Detailed Analysis for IEEE 802.11e EDCA in Non-saturated Conditions
– Frame-Transmission-Cycle Approach –

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Abstract—Theoretical analysis of the IEEE802.11e enhanced distributed channel access (EDCA) mechanism is now a mature research theme and is receiving considerable attention. Most previous studies have focused on the saturation throughput, but typical networks are non-saturated in practice. We herein propose an analytical model for IEEE 802.11e EDCA under non-saturated conditions based on the frame-transmission-cycle approach. The approach considered herein focuses on the status of each station at the start of the frame-transmission cycle, and describes the state transition by a Markov chain. Through comparison with simulation results obtained by ns2, we confirm that the proposed model accurately estimates the throughput performance of EDCA-based WLANs in which voice and best effort traffic are multiplexed.

Index Terms—wireless LAN, IEEE802.11e, EDCA, performance analysis, non-saturated

I. INTRODUCTION

The IEEE 802.11 has gained widespread popularity as a standard physical/MAC-layer protocol for wireless local area networks (WLANs). The IEEE 802.11 standard defines the Distributed Coordination Function (DCF) as a contention-based MAC mechanism, but the DCF does not have Quality-of-Service (QoS) functionality. The IEEE 802.11 standard group has specified the 802.11e standard to add a set of QoS enhancements to the original 802.11 protocol. In the IEEE 802.11e, the enhanced distributed channel access (EDCA) corresponds to a QoS enhancement of the DCF of IEEE 802.11.

The IEEE 802.11e classifies traffic into four access categories (ACs): voice, video, best effort, and background. ACs with higher priorities wait for shorter periods of time on average before frame transmission. This prioritization is achieved using a combination of the following parameters of the EDCA:

- arbitration inter-frame space (AIFS),
- minimum contention window size \((CW_{\min})\) and maximum contention window size \((CW_{\max})\),
- transmission opportunity (TXOP) limit.

If the above-mentioned parameters are ideally configured, the EDCA should provide better QoS than the DCF. In order to determine a better configuration of the parameters of the EDCA, an analytical model, which allows us to evaluate the performance under a given configuration of the EDCA parameters, is highly desired. Several analytical models have been proposed to evaluate the saturation throughput of the EDCA, but there is no decisive analytical model for EDCA-based WLANs with stations under non-saturated conditions, in which stations have no data ready to send. The purpose of the present study is to propose an analytical model for the EDCA with non-saturated stations. We also focus on wireless LANs, in which Voice over IP (VoIP) and best-effort traffic are multiplexed, and we apply the proposed analytical model to determine how the EDCA provides different levels of QoSs to VoIP and best-effort traffic.

The remainder of the present paper is organized as follows. In Section II, we present related research. In Section III, we explain the proposed analytical model, which is validated by comparison with simulation results in Section IV. In Section V, we present concluding remarks.

II. RELATED RESEARCH

The performance of the IEEE802.11 and IEEE802.11e EDCA has been widely studied. Bianchi [1] first proposed a two-dimensional Markov chain model by which to analyze the performance of the IEEE 802.11 DCF under saturation conditions. Bianchi’s approach focuses on the per-slot status of stations, especially the probability with which the station will start to send a frame. Most models for EDCA performance analysis originated from this approach.

Robinson et al. [2] proposed an extension of the slot-based model of Bianchi for the analysis of IEEE 802.11e EDCA under saturated conditions. They introduced the concept of contention zones to the model in order to take into consideration the fact that different ACs experience different collision probabilities. Kong et al. [3] and Inan et al. [4] took a different approach to evaluate the performance of IEEE 802.11e EDCA. They used a three-dimensional Markov chain, in which the status of a station was described in terms of three parameters: backoff stage, backoff counter, and the remaining AIFS time. Hui et al. [5] proposed an analytical model for the EDCA, called the generalized P-persistent CSMA/CA model, which approximately describes the behavior of stations in the EDCA by a classical p-persistent-like CSMA/CA model. These models make a common assumption that stations are saturated, which is not always the case in practice. Chen et al. [6] proposed an analytical model for the EDCA with...
non-saturated stations, but their model does not take into consideration the difference of AIFS.

Tinnirello et al. [7] developed a new model using an approach different from that of traditional per-slot modeling. They introduced the concept of the frame-transmission cycle and computed the stationary distribution of the status of each individual station at the beginning of the frame-transmission cycle. Although their model can easily take into account the difference of AIFS values of stations, their model only works under saturated conditions.

Ergen et al. [8] proposed a mathematical model for the EDCA by extending Bianchi’s DCF model. They introduced additional states to Bianchi’s Markov chain to represent idle states of a station. Malone et al. [9] developed a different extension of Bianchi’s DCF model that allows stations to have different packet-arrival rates. However, these models are not applicable to the EDCA.

