Energy-Efficient Multi-Hop Transmission for Machine-to-Machine Communications

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Abstract—Emerging machine-to-machine communication scenarios are envisioned to deal with more stringent quality-of-service demands. This relates mainly to outage and latency requirements, which are for example for safety-critical messages quite different than for traditional applications. On the other hand, it is widely accepted that machine-to-machine communication systems need to be energy-efficient because of the widespread use of battery-powered devices, but also due to their huge deployment numbers. In this paper, we address these issues with respect to multi-hop transmissions. Specifically, we deal with minimizing the consumed energy of transmitting a packet with end-to-end outage and latency requirements. We account for the cases in which the system can utilize solely average channel state information, or in addition obtain and profit from instantaneous channel state information. The developed solution is based on convex optimization. It is shown numerically that despite accounting for the energy consumption of acquiring instantaneous channel state information, especially as the outage and latency requirements become tough, it is by up to 100 times more energy efficient to convey a packet with instantaneous than with average channel state information.

I. INTRODUCTION

The human demand for communication has been the major source driving the evolution of wireless networks over the last decades. As communication over these networks became more and more popular, mass-market systems like cellular or local area networks evolved to meet this increasing demand. As a consequence, this has led to a steady increase in rate over the last 25 years. Associated with this increase in rate has been the ability of networks to support best-effort or delay-sensitive data flows as the major Quality-of-Service (QoS) classes.

In contrast, in the newly emerging area of machine-to-machine communication, we find scenarios with more challenging QoS requirements. In this case, the required level of reliability can become quite high, i.e., the acceptable probability that a message is corrupted is in the range of $10^{-5}$ and below while tight deadlines (in the range of milliseconds) have to be met. Applications with such QoS requirements are typically encountered in industrial automation or in different kinds of distributed control systems. To date, such communication is typically carried over wired networks due to reliability and security issues. However, recently there is more and more interest in feasible designs of wireless networks to substitute wired links in such application scenarios [1], [2]. Apart from the reliability requirements, it is also widely accepted that wireless networking solutions for machine-to-machine type of applications have to be energy-efficient. This is mainly due to the fact that networking devices might be battery-driven. Hence, energy-efficient operation ensures a long lifetime.

Surprisingly, energy-minimization under outage and latency requirements has not found much attention so far in related work. Especially regarding multi-hop networks, the typical assumption is that the sum transmit power is minimized while some end-to-end outage constraint needs to be fulfilled [3]–[5]. Such works typically show that transmit power can be saved if more hops are employed between a source and a destination. This implies, however, that the latency increases. As energy is the product of power and time, it is apparent that the implications for energy consumption under both outage and latency constraints are not clear from related work.

Hence, in this work, we study a relatively basic question: What is the minimum energy that it takes to transmit a packet of a certain size from a source to a destination such that on the one hand a certain end-to-end outage probability constraint is not violated, while on the other hand the transmission meets a given deadline. As mentioned, these parameters can become quite demanding in machine-to-machine type of applications for which high reliability levels have to be reached while meeting short deadlines. We study this question mostly with respect to multi-hop forwarding from a source to a destination. In this context, we study two different approaches: Forwarding the packet either based on average Channel State Information (CSI) or, instead, forwarding it based on instantaneous CSI. While in the first case the entire time span is available for payload transmission from source to destination, in the case of using instantaneous CSI the channel states have to be obtained first. This consumes time and transmit power (i.e., energy), but presents the nodes a big advantage for packet forwarding: The nodes are able to invest as much transmit power as necessary to achieve the QoS requirements, and therefore avoid doing power over-provisioning and wasting energy. Based on convex optimization, we develop for both cases the optimal allocation of transmit power along a multi-hop route, and afterwards we numerically study the comparison between both schemes. We show that the duration of the channel acquisition phase has a big impact on the energy consumption in case of exploiting instantaneous CSI. Moreover, for more demanding transmission scenarios (large packets, short deadlines, high
reliability requirements) the energy savings from working with instantaneous CSI are quite large (up to a factor of 100). Finally, we numerically show that for a given distance between source and destination there is an optimal number of hops to use with respect to minimizing transmit energy. Not using the optimal number of hops leads to a significant increase in the consumed energy.

Our work is structured as follows: In Section II, we present the system model and problem statement, and summarize related work. In Section III, we introduce the foundation of the power and energy minimization framework. Then, we present evaluations of the derived analytical and numerical expressions in Section IV. Section V concludes the paper.

