j+1} X_0(s)^{j+2} \quad (4)$$

From (4), the time-domain expression follows for $i = 1, \dots, K-1$:

$$\begin{aligned} X_i(t) &= p^i X_0(t) \sum_{j=0}^{i-1} v(i, i-j) \frac{[\lambda(1-p)]^{j+1} t^{j+1}}{(j+1)!} \\ &= p^i X_0(t) \sum_{j=0}^{i-1} \binom{i-1}{j} \frac{[\lambda(1-p)t]^{j+1}}{(j+1)!} = p^i X_0(t) \tilde{X}_i(t) \quad (5) \end{aligned}$$

In the following derivation it will be relevant to have a closed form expression for the dynamics of the fraction of

nodes having all the K packets, namely, X_K : by combining the above expressions we can derive

$$\begin{aligned} X_K(t) &= 1 - X_0(t) \left[1 + \sum_{r=1}^{K-1} p^r \sum_{j=1}^r \binom{r-1}{j-1} \frac{[\lambda(1-p)t]^j}{j!} \right] \\ &= 1 - X_0(t) \left[1 + \sum_{j=1}^{K-1} \frac{[\lambda(1-p)t]^j}{j!} \sum_{r=j}^{K-1} p^r \binom{r-1}{j-1} \right] \\ &= 1 - X_0(t) [1 + F_K(p)] \quad (6) \end{aligned}$$

where we used again the fact that $\sum_{l=k}^n \binom{l}{k} = \binom{n+1}{k+1}$ and we defined $F_K(p) = \sum_{j=1}^{K-1} \frac{[\lambda(1-p)t]^j}{j!} \binom{K-1}{j}$.

A. Success probability

Here, we are interested in the success probability of the full message, i.e., the probability by which the sequential transmission can deliver all the K packets to the destination by a given deadline. In particular, let T_K be the message delivery delay, i.e, the time elapsing from when the first packet is generated at the source to the time when the last packet K is delivered to destination. The distribution of the delay T_K is denoted by $D_X(t) = \mathbb{P}(T_K \leq t)$. Let $y_d(t)$ a random variable which represents the number of packets received by the destination during $[0, t]$. We can express the success probability in receiving the message by t as

$$\begin{aligned} D_X(t+dt) - D_X(t) &= \mathbb{P}(t < T_K \leq t+dt) \\ &= \sum_{i=0}^{K-1} \mathbb{P}(t < T_K \leq t+dt, y_d(t) = i) \\ &= \sum_{i=0}^{K-1} \mathbb{P}(T_K \leq t+dt | y_d(t) = i, T_K > t) \cdot \\ &\quad \cdot \mathbb{P}(y_d(t) = i | T_K > t) \mathbb{P}(T_K > t) \\ &= \sum_{i=0}^{K-1} dt \lambda (1 + N X_K(t)) \bar{p}_{K-i} \frac{N X_i(t)}{N - N X_K(t)} (1 - D_X(t)) \\ &= dt \lambda N \left(\frac{1}{N} + X_K(t) \right) \sum_{i=0}^{K-1} \frac{X_i(t) \bar{p}_{K-i}}{1 - X_K(t)} (1 - D_X(t)) \end{aligned}$$

In the above calculation, $\mathbb{P}(T_K \leq t+dt | y_d(t) = i, T_K > t)$ represents the probability that the destination receives all packets conditioning on the fact that it has already i packets at time t . This occurs if the destination meets a node having all packets and the contact is sufficient to exchange $K-i$ packets. The probability that the destination has i packets at time t is represented by the term $\mathbb{P}(y_d(t) = i | T_K > t)$; then $\mathbb{P}(y_d(t) = i | T_K > t)$ is the probability that the destination is among the nodes that have i packets at time t (observe that there are N such nodes).