III. Analytical Model

A. Frame Transmission Cycle

Consider the case in which $N$ stations, including the access point, contend for a wireless channel. In such a case, the time axis consists of two periods: one is the idle period, in which stations pause or decrease their backoff counters, and the other is the transmission period, in which a station successfully sends a frame or more than one station sends frames, resulting in a collision. An idle period and the following transmission period compose a frame transmission cycle [5], [7]. For example, in Fig. 1, the $i$th cycle starts just after station C successfully transmits a frame. When station A has successfully transmitted a frame, the $i$th cycle ends, and, at the same time, the $(i+1)$th cycle starts. More precisely, the frame transmission cycle is defined as a period that starts (and ends) when two slots have passed after the transmission of an ACK frame (Fig. 1). According to this definition, station $k$, the AIFS of which is larger than two, must wait for $\delta_k$ (defined below) slots before resuming its backoff counter.

$$\delta_k \overset{\text{def}}{=} AIFS_k - 2,$$

where $AIFS_k$ is the AIFS of station $k$.

B. Basic Assumptions

In [7], Tinnirello et al. presented an analytical model for the EDCA, using the concept of the frame transmission cycle. In the model, the state of station $k$ at the start of the $r$th transmission cycle was expressed in terms of two parameters: backoff stage $s_k(t)$ and backoff counter $b_k(t)$. They described the transition from $\{s_k(t), b_k(t)\}$ to $\{s_k(t+1), b_k(t+1)\}$ by a Markov chain and numerically obtained its stationary distribution, which in turn was used to obtain the saturation throughput. Their model uses a key assumption that the behavior of a station is statistically independent of other stations contending for access to the channel. The time evolution of a target station is governed by the Markov chain dedicated to the target station, which is independent of the time evolution of other stations. This assumption is known as decoupling approximation [1] or fixed point analysis [10]. Note that the transition probability of the Markov chain of station $k$ depends on the stationary distributions of the Markov chains of other stations. In this sense, stations contending for the common wireless channel are mutually dependent through the transition probabilities of their Markov chains.

In contrast to Tinnirello’s model, in which all stations are saturated, the model proposed herein needs to explicitly consider the arrival process of frames. In order to describe the transition from $\{s_k(t), b_k(t)\}$ to $\{s_k(t+1), b_k(t+1)\}$ by a Markov chain, the frame arrival process to station $k$ should be memoryless. Therefore, in the present study we assume that frames arrive according to a Poisson process. In the proposed model, each station could be in a post backoff state, in which the station has no data to send. We use the backoff-stage value to indicate whether station $k$ is in a post-backoff state. If $s_k(t) = -1$, station $k$ is in a post-backoff state at the start of the $r$th cycle; otherwise, station $k$ is in a backoff state.

We conventionally assume that the time axis is divided into slots, which are numbered sequentially, 0, 1, 2, ..., in each frame-transmission cycle. Even if a station is in a post-backoff state at the start of the frame-transmission cycle, the station transits to a backoff state during the current cycle when a new frame is received. Since frames are assumed to arrive according to a Poisson process, the probability that station $k$ receives the first frame at slot $i$ in a frame-transmission cycle is equal to $f_e(i)$ defined below:

$$f_e(i) \overset{\text{def}}{=} e^{-\lambda_k \sigma} - e^{-\lambda_k (i+1) \sigma} = e^{-\lambda_k \sigma} (1 - e^{-\lambda_k \sigma}),$$

where $\sigma$ is the slot-time (20 $\mu$s) and $\lambda_k$ is the frame arrival rate at station $k$. For convenience in the numerical analysis, we further assume that the inter-arrival time of frames is bounded by $L$, i.e.,

$$f_e(i) = \begin{cases} 
    e^{-\lambda_k \sigma} (1 - e^{-\lambda_k \sigma}) & \text{for } 0 \leq i < L, \\
    e^{-\lambda_k L \sigma} & \text{for } i = L, \\
    0 & \text{otherwise.}
\end{cases}$$
The cutoff time $L$ was set to 5,000 when we conducted the numerical analysis described in Section IV. We assume an ideal channel condition under which transmission failure occurs only due to collision, but the extension to cases in which transmission failure also occurs due to channel error is straightforward.

**C. Transition Probabilities of Markov Chain**

In this subsection, we derive the state transition probabilities of the Markov chain of a target station (station $k$). In order to derive the expression of the transition probabilities, we introduce the following variables:

$-Q_s(i)$: probability that no station other than station $k$ starts a frame transmission prior to slot $i$.

$-T_k(i)$: probability that (at least) one station other than station $k$ transmits a frame at slot $i$; $T_k(i)$ is related to $Q_s(i)$ through the following equation:

$$T_k(i) = Q_s(i) - Q_k(i + 1).$$

$-R_k$: retry limit. If the frame transmission of a station fails $R_k$ times, the frame is discarded.