II. PRELIMINARIES

In this section, we first present the system model. Then, we give a more formal description of the problem we are interested in and at the end discuss related work.

A. System Model

We consider the transmission of packets of size $D$ from a source to a destination over a set of $n$ links ($n-1$ intermediate nodes). All nodes in the system are static. The packets might belong to a flow, however, we only focus on a single packet transmission. This transmission is constrained by QoS parameters. Namely, we define by $P$ the required success probability of the transmission, and by $T$ an associated deadline. A successfully transmitted packet implies that it reaches the destination within the time span $T$ error-free. In all other cases (late arrival, bit errors) an outage occurs.

In order to forward the packet, all transceivers use certain resources. First of all, as there are $n$ links in the multi-hop route, each node can use a specific, bounded time for forwarding the packet to the next node. We refer to this time unit as slot in the following. During its slot, node $i$ utilizes a transmit power of $P_i$ to forward the packet. Finally, all nodes utilize the same bandwidth of $B$ Hz. This spectrum is not subject to any external interference.

The major source of unreliability in the network stems from the random behavior of the wireless links along the path. Forwarding link $i \in \{1, \ldots, n\}$ (from node $i$ to $i+1$) is characterized by an instantaneous channel gain $h_i^2$. This instantaneous channel gain is composed of an average channel gain $\bar{h}_i^2$ as well as a random fading component. The average channel gain $\bar{h}_i^2$ consists of a path loss factor and a random (but constant) shadowing factor. For the path loss we assume a straightforward model in which the gain is given by $d_i^{-\alpha}$ with $d_i$ being the distance between the transmitter and the receiver of link $i$. Link lengths $d_i$ are arbitrary and bridge a total distance $d$. For the shadowing component we assume a lognormal distribution with standard deviation $\sigma_{\text{SH}}$ and mean of $\mu_{\text{SH}} = 0$. Any instantaneous channel gain sample $h_i^2$ deviates from the average gain due to random small-scale fading. This fading is modeled by a stationary Rayleigh process such that the instantaneous Signal-to-Noise Ratio (SNR) $\gamma_i$ is an exponentially distributed random variable with mean

$$E[\gamma_i] = \bar{\gamma}_i = P_i \cdot \bar{h}_i^2 / \sigma^2,$$

where $\sigma^2$ denotes the noise power. We assume a slowly varying block-fading process such that over the time span $T$ the instantaneous channel gains remain constant.

Due to fading, a packet transmission is potentially subject to errors. If the transmitter $i$ does not have information about the instantaneous channel state $\gamma_i$, we account for the transmission errors by a threshold error model [6]. Given the random SNR $\gamma_i$ the corresponding error-free transport capacity $c_i$ of the link is a random variable as well. Taking the slot duration $T_i$ into account, the instantaneous transport capacity is given by

$$c_i = T_i \cdot B \cdot \log_2[1 + \gamma_i \beta].$$

Transport capacity $1$ represents the amount of data (in bits) that can be sent error-free for an SNR of $\gamma_i$ over the corresponding link and depends directly on the applied transmit power $P_i$. For the transmission of a packet of size $D$, the packet is lost on link $i$ whenever $c_i < D$. Based on the definition of transport capacity and the stochastic SNR model, we can derive the success probability $p_i$, which is the probability that the random transport capacity is bigger than the packet size $D$. To determine this, we first need to derive the probability density function (PDF) of $c_i$ based on the exponential distribution of the link SNR. This can be obtained by PDF transformation as

$$f_{c_i}(x) = \ln[2] \frac{2^{x/(T_i \cdot B)}}{T_i \cdot B \cdot \bar{\gamma}_i} \exp\left[-\frac{2^{x/(T_i \cdot B)} - 1}{\bar{\gamma}_i}\right].$$