Thus we obtain the separable ODE

$$\frac{dD_X(t)}{dt} = \lambda N \left(\frac{1}{N} + X_K(t) \right) \sum_{i=0}^{K-1} \frac{X_i(t) \bar{p}_{K-i}}{1 - X_K(t)} [1 - D_X(t)] \quad (7)$$

n	k	$p(n)$	partition	ordered partitions	$v(n, k)$
1	1	1	(1)	(1)	1
2	1	2	(2)	(2)	1
	2		(1, 1)	(1, 1)	1
3	1	4	(3)	(3)	1
	2		(2, 1)	(2, 1) (1, 2)	2
	3		(1, 1, 1)	(1, 1, 1)	1
4	1	5	(4)	(4)	1
	2		(3, 1)	(3, 1) (1, 3)	3
			(2, 2)	(2, 2)	
	3		(2, 1, 1)	(2, 1, 1) (1, 2, 1) (1, 1, 2)	3
	4		(1, 1, 1, 1)	(1, 1, 1, 1)	1

TABLE I
THE NUMBER OF PARTITIONS AND THE VALUE $v(n, k)$ FOR SMALL VALUES OF k .

which can be easily be integrated as

$$\int_0^t \frac{dD}{1-D} = \lambda N \int_0^t \left[\frac{1}{N} + X_K(s) \right] \sum_{i=0}^{K-1} \frac{X_i(s) \bar{p}_{K-i}}{1-X_K(s)} ds$$

Finally, we can derived a closed form expression of $D_X(t)$ by integrating by parts and noticing that $\dot{X}_K(s) = \lambda \sum_{i=0}^{K-1} X_i(s) \bar{p}_{K-i}$ in order to rearrange the following calculation

$$\begin{aligned} D_X(t) &= 1 - \exp \left\{ -\lambda N \sum_{i=0}^{K-1} \int_0^t ds \left[\frac{1}{N} + X_K(s) \right] \frac{X_i(s) \bar{p}_{K-i}}{1-X_K(s)} \right\} \\ &= 1 - \exp \left\{ -N \int_0^t \frac{\left[\frac{1}{N} + X_K(s) \right] \dot{X}_K(s)}{1-X_K(s)} ds \right\} \\ &= 1 - \exp \left\{ N(X_K(t) - X_K(0)) + (N+1) \log \left(\frac{1-X_K(t)}{1-X_K(0)} \right) \right\} \end{aligned}$$

where we used $\int \frac{x}{1-x} = -x - \log(1-x)$.

The main result from the above analysis is that under a sequential packet delivery protocol, the success probability $D_X(t)$ at any given time does not depend on the trajectory of each packet, i.e., on the X_i s. But, it only depends on the final state of $X_K(t)$. From this last observation it is possible to verify with a simple calculation that the expression above is increasing in the X_K variable. Thus, we can state

Lemma 3.2: $D_X(t)$ is a function of $X_K(t)$ only and it is increasing in X_K .

As a consequence of Lemma 3.2, when we want to characterize the success probability as a function of the distribution of the number of packets that can be transmitted during a contact, by the monotonicity property we can simply focus on the effect of such distribution on the variable X_K .

B. Fragmentation

In the following we will develop the analysis of X_K as a function of K . Looking at (6), we shall be studying the impact of the distribution of the packets transmitted per contact on the dynamics of X_K . Namely, since we assume a geometric distribution, we need to study the impact of the parameter p , i.e., the success probability of transmitting a packet over a contact; we write $p = p_K$ to recall that such a probability is a function of the number of fragments generated from the original message and we make a rather natural assumption here:

Assumption 3.1: p_K is non decreasing with K : $p_{K+1} \geq p_K$ for $K = 1, 2, \dots$

Clearly, if the probability of delivering a shorter packet over a single contact is not better off than delivering using a larger packet size, the increased number of packets to be delivered to the destination becomes a penalty. In the extreme case when p_K is a constant, the message delivery probability is decreasing compared to the case when the message is not fragmented. However, this is the trivial case when there is no benefit in fragmenting the message.