$-W_{k,s}$: contention window size of station $k$ when its backoff stage is $s$.

$-r_s$: probability that station $k$ keeps the next frame to send after a successful frame transmission or a frame discard because the retry limit has been exceeded.

$-D_{k,acc} (D_{k,off})$: duration of a successful frame transmission (frame collision). We assume that $D_{k,acc} = D_{k,off}$.

Moreover, we use the following notation:

$$F_s(i) = \sum_{j=0}^{i-1} f_s(j), \quad F_s(i) = 1 - F_s(i), \quad D_s = E[D]/\sigma,$$

where $E[D]$ denotes the expectation of the duration of a successful transmission or collision. The calculation for $E[D]$ will be discussed in Section III-F. If all frames are of the same size, $E[D] = D_{k,acc} = D_{k,off}$.

Let $P(s, i; s, j)$ denote the transition probability from $[s_k(t) = s, b_k(t) = i]$ to $[s_k(t+1) = u, b_k(t+1) = j]$. The derivation of the state transition probabilities $P(s, i; u, j)$ is rather complicated and is summarized in the Appendix. Here, as an example, we present the expression of $P(s, i; s, j)$ for $s > 0$, i.e., the probability of transition from a backoff state to another backoff state with the same backoff stage.

First, we consider the case of $s > 0$. If $i > j$, such a transition event occurs only when (at least) one station other than station $k$ transmits a frame at slot $i-j-1+\delta_k$ and thus the $r$th cycle ends just after the backoff counter of station $k$ reaches $j$ (Fig. 2). Since one station other than station $k$ transmits a frame at slot $j-i+1+\delta_k$ with probability $T_k(j-i+1+\delta_k)$, we have

$$P(s, i; s, j) = T_k(j-i+1+\delta_k), \quad \text{for } i > j.$$ 

If $i = j$, the transition is allowed only when one of other stations sends a frame prior to slot $\delta_k$ and then station $k$ does not resume its backoff counter (Fig. 3). Thus, we have

$$P(s, i; s, i) = \sum_{r=0}^{\delta_k-1} T_k(r) = 1 - Q_k(\delta_k).$$

For the case of $s = 0$, we need to consider the following additional transition event, in which station $k$ successfully transmits a frame at the end of the $r$th cycle, and a new backoff counter value is set to $j$. This transition event occurs when the following three conditions are jointly satisfied. (1) No other station sends frames prior to slot $i+\delta_k + 1$, the probability of which is $Q_k(i+1+\delta_k)$. (2) Station $k$ has the next frame to send just after the successful frame transmission, the probability of which is $r_s$. (3) The new backoff-counter value after the frame transmission is $j$, the probability of which is $1/(1+W_{k,0})$. Since this additional event occurs with probability $r_sQ_k(i+1+\delta_k)/(1+W_{k,0})$, for $i > j$, we finally obtain

$$P(0, i; 0, j) = T_k(j-i+1+\delta_k) + r_sQ_k(i+1+\delta_k)/(1+W_{k,0}),$$

and

$$P(0, i; 0, i) = 1 - Q_k(\delta_k) + r_sQ_k(i+1+\delta_k)/(1+W_{k,0}).$$

In summary,

$$P(s, i; s, j) \begin{cases} T_k(j-i+1+\delta_k) & \text{for } i > j, s > 0, \\ 1 - Q_k(\delta_k) & \text{for } i = j, s \neq 0, \\ T_k(j-i+1+\delta_k) + r_sQ_k(i+1+\delta_k)/(1+W_{k,0}) & \text{for } i > j, s = 0, \\ 1 - Q_k(\delta_k) + r_sQ_k(i+1+\delta_k)/(1+W_{k,0}) & \text{for } i = j, s = 0. \end{cases}$$

**D. Balance Equations**

Let $\pi(s, j)$ denote the stationary distribution of the Markov chain, i.e.,

$$\pi(s, j) = \lim_{t \to \infty} P[s_k(t) = s, b_k(t) = i].$$

![Fig. 2. Transition from $[s_k(t) = s, b_k(t) = i]$ to $[s_k(t+1) = s, b_k(t+1) = f]$](image)

![Fig. 3. Transition from $[s_k(t) = s, b_k(t) = i]$ to $[s_k(t+1) = s, b_k(t+1) = i]$](image)
The stationary distribution can be computed by solving the following balance equations:

[Transition to a post backoff state]

\[
\pi_k(-1, j) = \pi_k(-1, j)F_e(\delta_k + D_s)[1 - Q_k(\delta_k)] \\
+ \sum_{i=1}^{W_{k,i}} \pi_k(-1, i)F_e(i + 1 + \delta_k + D_s)T_k(i + 1 + \delta_k) \\
+ \frac{1 - r_k}{1 + W_{k,0}} \left\{ \sum_{i=0}^{R_k - 1} \pi_k(R_k, i)Q_k(i + \delta_k) \\
+ \sum_{i=0}^{W_{k,0}} \pi_k(s, i)Q_k(i + 1 + \delta_k) \right\} \\
+ \frac{1}{1 + W_{k,0}} \sum_{i=0}^{W_{k,0}} (i + \delta_k)\pi_k(-1, i) \\
\times f_e(i + \delta_k - 1 + D_s)Q_k(i + 1 + \delta_k) \\
+ \frac{1}{1 + W_{k,0}} \sum_{i=0}^{W_{k,0}} \pi_k(-1, i)F_e(D_s) \sum_{l=i+D_s}^{L} f_e(l)Q_k(l + 2) \\
+ 1(j = 0) \sum_{i=0}^{W_{k,0}} \pi_k(-1, i) \sum_{l=i+D_s}^{L} F_e(l + D_s)T_k(l),
\]  

(2)

where \(1(A)\) is the indicator function that is equal to 1 (0) when \(A\) is true (false).

[Transition to a backoff state (backoff stage = 0)]

\[
\pi_k(0, j) = (\pi_k(0, j) + \pi(-1, j)F_e(\delta_k + D_s))[1 - Q_k(\delta_k)] \\
+ \sum_{i=1}^{W_{k,i}} (\pi_k(0, i + 1) + 1(j \neq 0)\pi(-1, j + i)F_e(i + \delta_k + D_s - 1) \\
+ 1(j = 0)\pi(-1, i)F_e(i + \delta_k)T_k(i + 1 + \delta_k) \\
+ \frac{r_k}{1 + W_{k,0}} \left\{ \sum_{i=0}^{W_{k,0}} \pi_k(R_k, i)Q_k(i + \delta_k) \\
+ \sum_{i=0}^{W_{k,0}} \pi_k(s, i)Q_k(i + 1 + \delta_k) \right\} \\
+ \frac{1}{1 + W_{k,0}} \sum_{i=0}^{W_{k,0}} \pi_k(-1, i)Q_k(i + 1 + \delta_k) \\
\times \sum_{l=i+\delta_k}^{D_s} f_e(l)F_e(i + \delta_k - l - 1 + D_s) \\
+ \frac{1}{1 + W_{k,0}} \sum_{i=0}^{W_{k,0}} \pi_k(-1, i)F_e(D_s) \sum_{l=i+D_s}^{L} f_e(l)Q_k(l + 2) \\
+ \frac{1}{1 + W_{k,0}} \sum_{i=0}^{W_{k,0}} \pi_k(-1, i) \sum_{l=i+\delta_k}^{D_s} f_e(l) \sum_{m=(l-1)/\delta_k+(l-D_s)}^{l} T_k(m),
\]

(3)

[Transition to a backoff state (backoff stage = 1)]

\[
\pi_k(1, j) = \pi_k(1, j)[1 - Q_k(\delta_k)] \\
+ \sum_{i=1}^{W_{k,i}} \pi_k(1, i)Q_k(i - 1 + \delta_k) \\
+ \frac{1}{1 + W_{k,1}} \sum_{i=0}^{W_{k,0}} \pi_k(0, i)Q_k(i + \delta_k) \\
+ \frac{1}{1 + W_{k,1}} \sum_{i=0}^{W_{k,0}} \pi_k(-1, i) \\
\times \left\{ F_e(i + \delta_k)T_k(i + \delta_k) + \sum_{l=i+\delta_k}^{L} f_e(l)T_k(l + 1) \right\}.
\]

(4)

[Transition to a backoff state (backoff stage \(s \geq 2\)]

\[
\pi_k(s, j) = \pi_k(s, j)[1 - Q_k(\delta_k)] \\
+ \sum_{i=1}^{W_{k,i}} \pi_k(s, i)Q_k(i - 1 + \delta_k) \\
+ \sum_{i=0}^{W_{k,0}} \pi_k(s - 1, l)T_k(i + \delta_k).
\]

(5)

In these equations, \(Q_k(i)\) and \(T_k(i)\) depend on the stationary distributions of other stations. In order to obtain expressions for \(Q_k(i)\) and \(T_k(i)\), we define the following variable:

\[
B_{n}(j) \overset{\text{def}}{=} \sum_{i=0}^{R_n} \pi_n(s, i) + \sum_{i=1}^{\delta_n} \pi_n(-1, l)f_e(j + \delta_n - 1) \\
+ \sum_{i=1}^{\delta_n} \pi_n(-1, j)F_e(j + \delta_n).
\]

Note that \(B_{n}(j)\) is the probability that station \(n\) attempts frame transmission at slot \(j + \delta_n\). The probability that station \(n\) attempts frame transmission prior to slot \(i\), denoted by \(\beta_n(i)\), is given by:

\[
\beta_n(i) \overset{\text{def}}{=} \sum_{j=0}^{\delta_n - 1} B_{n}(j) \quad i > \delta_n.
\]

Using \(\beta_n(i)\), \(Q_k(i)\) is expressed as

\[
Q_k(i) = \frac{\prod_{n=1}^{N}[1 - \beta_n(i)]}{1 - \beta(i)},
\]

(6)

from which we obtain \(T_k(i)\) using (1).