Given this characterization of random transport capacity, the success probability $p_i$ is obtained as

$$p_i = \Pr \{c_i \geq D\} = \int_D^\infty f_{c_i}(x) \, dx = \exp\left[-\frac{1}{\bar{\gamma}_i} \left(2^{\sigma^2/(\bar{\gamma}_i \cdot B)} - 1\right)\right] = \exp\left[-\frac{K[D, T_i]}{P_i \cdot \bar{h}_i^2}\right],$$

in which $K[\cdot, \cdot]$ is a strictly positive scaling factor given as

$$K[\delta, \tau] = \sigma^2 \left(2^{\beta/(\tau \cdot B)} - 1\right).$$

B. Problem Statement

In this paper we are interested in fundamental insights how to transmit a packet over multiple hops in an optimal way. Optimality refers in the following to the total consumed energy whereas the transmission is constrained by the QoS pair $(P, T)$ of target success probability and deadline. As we consider the same slot duration $T_s$ for every link along the path, the total energy $E$ is directly given by

$$E = P \cdot T_s,$$

where $P$ denotes the sum of the individual transmit powers $P_i$. Energy necessary for packet reception (and idling) at each node is denoted by $D$. The $\beta$ factor accounts for different modulation / coding types [6]–[8], that practical systems are able to use. This gap factor can be used to match Shannon rates to practical systems. For the sake of simplicity, throughout the paper we assume $\beta = 1$. 

\[1\]
node will not be considered in our work as these values are strongly dependent on the implementation of the transceiver. However, note that such terms could be added to Eq. (4).

A key aspect to reduce the energy consumption is the availability of Channel State Information (CSI) at the nodes. There are multiple ways of characterizing the state of a channel. We define the CSI as the channel gain of a forwarding link, and focus on the average CSI and instantaneous CSI. By knowing the channel conditions, nodes are able to adjust the transmit power to the minimum level that still fulfills the QoS constraints. We assume that the source node possesses knowledge about all average channel states of the links along the route to the destination. This information might be available through a previous routing decision. Important in our consideration is that this information does not need to be acquired separately for each packet sent along the route, but is updated infrequently. Both approaches considered differ in the type of channel state information available to the nodes:

- **Average CSI Approach**: In this case, the source can determine transmit powers for all links only based on average channel states. The transmit powers have to be selected such that the required energy is minimized while satisfying the demanded end-to-end QoS constraints. Note that in this case all forwarding nodes have to perform significant power over-provisioning to cope with the random channel behavior. On the other hand, the entire time span up to the deadline can be dedicated exclusively to packet forwarding. We deal with energy minimization for this approach in Subsection III-A.

- **Instantaneous CSI Approach**: In this case, all links first acquire the actual channel states by exchanging control packets. Note that this consumes energy, as well as a certain amount of time from the overall time span. Once this information has been successfully acquired, the payload packet is sent from the source to the destination by adapting the transmission to the current channel states. We consider the energy minimization problem of this case in Subsection III-B.

A part of the novelty of our work comes from the consideration of both end-to-end success probability \( P \) and deadline \( T \) as QoS parameters. Applications for such a QoS model are machine-to-machine communications. In other QoS models, data rate is another important factor, e.g., for multimedia flows. In this work, data rate is sacrificed to a certain extent to meet the demanded high reliabilities and short deadlines, e.g., in safety critical applications. A major motivation for our investigations is given by such scenarios.

### C. Related Work

Optimizing power efficiency under QoS constraints in wireless networks has received significant attention over the last years [9]–[12]. In the context of multi-hop networks, only few works have been considering related issues. Efficient resource allocation schemes in wireless multi-hop networks are discussed in [3]–[5]. In [3], energy-constrained multi-hop links subject to an end-to-end outage constraint are considered. A closed-form expression for the minimum total transmit power is derived. The authors show an \( n \)-fold reduction of the total power for an \( n \)-hop route compared to a power distribution according to individual outage requirements. However, they do not consider an end-to-end deadline as QoS parameter nor do they include channel state information in the system model. In [4], the authors define an optimal power allocation scheme in a multi-hop amplify-and-forward system considering an end-to-end instantaneous SNR as target QoS requirement. The paper analyzes a QoS-aware multi-branch relaying power allocation problem which aims at minimizing the total power consumption of all transmission nodes. It demonstrates a gain of up to 5 dB when applying the optimal power allocation as compared to an equal power allocation over all links. However, no end-to-end deadline is considered. Power allocation for regenerative and non-regenerative relayed systems is investigated in [5]. The authors derive a closed-form expression by means of convex optimization for the power per hop while minimizing the end-to-end outage probability subject to various power constraints. The results show that optimizing the power allocation is required for systems with highly unbalanced links or with a large number of hops. Thereby, a gain of up to 2 dB over an equal power allocation has been achieved, but again, not considering end-to-end deadlines nor instantaneous CSI. Finally, [13] investigates the minimization of end-to-end outage probability under different assumptions regarding the knowledge of the instantaneous channel state. It shows that optimal power allocation derived by means of convex optimization leads to significant enhancements in outage probability if instantaneous channel state information can be used. However, the authors do not consider the cost of obtaining those instantaneous channel states.