In general, the assumption above states that we expect that $p = p_K$ is a non decreasing function of K , according to the intuition that it is easier to transmit a single packet during a contact if the transmission time is smaller. I.e, typically we expect that the effect of fragmentation is beneficial in order to deliver more packets over single contacts, so that the dependence of p on K should not be a constant. For the sake of notation, in what follows, we will still denote X_K be the fraction of nodes having K packets when exactly K fragments are released from the original message: however, starting from (6) we should account for the fact that X_K depends on K also through p_K . The above observations are formally captured by the following

- Theorem 3.1:** i. If $p_K < p_{K+1}$, then $X_{K+1} > X_K$
ii. If $p_K = p_{K+1}$, $X_K < X_{K+1}$

Proof: i.) It follows by inspection of (6). ii.) In order to prove our statement, we adopt a variational argument by using a linear interpolated function of the original expression as it appears in (6). In particular, let us define the transformation of $\bar{X} : [0, 1] \rightarrow [0, 1]$

$$\begin{aligned} \bar{X} : y \rightarrow \bar{X}_K(y) &= 1 - e^{-xp(y)} \Delta_k y - \\ &e^{-xp(y)} \left[1 + \sum_{r=1}^{K-1} p(y)^r \sum_{j=1}^r \binom{r-1}{j-1} \frac{[\lambda(1-p(y))t]^j}{j!} \right] \end{aligned}$$

where we let, $x = \lambda\tau$, $\Delta_k = p_{K+1} \frac{x^K (1-p_{K+1})^K}{K!}$. In particular, let us define

$$p(y) = p_K + \delta_{p_K} y, \quad \text{where} \quad \delta_{p_K} = p_{K+1} - p_K > 0$$

We observe that $\bar{X}_K(0) = X_K(t)$ and $\bar{X}_K(1) = X_{K+1}(t)$: we are going to prove that the transformation is increasing in the parameter y so that $X_K(t) = \bar{X}_K(0) < \bar{X}_K(1) = X_{K+1}(t)$.

In fact, we can simply observe that

$$\begin{aligned} \frac{d}{dy} \overline{X}_K(y) &= \delta \frac{d}{dp} X_K(p) - \frac{d}{dy} \left(e^{-p(y)x} \Delta_k y \right) \\ &= \delta \frac{d}{dp} X_K(p) + \Delta_k e^{-p(y)x} (-1 + x\delta y) \end{aligned} \quad (8)$$

and we can derive

$$\begin{aligned} X_{K+1}(t) - X_K(t) &= \overline{X}_K(1) - \overline{X}_K(0) \\ &= \int_0^1 \delta \frac{d}{dp} X_K(p) dy + \Delta_k e^{-pKx} \int_0^1 e^{-x\delta y} (-1 + x\delta y) \\ &\geq \Delta_k e^{-pKx} \int_0^{x\delta} e^{-v} (-1 + v) = 1 - x\delta e^{-x\delta} > 0 \end{aligned}$$

The first inequality follows from the fact that $\frac{d}{dp} X_K(p) \geq 0$, whereas the final strict inequality can be written once we observed that for $x \geq 0$, the function xe^{-x} attains its maximum at $x = 1$, where its value is $e^{-1} < 1$. Since the reasoning holds irrespective of the packet number K , this concludes the proof. \blacksquare

Remark 3.1: Proof of Thm. 3.1 provides us with insight into the impact of fragmentation: there are two opposite effects that can be visualized in (6). One is a local effect and the other is a global effect. The global effect is the polynomial increase in the product term that appears within the summation: it accounts for the increased number of packets and tends to decrease the message delivery probability simply because one more packet has to be delivered. Conversely, the local effect is the increased probability of transmitting a packet during a contact, which causes a decrease of the exponential term that multiplies the summation. Basically, Thm. 3.1 tells us that when case ii holds, the local effect takes over the global one, whereas in the case i. the global effect dominates.

Now, combining Lemma. 3.2 and Cor. 3.1, we obtain a result that characterizes the efficiency of the fragmentation mechanism.