Remark 1: If we apply an approximation whereby each station in a post-backoff state sends a frame just after its backoff counter expires, we can obtain a much simpler balance equation [11]. Applying the approximation is equivalent to discarding the uniform backoff-counter-value selection for post-backoff stations. Instead, new backoff counter values of post-backoff stations are probabilistically selected so as to follow an exponential distribution. This approximation yields only a very small difference in the numerical results.
E. Calculation of Stationary Distribution

The stationary distribution can be numerically obtained in the manner described in [7]. Starting from an initial assumption on \( \{\pi_k(s, j)\} \), we compute \( Q_k(i) \) and \( T_k(i) \) from (1) and (6). These parameters are used to compute the stationary distribution \( \{\pi_k(s, j)\} \) through the set of linear equations (2), (3), (4), and (5) with the normalization condition:

\[
\sum_{j=0}^{L} \pi_k(-1, j) + \sum_{i=0}^{L} \sum_{j=0}^{W_k} \pi_k(s, j) = 1.
\]

The obtained stationary distribution is in turn used to compute \( Q_k(i) \) and \( T_k(i) \). This cycle is repeated until a convergence criterion is met.

F. Throughput Evaluation

The throughput of station \( k \) is given as

\[
Throughput_k = \frac{P_k^{\text{succ}} \cdot DATA}{E[CL]},
\]

where

\[
P_k^{\text{succ}} = \sum_{i=0}^{L} B_k(i) Q_k(i + 1 + \delta_k)
\]

is the probability that station \( k \) successfully transmits a frame in a cycle. \( DATA \) is the mean payload length of data frames, and \( E[CL] \) denotes the mean duration of a cycle given by

\[
E[CL] = E[IL] \sigma + E[D],
\]

where \( IL \) is the length (in slots) of the idle period in a cycle. Note that \( E[IL] \) is obtained by the following calculation:

\[
E[IL] = \sum_{j=1}^{L} P[I\L \geq j] = \sum_{j=1}^{N} \prod_{k=1}^{L} (1 - \beta_k(j)).
\]

If all frames transmitted in the WLAN are of the same size, \( E[D] = D_k^{\text{succ}} \); otherwise

\[
E[D] = \sum_{k=1}^{N} P_k^{\text{succ}} D_k^{\text{succ}} + \sum_{r=2}^{N} \sum_{2 \leq k_1 < \cdots < k_r \leq N} \max\{D_k^{\text{succ}}, \ldots, D_{k_r}^{\text{succ}}\} \cdot P_{k_1 \cdots k_r}^{\text{coll}},
\]

where

\[
P_{k_1 \cdots k_r}^{\text{coll}} = \prod_{j=1}^{r} P_j^{\text{succ}} \prod_{j=k_1, \ldots, k_r} P_j^{\text{coll}} = \sum_{i=0}^{L} B_k(i) T_k(i + \delta_k).
\]

Note that the mean inter-frame-transmission time of station \( k \) should be equal to the mean inter-arrival time of frames at station \( k \) if the transmission buffer of station \( k \) is not overloaded (i.e., the queue in the transmission buffer is stable), and thus

\[
\lambda_k \cdot DATA = Throughput_k,
\]

or

\[
\lambda_k = P_k^{\text{succ}} / E[CL].
\]

The right-hand side of (8) is an increasing function of \( r_k \), and we can numerically determine the value of \( r_k \) such that (8) holds. If the right-hand side of (8) is lower than \( \lambda_k \) when \( r_k = 1 \), then station \( k \) is overloaded (saturated) and frames are lost due to the overflow of the transmission buffer of station \( k \) [12].

IV. SIMULATION EXPERIMENTS

In this section, we validate the accuracy of the proposed analytical model through simulation experiments using the ns2 network simulation tool under two different scenarios.

A. VoIP Only

In the first simulation, a wireless station conducted a bidirectional voice conversation through the AP with a node outside the WLAN. If there were \( n \) wireless stations conducting the communication, there should be \( n \) uplink and \( n \) downlink voice flows. Each voice flow generated G.711-codec traffic, and a 200-byte packet (160-byte data and 40-byte RTP/UDP/IP header) was generated every 20 ms, meaning that the data rate of each voice flow was 64 kbps.