### III. ENERGY MINIMIZATION FRAMEWORK

In this section, we present details on how to minimize energy consumption for both transmission approaches (average CSI and instantaneous CSI). As introduced in Subsection II-A, we deal with a QoS-constrained packet transmission over \( n \) hops. The objective is to minimize the required transmit energy. The QoS constraints are composed of a minimum success probability \( P \) and an associated deadline \( T \). We start with minimizing energy consumption for the average CSI case in Subsection III-A, before we present the energy minimization in case of utilizing instantaneous CSI in Subsection III-B.

#### A. Optimal Energy Consumption with Average CSI

If the nodes only utilize average channel state information to ensure the QoS constraints, the entire available time can be spent on the payload transmission. However, each forwarding link will need to perform a significant over-provisioning of the transmit power (and hence the consumed energy) to account for actual link states of potentially very bad quality. We obtain for the minimization of the energy the following theorem.

*Theorem 1*: In case of using only average channel state information, the minimum total energy \( E \) required for a transmission of a packet of size \( D \) with probability of successful
transmission $\mathcal{P}$ and time slots of length $T_i$ over $n$ hops is given as

$$E = \frac{-\sigma^2}{\ln |\mathcal{P}|} \left( \frac{2^D}{(T_i \cdot B)} - 1 \right) \left( \sum_{i=1}^{n} \frac{1}{h_i} \right)^2 \cdot T_i. \quad (5)$$

**Proof:** Since the average CSI knowledge is given a-priori, the entire deadline $T$ is used for data transmission. This time span is divided into time slots of length $T_i = \frac{T}{n}$. Accounting for these time slots of equal length, Theorem 1 can be proven by considering the generalized power allocation problem in the Appendix and letting $k = 0$ in Eq. (17). Finally, the result is multiplied by $T_i$ to obtain the minimum transmit energy. ■

### B. Optimal Energy Consumption with Instantaneous CSI

In contrast to using only average CSI for transmission, nodes might also send data along the route based on instantaneous CSI. If a node knows the actual channel state of its link, it can forward the data packet without a transmission error due to the threshold error model introduced in Subsection II-A. Therefore, each node along the route has to first acquire the instantaneous channel state. In the following, we account for this acquisition by a dedicated two-phase model. Prior to the actual data transmission, nodes exchange small control messages in order to estimate the current channel state in a two-way handshake fashion. Thus, the transmission of these control packets becomes subject to the end-to-end success probability constraint. Due to the control packet exchange, the available time $T$ to pass the payload data packet from source to destination is shortened. Both phases are given as follows:

- **Channel Acquisition Phase:** A control packet of size $D_c$ is successively exchanged between every node $i$ and its successor based on average CSI. If channel reciprocity is given, the response control packet might already be sent with instantaneous CSI on its way back (mode $m = 1$); otherwise with average CSI as well (mode $m = 2$). In the first mode, sending the control packet with instantaneous CSI is assumed to cause no errors; whereas in the latter mode, setting per-link success probabilities needs to respect the fact that the control packet traverses each link twice. This phase ends at time instance $T_c$, resulting into slot lengths of $T_i = T/2n$.

- **Payload Transmission Phase:** In the remaining time, $T - T_c$, the data packet of size $D$ is forwarded such that it always reaches the next hop reliably. This is possible as each node now holds the exact channel state information and can set the transmit power accordingly. The payload transmission phase is divided into equally sized time slots, as well.

We now proceed to determine the associated energy consumption of this scheme. Instantaneous channel gains $h_i^2$ allow a node to perfectly adapt to the current channel state by inverting the gains and transmitting with a modified power level. The required power level of node $i$ is derived from Eq. (2). Hence, the total energy consumed during the payload transmission phase is given by

$$E_{\text{data,inst}} = K \left[ D, \frac{T_c}{n} \right] \left( m \cdot \frac{1}{(\ln |\mathcal{P}|)^{1/m}} \sum_{i=1}^{n} \frac{1}{h_i^2} \right)^2 \cdot T_c. \quad (6)$$