Theorem 3.2: Let $p^* = \sup\{p_K, K = 1, 2, \dots, \infty\}$ and denote $K^* = \inf\{K : p_K = p^*\}$, then

$$D_X(K^*) = \sup\{D_X(K) : K = 1, 2, \dots, \infty\}$$

From the above result, we understand that the sup is attained only in the case when the intermeeting process is such that there exists a finite K^* such that $p_{K^*} = p_K$ for all $K \geq K^*$, i.e., above K^* the success of a single packet transmission during a contact is saturated at some maximum value p^* .

In that case, this also corresponds to the optimal number of packets by which the message should be fragmented. Denote T_m the transmission time of the message, and let $T_m(1 + K\zeta)$ be the transmission time of the message once it is fragmented. The term $\zeta > 0$ accounts for the overhead, which is possibly due to transmission time and to the protocol operations, e.g., beaconing interval and/or handshake procedures.

Then we recall that in the geometric case $p = \overline{p}_1 = \mathbb{P}\{T_c \geq t_1\}$ so that

$$p = \mathbb{P}\left\{T_c \geq \zeta + \frac{T_m}{K}\right\} = 1 - F_{T_c}\left(\zeta + \frac{T_m}{K}\right)$$

where $F_{T_c}(\cdot)$ is the CDF of the contact time. Thus, if $K^* < \infty$, we can translate the optimal packet number K^* as the one for which it holds

Corollary 3.1: Given per contact overhead $\zeta \geq 0$, the optimal fragmentation

$$K^* = \min\{K : p_K = 1 - F_{T_c}(\zeta)\} \quad (9)$$

That is optimal fragmentation is the smallest one that saturates the success probability to the value that corresponds to the bare overhead transmission over a contact.

IV. FOUNTAIN CODES

Now, let us assume that the source uses additional redundant packets to improve the efficiency of the DTN. Using fountain codes, for any ϵ , the destination is able to decode the K packets with at least probability $1 - \epsilon$ if it has received at least $M := K \log(\frac{K}{\epsilon})$ coded packets. Saying it in a different way, this means that any $M := K \log(K/\delta)$ encoded packets let decode the original K packets with at least the probability $P_s = 1 - \delta$ by on average $O(K \log(1/\delta))$ operations [18]. Using fountain coding, the source node is able to release newly generated packets at each contact with relays, so that the sequential order of packets becomes irrelevant in this case: in particular, this means that the analysis of the sequential transmission scheme still holds, but the decoding is performed irrespective of the order by which packets have been received at the destination.

We define the network state, the coded packet distribution on relay nodes by a M -tuple (X_1, \dots, X_M) , where $X_i(t)$, $i = 0, \dots, M - 1$ denotes the number of nodes having i coded packets and $X_M(t)$ denoted the number of nodes having more than M coded packets. Then we introduce the same standard fluid approximation (based on mean field analysis) done in the first case, but we change the K values for the M values, i.e., we consider δ fixed.

Now we want to write the number of nodes having at least i packets as a function of the number of nodes having exactly i packets to get the closed form as it follows. By the previously stated equations we have that:

$$\begin{aligned} Y_0 &= Y_1 + X_0 \\ Y_h(t) &= Y_{h+1}(t) + X_h(t) \\ Y_{M-1}(t) &= Y_M(t) + X_{M-1}(t) \\ Y_M(t) &= X_M(t) \end{aligned}$$

So, as $X_M(t)$ is given by equation (6) we can perform the recursive backwards calculation and achieve all the Y_i values.¹ Let T_M be the message delivery delay, i.e, the time from when the first packet is generated at the source to the time when the M th coded packet is delivered to the destination. The distribution of the delay T_M is denoted by $D_Y(t) = \mathbb{P}(L_M \leq t)$. Let $y_d(t)$ a random variable which represents the number of coded packets received by destination during $[0, t]$. Let $Y_i(t)$ denote the number of nodes having at

¹We are implicitly assuming that let B is the buffer size of a node, then $B > M$.