We first conducted the simulation using IEEE 802.11 DCF as a MAC layer protocol. Figure 4 shows the application-level throughputs defined by (7), where we exclude the lengths of PHY, MAC, IP, UDP, and RTP headers from the length of the data frame. The throughput obtained by our analytical model was almost identical to the results obtained by simulation. Figure 4 also shows that uplink flows obtained larger throughput than downlink flows, which is typical of the unfairness between uplink and downlink flows caused by the congestion of the AP.

Figure 5 shows the throughput performance when VoIP traffic was transmitted in the voice category of the EDCA (\( CW_{min}=7, CW_{max}=15; \text{AIFS}=2 \)). The throughput performance was found to be worse than in the DCF case (Fig. 4). In particular, the performance deteriorated significantly when the number of stations was larger than fifteen. This result indicates...
that the voice category of the EDCA is not a good choice when only voice traffic is multiplexed in the WLANs.

Figure 6 shows the results when the AP transmitted traffic in the voice category, while wireless stations transmitted traffic in the best effort category of the EDCA \( (CW_{\text{min}}=31, CW_{\text{max}}=1023; \text{AIFS}=3) \). Under this configuration, the uplink and downlink voice flows achieved almost the same level of throughput performance, significantly mitigating the unfairness between uplink and downlink flows.

**B. VoIP with UDP flows**

Next, we conducted another simulation experiment in more practical environments, in which wireless stations sending UDP traffic used WLANs together with stations sending VoIP traffic. Bidirectional voice traffic was transmitted between the AP and wireless stations as in the first simulation, whereas the WLAN also transmitted uplink saturated UDP flows (frame size: 1,500 bytes).

Figure 7 compares the sum of throughputs of uplink voice flows, the sum of throughputs of downlink voice flows, and the sum of throughputs of saturated UDP flows when IEEE 802.11 DCF was used as the MAC-layer protocol. As shown in the figure, the throughput computed by our model agrees well with the simulation results. Since no prioritization was applied, the saturated UDP flows occupied most of the wireless bandwidth.

Figure 8 shows the results when voice flows were transmitted in the voice category while saturated three UDP flows were transmitted in the best-effort category of the EDCA. The total throughput of saturated UDP flows was lower than in the DCF case because the bandwidth occupied by the saturated UDP flows was restricted by lower priority parameters. The figure shows that as the number of voice flows increased, opportunities for the UDP flows to send their frames became sparse.

Figure 9(a) shows the average throughput of a voice flow for the DCF condition. The figure shows that a downlink voice flow did not achieve 64-kbps throughput on average when more than two voice flows were multiplexed, meaning that the QoS of voice conversation was not guaranteed. Figure 9(b) shows the results obtained for the EDCA case, where a downlink voice flow achieved 64-kbps throughput if the number of voice flows was less than eight. This result verifies EDCA as effective in suppressing saturated background UDP traffic to provide better QoS for voice traffic.

**V. Conclusion**

In the present paper, we proposed an analytical model based on a frame-transmission-cycle approach, which allows us to
evaluate the performance of each flow under the IEEE 802.11e EDCA with non-saturated stations, and we compared the throughput performance achieved by the proposed model with the simulation results in order to verify the accuracy of the proposed model. Using the proposed model, it is easy to see how the EDCA parameters affect the network performance under a relatively practical condition (in which voice and UDP flows exist simultaneously). The proposed model is also expected to be useful in the evaluation of the performance of TCP flows, which are multiplexed over a WLAN with streaming applications. Analysis of the TCP throughput over a WLAN remains an important task.

**Appendix**

**A. Transition probabilities** \(P(s,i; s + 1, j)\) \((0 \leq s < R_k)\)

The transition from \(\{s_k(t) = s, b_k(t) = i\}\) to \(\{s_k(t) = s + 1, b_k(t) = j\}\) occurs if the following two conditions hold jointly: 1) the transmission of station \(k\) fails due to a collision, and 2) the new backoff-counter value after the collision of station \(k\) is \(j\). The first condition holds when at least one station other than \(k\) sends a frame at slot \(j + \delta_k\), and its probability is equal to \(T_k(j + \delta_k)\). The second condition holds with probability \(1/(1 + W_{k,s+1})\). Thus, we obtain

\[
P(s,i; s + 1, j) = \frac{1}{1 + W_{k,s+1}} T_k(i + \delta_k).
\]