The minimum energy for transmitting a control packet with average CSI only one-way ($m = 1$) or both ways ($m = 2$) is calculated according to Eq. (5). The resulting energy is

$$E_{c,\text{avg}}(m) = -K \left[ D_c, \frac{T_c}{2n} \right] \frac{m}{(\ln |\mathcal{P}|)^{1/m}} \sum_{i=1}^{n} \frac{1}{h_i^2} \cdot T_c. \quad (7)$$

In case control packets are sent with instantaneous CSI on their way back, the energy for this part is given as

$$E_{c,\text{inst}} = K \left[ D_c, \frac{T_c}{2n} \right] \sum_{i=1}^{n} \frac{1}{h_i^2} \cdot T_c. \quad (8)$$

For both operation modes described above, a further issue is how to divide the total available time $T$ into channel acquisition and payload transmission phase. The corresponding minimization problems can be stated as follows

$$\arg\min_{0<T_c<T} (E_{c,\text{avg}}(m = 1) + E_{c,\text{inst}} + E_{\text{data,inst}}). \quad (9)$$

$$\arg\min_{0<T_c<T} (E_{c,\text{avg}}(m = 2) + E_{\text{data,inst}}). \quad (10)$$

Unfortunately, both problems can only be solved numerically, because they are inherently non-linear in $T_c$. Furthermore, even if a closed-form expression that approximately resembles the analytical solution was found, an evaluation would be causally infeasible since determining the optimal channel acquisition duration requires a-priori knowledge of instantaneous CSIs, and thus, renders the channel acquisition phase meaningless.

### IV. Numerical Evaluation

In this section we evaluate the two different approaches against each other. We first give a brief overview of the applied methodology, and then present results regarding the energy consumption for several different parameter variations.

#### A. Methodology

Both approaches (instantaneous CSI and average CSI) are evaluated and compared against each other. In case of instantaneous CSI, a further important question to investigate relates to the optimal length of the channel acquisition phase. Our primary metric of interest is the total energy consumption required to transmit a packet with respect to the QoS constraints. We consider a transmission of a packet of 2500 bit within a deadline of $T = 10 \text{ ms}$ requiring a high success probability of $\mathcal{P} = 1 - 10^{-5}$. Source and destination are separated by $20 \text{ m}$.
and the path consists of \( n = 3 \) links. This is our reference scenario. More parameters are presented in Table I. Starting from the reference scenario, we vary several parameters sequentially to investigate the total energy consumption. We thereby consider different packet sizes, deadlines and number of links in the path, as well as different bandwidths.

In case of employing average CSI we conduct numerical evaluations of the equations from Subsection III-A for at least 1,000 randomly generated path instances with a fixed total distance. Afterwards, the results are averaged. The evaluation of the instantaneous CSI approach is more involved as we have to account for the instantaneous channel states. Hence, we create for each generated path 1,000 sets of instantaneous channel states and calculate the corresponding average energy consumption. Finally, everything is aggregated again by averaging over the path instances. Intervals for the confidence level are omitted in the figures since their size does not exceed 0.1\% of the average value. Evaluations are done with the software package MATLAB. Figure legends show the curves in their order of appearance (top to bottom).

### B. Results

We start in Fig. 1 with a basic comparison of the approaches of average CSI and instantaneous CSI for the reference scenario. The figure shows the total required energy for packet transmission versus the choice of the channel acquisition phase duration \( T_c \). As can be seen, the average CSI approach (black dashed line) is insensitive to \( T_c \) selection since it does not involve obtaining instantaneous channel states. It ends up at around \( 7 \cdot 10^{-9} \) Joule. However, the duration of the channel acquisition phase has a significant impact on the energy consumption if instantaneous CSI (black solid and dash-dotted curves) is going to be used. Then, the energy consumption varies extensively, and reaches its minimum (marked with a cross) at approximately \( 10^{-9} \) Joule, with the minimum achieved at an acquisition phase duration of about 7.5 ms (while the deadline \( T = 10 \) ms). Energy consumption can be further reduced if the channel states are reciprocal (black solid curve). In the remainder of the paper, this mode of the instantaneous CSI approach will be used as reference. Two remarks should be noted. First of all, acquiring CSI performs better than using average CSI by a factor of about 18. However, if the optimal duration of the channel acquisition phase is not respected, the relationship might easily turn around. Second, for the minimum energy consumption with instantaneous CSI the optimal duration of the channel acquisition is comparably long. This shows that most of the time is spent on reliably acquiring CSI and obtaining channel states dominates the total energy consumption for this approach. This can also be seen by referring to the green curves in Fig. 1. They depict the fractions of the reference scenario’s total energy that are spent on transmitting the control packet based on average CSI (green dash-dotted curve) in the forward direction, backwards based on instantaneous CSI (green dotted curve), and the payload transmitted with instantaneous CSI (green dashed curve).