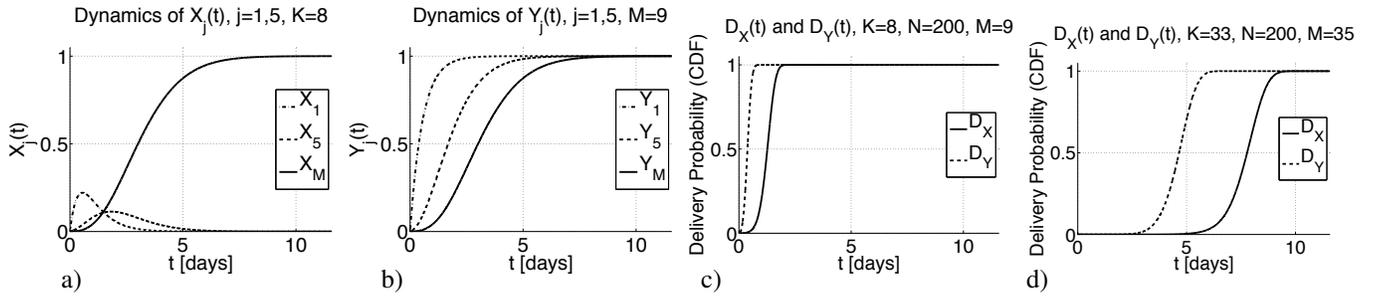


Fig. 1. (a) Dynamics of $X_j(t)$, (b) Dynamics of $Y_j(t)$ for $j = 1, 5, 8$ and (c) message delivery probability $D_X(t)$ and $D_Y(t)$ for $K = 8 = M - 1$. (d) Message delivery probability $D_X(t)$ and $D_Y(t)$ for $K = 33 = M - 2$.

least i packets, i.e., $Y_i(t) = \sum_{j=i}^M X_j(t)$. Hence it follows that $X_i(t) = Y_i(t) - Y_{i+1}(t)$. The success probability calculates similarly to what done before in the uncoded case.

$$\begin{aligned} dD_Y &= \sum_{i=0}^{M-1} \mathbb{P}(t < T_M \leq t + dt, y_d(t) = i) \\ &= dt N \lambda \sum_{i=0}^{M-1} \bar{p}_{M-i} \left(\frac{1}{N} + Y_{M-i}(t) \right) \frac{(Y_i(t) - Y_{i+1}(t))}{1 - Y_M(t)}. \end{aligned}$$

where we recall that $\bar{p}_i = \mathbb{P}(T_c \geq t_i)$. The separable expression integrates to provide the final expression

$$\begin{aligned} D_Y(t) &= 1 - \exp \left(N \lambda \sum_{i=0}^{M-1} \int_0^t ds \bar{p}_{M-i} \left(\frac{1}{N} + Y_{M-i}(s) \right) \right. \\ &\quad \left. \cdot \frac{(Y_i(s) - Y_{i+1}(s))}{1 - Y_M(s)} \right) \end{aligned}$$

where in the geometric case $\bar{p}_{M-i} = p^{M-i}$.

V. NUMERICAL RESULTS

In this section we provide a numerical characterization of the models obtained before. We reported in Fig. 1 the behavior of the sequential transmission and of fountain coding schemes predicted by the model in the case of two sample number of packets, i.e., $K = 8$ and $K = 33$ and for the same value $p = 0.4$. In particular, in Fig. 1a) we observe the transient behavior of the fraction of infected nodes having $j = 1, 5$ and $K = 8$ packets for sequential delivery and in Fig. 1b) the predicted dynamics of the corresponding Y_i variables when using fountain codes. As it can be seen there, all X_i s for $0 < i < K$ peak in order and then decrease as expected; conversely, the Y_i variables are monotonically increasing. Thus, despite the final number of infected $X_M = Y_M$ is the same for both the schemes, the coded scheme provides a sharp improvement: the key gain in performance is indeed due to the fact that the coded forwarding scheme does not constrain packets forwarding to occur in order and thus enables much faster message reconstruction at the destination node.