**B. Transition probabilities** \(P(s,i; -1, j)\) and \(P(s,i; 0, j)\)

We first consider the case in which \(0 \leq s < R_k\). The transition from \(\{s_k(t) = s, b_k(t) = i\}\) to \(\{s_k(t) = -1, b_k(t) = j\}\) or \(\{s_k(t) = 0, b_k(t) = j\}\) is allowed when station \(k\) successfully transmits a frame. The probability that station \(k\) successfully transmits a frame is equal to \(Q_k(i + 1 + \delta_k)\), as shown in Section III-C. After the frame transmission, station \(k\) enters a backoff state with probability \(r_k\) or enters a post-backoff state with probability \(1 - r_k\), depending on whether station \(k\) keeps the next frame to be sent. A new backoff-counter value is set at \(j\) with probability \(1/(1 + W_{k,0})\). Thus, we have

\[
P(s,i; -1, j) = \frac{1}{1 + W_{k,0}} (1 - r_k) Q_k(i + 1 + \delta_k).
\]

\[
P(s,i; 0, j) = \frac{1}{1 + W_{k,0}} r_k Q_k(i + 1 + \delta_k).
\]

The transition from \(\{s_k(t) = R_k, b_k(t) = i\}\) to \(\{s_k(t) = -1, b_k(t) = j\}\) or \(\{s_k(t) = 0, b_k(t) = j\}\) occurs when the backoff counter of station \(k\) expires and then station \(k\) tries to transmit a frame, independent of whether the transmission is successful. The probability that the backoff counter of station \(k\) expires prior to the frame transmission of other stations is \(Q_k(i + \delta_k)\). Thus, we have

\[
P(R_k,i; -1, j) = \frac{1}{1 + W_{k,0}} (1 - r_k) Q_k(i + \delta_k).
\]

\[
P(R_k,i; 0, j) = \frac{1}{1 + W_{k,0}} r_k Q_k(i + \delta_k).
\]

**C. Transition probability** \(P(-1,i; -1,j)\)

We first consider the case in which \(j \neq 0\). There are three possible events that trigger the transition from \(\{s_k(t) = -1, b_k(t) = i\}\) to \(\{s_k(t) = -1, b_k(t) = j\}\) when \(j \neq 0\). The first event, in which the backoff counter of station \(k\) is not expired, consists of the following two sub-events: (1) one of stations other than station \(k\) sends a frame at slot \(i - j - 1 + \delta_k\), the probability of which is \(T(i - j - 1 + \delta_k)\); (2) no frame arrives at station \(k\) during the current frame-transmission cycle, the probability of which is \(\tilde{F}_{k}\). Thus, the probability of the first event is \(\tilde{F}_{k}(i - j - 1 + \delta_k + D_k)\). Hence, the probability of the first event is \(\tilde{F}_{k}(i - j - 1 + \delta_k + D_k)\). The second event, in which station \(k\) successfully sends a frame and begins the stage-0 backoff with a new backoff counter, consists of the following four sub-events: (1) a frame

\[
P(-1,i; -1,j) = \frac{1}{1 + W_{k,0}} (1 - r_k) Q_k(i + \delta_k).
\]

\[
P(R_k,i; 0, j) = \frac{1}{1 + W_{k,0}} r_k Q_k(i + \delta_k).
\]

\[
P(R_k,i; -1, j) = \frac{1}{1 + W_{k,0}} (1 - r_k) Q_k(i + \delta_k).
\]
arrives at station \( k \) at slot \( l \), where \( 0 \leq l \leq i + \delta_k - 1 \); (2) station \( k \) successfully sends the frame at slot \( i + \delta_k \), the probability of which is \( Q(i + \delta_k + 1) \); (3) no new frame arrives at station \( k \) from slot \( l + 1 \) to slot \( i + \delta_k + D_s - 1 \), the probability of which is \( F_c(i + \delta_k + D_s - l - 1) \); and (4) the new backoff counter value of station \( k \) is set to \( j \), the probability of which is \( 1/(1 + W_{k,0}) \).

Hence, the probability of the second event is
\[
\frac{1}{1 + W_{k,0}} \sum_{l=0}^{i+\delta_k-1} f_e(l)Q(i + \delta_k + 1)F_c(i + \delta_k + D_s - l - 1) = \frac{1}{1 + W_{k,0}}(i + \delta_k)Q(i + \delta_k + 1)f_e(i + \delta_k + D_s - 1).
\]

The third event is similar to the second event, but a frame arrives at station \( k \) after its backoff counter is expired. The third event consists of four sub-events: (1) a frame arrives at station \( k \) at slot \( l \), where \( i + \delta_k \leq l \leq L \); (2) station \( k \) successfully sends the frame at slot \( i + \delta_k + D_s \), the probability of which is \( Q(i + \delta_k + 2) \); (3) no new frame arrives at station \( k \) during its frame transmission, the probability of which is \( F_c(D_s) \); and (4) the new backoff counter value of station \( k \) is set to \( j \), the probability of which is \( 1/(1 + W_{k,0}) \). Hence, the probability of the third event is
\[
\frac{1}{1 + W_{k,0}}F_c(D_s)\sum_{l=i+\delta_k}^{L} f_e(l)Q_k(l + 2)
\]