Next, in Fig. 2, we study various modifications regarding the parameters of the reference scenario. The plot again shows total energy consumption versus the length of the channel acquisition phase. We first study a payload size reduction to 300 bit (green curves). In this case, the total amount of consumed energy is in general low as compared to the reference scenario (for the average CSI approach significantly). However, if the payload size is that small, it does not pay off to work with instantaneous CSI (solid green curve) anymore. Instead, at the optimal duration of the channel acquisition phase (which is close to 10 ms!), the energy consumption is worse than that of the average CSI scheme (horizontal green dashed line). On the other hand, if we vary other parameters, e.g., decreasing the bandwidth (magenta curve), the achievable gain increases (factor of about 100). A comprehensive list of

### Table I

<table>
<thead>
<tr>
<th>Abbrev.</th>
<th>Explanation</th>
<th>Value</th>
</tr>
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<tbody>
<tr>
<td>( D )</td>
<td>Message size</td>
<td>2500 bit</td>
</tr>
<tr>
<td>( D_c )</td>
<td>Control packet size</td>
<td>( 250 + 8 \cdot n ) bit</td>
</tr>
<tr>
<td>( P )</td>
<td>End-to-end success probability</td>
<td>( 1 - 10^{-5} )</td>
</tr>
<tr>
<td>( T )</td>
<td>End-to-end deadline</td>
<td>10 ms</td>
</tr>
<tr>
<td>( B )</td>
<td>Bandwidth</td>
<td>300 kHz</td>
</tr>
<tr>
<td>( n )</td>
<td>Number of links</td>
<td>3</td>
</tr>
<tr>
<td>( d )</td>
<td>Total distance</td>
<td>20 m</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>Path loss coefficient</td>
<td>3</td>
</tr>
<tr>
<td>( \sigma_{\text{SH}} )</td>
<td>Shadowing variance</td>
<td>0 dB</td>
</tr>
</tbody>
</table>
gain factors (ratios of average CSI approach to the minimum of the instantaneous CSI approach) for all plotted curves can be retrieved from Table II. Further observation of Fig. 2 reveals that the sensitivity of determining $T_c$ depends on system parameters. For some, the corresponding curve is V-shaped, resulting into a remarkable degradation in energy efficiency if the optimal duration of the channel acquisition phase is not selected properly; while for other parameters, the results are U-shaped, and hence, almost insensitive to the selection of the channel acquisition time over a wide range of values.

In Fig. 3, we consider different deadlines for the packet transmission. Again, the plot compares the total energy consumption against the length of the channel acquisition phase. However, note that the scaling of the x-axis is different, as it shows the duration in percentage of the total packet deadline. The plot shows that, as the deadline becomes more restrictive, the energy consumption increases (green curve), while the gain between the instantaneous CSI and the average CSI approach increases by a factor of 95 (see Table II). If we increase the deadline instead (cyan and red curves), the total energy consumption decreases for both schemes, while the gains remain around a factor of 10. Interestingly, the consumption converges against a threshold for both approaches if the available deadlines $T$ are extended: The achieved power level reduction is exactly compensated by the longer transmission time. Considering the optimal channel acquisition phase duration, the percentages decrease and increase, respectively, from 72% for the reference scenario to 36% if $T = 2 \text{ ms}$ and 95% if $T = 500 \text{ ms}$. The reason for this is twofold. First, enlarging the time available for packet transmission with average CSI linearly, exponentially reduces the required energy (see Eq. (5)). Since the energy budget of sending the control packets based on average CSI is the dominant part (compare green curves in Fig. 1), extending the channel acquisition phase tends to improve energy efficiency. Second, given a certain set of instantaneous channel states, the system’s only choice is to deal with the actual states by inverting gains. This may lead to high power levels. Reducing the associated energy can only be accomplished by transmitting for a diminishing time span. Consequently, sending the payload at an acceptable energy level requires a short, almost constant amount of time, regardless of the total available time.