In particular, we can compare the behavior of the dynamics in the case of $K = 8$ and $K = 33$, Fig. 1c) and Fig. 1d), respectively: clearly, as from Thm. 3.2, since we used $p_8 = p_{35} = 0.4$, the increase in the number of packets has a detrimental effect in the delivery probability. But, we can see

that the relative gain of coding combined with the unordered delivered of packets provides indeed a larger relative gain for larger number of packets.

The effect of p on the delivery probability is reported in Fig. 2a): the increase of p from 0.1 to 0.4 configures as an acceleration of the dynamics, which in turn reflects onto the delivery probability. We observe again how the gain of D_Y over D_X in terms of delivery probability is much larger for lower values of p , which follows from the fact that the delivery is unordered in the coded scheme. Finally, the effect of the increase of p is reported for a given value of $x = \lambda\tau$: in that case, we can observe a sharp transition of the delivery probability as a function of p , i.e., the packet transmission probability per contact. This result resembles phase-transitions effects showed in [2], where the parameter was actually the forwarding control operated at the source node in order to reduce the number of infected nodes and spare network resources. However, in that context nodes were modeled as having single packet buffer.

VI. CONCLUSIONS AND DISCUSSION

In this paper we have presented an analysis of the combined effect of buffering and fragmentation in DTNs. We considered the case when sequential packet forwarding is operated after a message is fragmented at the source node into K packets: this is done in order to cope with finite contact durations. The model can be extended immediately to the case when fountain codes are employed at the source node in order to increase the delivery probability.

Given a certain contact duration distribution, the success probability can be expressed in closed form, providing an explicit relation with the probability of transmission of a packet during a single contact, which is a local property of the contact pattern, and the full message delivery probability, which is a global property of the forwarding process. In turn, this relates to the optimal fragmentation level which becomes a function of the contact duration distribution through the overhead for single packet transmission.

Finally, in the derivation we leveraged on simple memoryless models for the intermeeting process, i.e., intermeeting times were assumed memoryless. But, general contact time duration distribution could be accounted for: for instance, depending on the mobility, contact durations may possess

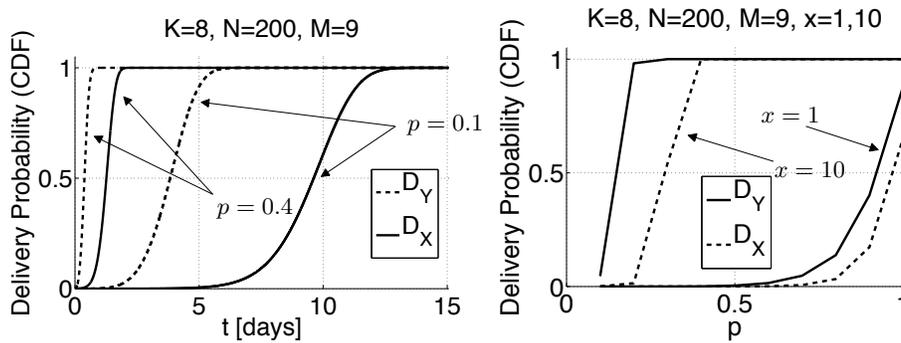


Fig. 2. a) Dependence of D_X and D_Y on the per packet transmission probability $p = 0.1$ and $p = 0.4$, respectively. b) Dependence of D_X and D_Y on the packet delivery probability p for two values of $x = \lambda\tau$

heavy-tail distributions or finite support distributions. For such a reason, an interesting research direction would extend the monotonicity result on the impact of fragmentation onto the message delivery probability to the case of general intermeeting times. In the general case, in fact, in order to understand the impact of the distribution of contact durations on the intermeeting process, one should be able to characterize the interplay of the local and global effect. When the exponential distribution rules the local effect onto the increase of delivery probability, we found that all contact times distributions are equivalent as long as the evaluation corresponding to the overhead coincides. In the case of general intermeeting processes, one should determine how the p_K should increase in order to make fragmentation convenient. Future work will validate the model for general mobility and contact traces to compare the cases where such assumptions of the model hold, e.g., in the case of RWP, and where they do not, e.g., real-world traces.

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