When \( j = 0 \), in addition to the three events described above, we need to consider an additional event, in which no frame arrives at station \( k \) during the current cycle and the backoff counter is expired. This event occurs when the following sub-events jointly occur: (1) a station other than station \( k \) sends a frame at slot \( l \), where \( i + \delta_k \leq l \leq L \); (2) no new frame arrives at station \( k \) during the current frame-transmission cycle. The probability that this event occurs is
\[
\sum_{l=i+\delta_k}^{L} F_c(l + D_s)T_k(l).
\]

When \( i = j \), we need to consider one more event, in which station \( k \) does not resume its backoff counter within the current cycle and no frame arrives at station \( k \). The probability that this event occurs is \( \hat{F}_c(\delta_k + D_s)[1 - Q_k(\delta_k)] \). Thus, we finally have

\[
P(-1, i; -1, j) = \hat{F}_c(i + l - 1 + \delta_k + T)T_k(i - l + 1 + \delta_k) + \frac{1}{1 + W_{k,0}}(\delta_k + i)f_e(\delta_k + i - l + 1 + \delta_k)Q(i + 1 + \delta_k)
\]
\[
+ \frac{1}{1 + W_{k,0}}\hat{F}_c(T)\sum_{l=0}^{i+\delta_k+1} f_e(l)Q_k(l + 2) + 1(j = 0)\sum_{l=i+\delta_k}^{L} F_c(l)T_k(l) + 1(i = j)\hat{F}_c(\delta_k + D_s)[1 - Q_k(\delta_k)].
\]

D. Transition probability \( P(-1, i; 0, j) \)

There are basically three possible events that trigger the transition from \( \{s_k(t) = -1, b_k(t) = i\} \) to \( \{s_k(t) = 0, b_k(t) = j\} \).

The occurrence probability of the first event, in which the backoff counter of station \( k \) is not expired, is
\[
F_c(i - j + 1 + \delta_k + D_s)T_k(i - j + 1 + \delta_k).
\]

The occurrence probability of the second event, in which station \( k \) successfully sends a frame and begins the stage-0 backoff with a new backoff counter, is
\[
\frac{1}{1 + W_{k,0}}\sum_{l=0}^{i+\delta_k} f_e(l)F_c(i - l + 1 + D_s + 1)Q_k(i + 1 + \delta_k)
\]
\[
+ \frac{1}{1 + W_{k,0}}F_c(D_s)\sum_{l=0}^{i+\delta_k+1} f_e(l)Q_k(l + 2).
\]

The third event triggering the transition is the event in which station \( k \) receives a frame after its backoff timer has expired, but the wireless medium is busy. In such a case, station \( k \) begins the stage-0 backoff with new backoff counter. The probability of the third event is
\[
\frac{1}{1 + W_{k,0}}\sum_{l=0}^{i+\delta_k}\sum_{s=0}^{L} f_e(l)\sum_{m=0}^{L} T_k(m).
\]

When \( i = j \), we need to consider an additional event, in which station \( k \) does not resume its backoff counter within the current cycle, and its probability of occurrence is \( F_c(\delta_k + D_s)[1 - Q_k(\delta_k)] \). Thus, we finally obtain
\[
P(-1, i; 0, j) = F_c(i - j + 1 + \delta_k + D_s)T_k(i - j + 1 + \delta_k)
\]
\[
+ \frac{1}{1 + W_{k,0}}\sum_{l=0}^{i+\delta_k} f_e(l)F_c(\delta_k + i - l + 1 + D_s + 1)Q_k(i + 1 + \delta_k)
\]
\[
+ \frac{1}{1 + W_{k,0}}F_c(D_s)\sum_{l=0}^{i+\delta_k+1} f_e(l)Q_k(l + 2)
\]
\[
+ \frac{1}{1 + W_{k,0}}\sum_{l=0}^{i+\delta_k}\sum_{s=0}^{L} f_e(l)\sum_{m=0}^{L} T_k(m)
\]
\[
+ 1(i = j)F_c(\delta_k + D_s)[1 - Q_k(\delta_k)].
\]

E. Transition probability \( P(-1, i; 1, j) \)

The transition from \( \{s_k(t) = -1, b_k(t) = i\} \) to \( \{s_k(t) = -1, b_k(t) = j\} \) occurs when station \( k \) receives a new frame but the transmission fails due to a collision. When a frame arrives before the expiration of the backoff counter, the collision occurs if a station other than \( k \) transmits a frame at slot \( i + \delta_k \), when a frame arrives at slot \( l \), where \( l \geq i + \delta_k \), the collision occurs if a station other than \( k \) transmits a frame at slot \( l + 1 \). Thus, we obtain
\[
P(-1, i; 1, j) = \frac{1}{1 + W_{k,1}}\left( F_c(i + \delta_k)T_k(i + \delta_k) + \sum_{l=i+\delta_k}^{L} f_e(l)T_k(l + 1) \right).
\]