In contrast to the previous investigations, we now consider the impact of the number of intermediate (relay) nodes to bridge a fixed distance. In Fig. 4, the total energy is plotted against the number of links $n$ for both approaches (black curves) and for a different channel acquisition phase of $T_c = 0.75 \text{ ms}$ (red dotted curve). As can be seen, mistakenly selecting too few or too many relay nodes leads on average to an increased energy consumption. The optimal number of intermediate nodes depends on the approach and scenario parameters, and can be determined efficiently (results exhibit a convex shape). Moreover, all previously presented results not only hold for the multi-hop case, but also for single-hop and dual-hop transmissions ($n = 1$ and $n = 2$ in Fig. 4).

As part of a larger numerical analysis, we show in Table II exemplary results of optimal $T_c$ values and the associated gain factors (ratios). As can be seen, different paths (characterized by total and individual path lengths, pathloss, shadowing) have no impact on the optimal duration of the channel acquisition phase ($T_c = 7.2 \text{ ms}$) and on the ratio that can be achieved by exploiting instantaneous CSI if $T_c$ is chosen optimally (factor of $1.77$). However, note that absolute energy values may differ.

![Fig. 3. Dependency of optimal $T_c$ on available time $T$.](image)

![Fig. 4. Dependency of energy minimum on the number of links $n$.](image)

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Fluctuating links</th>
<th>Equal links</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$d_i \sim \exp$</td>
<td>$d_i = d/n$</td>
</tr>
<tr>
<td>$\sigma_{SH} = 3 \text{ dB}$</td>
<td>$\sigma_{SH} = 0 \text{ dB}$</td>
<td></td>
</tr>
<tr>
<td>$T_c [\text{ ms}]$</td>
<td>Gain factor</td>
<td>$T_c [\text{ ms}]$</td>
</tr>
<tr>
<td>Reference ($m = 1$)</td>
<td>7.26</td>
<td>17.7</td>
</tr>
<tr>
<td>Reference ($m = 2$)</td>
<td>7.70</td>
<td>4.5</td>
</tr>
<tr>
<td>$D = 300 \text{ bit}$</td>
<td>9.65</td>
<td>0.96</td>
</tr>
<tr>
<td>$B = 50 \text{ kHz}$</td>
<td>3.32</td>
<td>96.8</td>
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<tr>
<td>$n = 1 \text{ link}$</td>
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<td>11.9</td>
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<tr>
<td>$n = 15 \text{ links}$</td>
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<td>$T = 2 \text{ ms}$</td>
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<td>95.8</td>
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<td>$T = 50 \text{ ms}$</td>
<td>44.85</td>
<td>10.2</td>
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<tr>
<td>$T = 500 \text{ ms}$</td>
<td>474.5</td>
<td>9.0</td>
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<tr>
<td>$d = 20,000 \text{ m}$</td>
<td>7.26</td>
<td>17.6</td>
</tr>
<tr>
<td>$\alpha = 2$</td>
<td>7.25</td>
<td>17.7</td>
</tr>
<tr>
<td>$p = 1 - 1 \cdot 10^{-10}$</td>
<td>8.37</td>
<td>20.1</td>
</tr>
</tbody>
</table>
Important system design aspects can be summarized as follows. Although the optimal length of the channel acquisition phase can only be obtained numerically, the scheme can be applied to real systems efficiently. Since the optimal solution only depends on system and transmission parameters and not on actual channel states, proper durations can be stored in look-up tables. Certain parameters, such as bandwidth, have to be respected in particular due to a higher sensitivity. We further reveal that for some scenarios, e.g., small packets, conducting a transmission based solely on average CSI is outperforming the one utilizing instantaneous CSI. In that case, this observation holds for any length of the channel acquisition phase. Concerning the total available transmission time, the required energy converges against a threshold after which each achievable power reduction gets compensated by the longer invested transmit duration. Moreover, for scenarios in which the number of intermediate nodes can be varied, e.g., in networks, it is not necessarily beneficial for the energy consumption to choose the largest possible number of relays, especially in case of tight deadline constraints.

V. CONCLUSIONS

We have presented an approach which minimizes the transmit energy required to carry out a QoS constrained transmission. Results can be applied to single-hop, as well as multi-hop scenarios. The time domain plays an important role to minimize energy consumption. Two different approaches are compared: Sending the actual payload while possessing only knowledge about average CSI, or first acquiring instantaneous CSI, and then sending the actual payload. Results can be applied to single-hop, as well as multi-hop scenarios. The time domain plays an important role to minimize energy consumption. Two different approaches are compared: Sending the actual payload while possessing only knowledge about average CSI, or first acquiring instantaneous CSI, and then sending the actual payload. Results can be applied to single-hop, as well as multi-hop scenarios. The time domain plays an important role to minimize energy consumption. Two different approaches are compared: Sending the actual payload while possessing only knowledge about average CSI, or first acquiring instantaneous CSI, and then sending the actual payload.

APPENDIX

GENERALIZED POWER OPTIMIZATION PROBLEM

In the following we first formulate the power allocation problem as a convex optimization problem and present afterwards a solution based on Lagrangian duality theory. We generalize the problem with respect to practical systems: The applicable transmit power per link is always upper bounded by a maximum $P_{\text{max}}$ due to technical limitations.

Without loss of generality, we assume that all $n$ links of the path are sorted in ascending order according to their channel gain, i.e., $h_1^2 \leq \ldots \leq h_k^2 \leq \ldots \leq h_n^2$. For the sake of clear notation, we simplify the $K[\cdot,\cdot]$ expression by dropping its arguments. We start with deriving the required transmit power $P_i$ for link $i$ to achieve a packet success probability of $p_i$ according to Eq. (3). Hence, the total transmit power along the path can be formulated depending on the per-link success probabilities as

$$P = \sum_{i=1}^{n} P_i = \sum_{i=1}^{n} \frac{-K}{h_i^2 \ln [p_i]}.$$  \hspace{1cm} (11)

Taking into account the maximum transmit power restriction per node, $P_{\text{max}}$, we obtain the following optimization problem

minimize $\sum_{i=1}^{n} P_i$ \hspace{1cm} (12)
subject to $\prod_{i=1}^{n} p_i \geq \mathcal{P}$ \hspace{1cm} (12a)
$P_i \leq P_{\text{max}} \quad \forall i \in \{1,\ldots,n\}$. \hspace{1cm} (12b)

Constraint (12a) preserves the end-to-end reliability $\mathcal{P}$. Any optimal solution to problem (12) will fulfill this constraint with equality.

Theorem 2: Problem (12) is feasible if and only if

$$\sum_{i=1}^{n} \frac{1}{h_i^2} \leq -\ln[\mathcal{P}] \frac{P_{\text{max}}}{K}. \hspace{1cm} (13)$$

An optimal solution to the minimization problem is specified by the number of nodes $k$ operating at $P_{\text{max}}$, i.e.,

$$P_i^* = P_{\text{max}} \quad \forall i \in \{1,\ldots,k\}. \hspace{1cm} (14)$$

For a given $k \in \{0,\ldots,n\}$, the optimal power assignment $P_i^*$ at the remaining nodes, which minimizes problem (12), is given by

$$P_i^* = \frac{c}{h_i^2} \sum_{j=k+1}^{n} \frac{1}{h_j^2} \quad \forall i \in \{k+1,\ldots,n\}, \hspace{1cm} (15)$$

in which the (positive) constant $c$ is defined as

$$c = -\left( \frac{\ln[\mathcal{P}]}{K} + \frac{1}{P_{\text{max}}} \sum_{j=1}^{k} \frac{1}{h_j^2} \right)^{-1} \hspace{1cm} (16)$$

A closed-form expression for the minimum total transmit power along the path, $P^*$, is given by

$$P^* = kP_{\text{max}} + c \left( \sum_{i=k+1}^{n} \frac{1}{h_i^2} \right)^2 \hspace{1cm} (17)$$

Proof: Problem (12) can be stated as a convex optimization problem. By taking the logarithm of Eq. (12a) we obtain the following problem

minimize $\sum_{i=1}^{n} P_i$ \hspace{1cm} (18)
subject to $\sum_{i=1}^{n} \frac{-K}{h_i^2 P_i} \geq \ln[\mathcal{P}]$ \hspace{1cm} (18a)
$P_i \leq P_{\text{max}} \quad \forall i \in \{1,\ldots,n\}$. \hspace{1cm} (18b)
The feasibility of the problem is easily checked by setting $P_i = P_{\text{max}}$ for all $i \in \{1, \ldots, n\}$ and evaluating Eq. (18a). This immediately yields Eq. (13), which ensures that the solution space is nonempty, guaranteeing the existence of an optimal solution. Otherwise, success probability $P$ cannot be achieved without violating the transmit power constraint $P_{\text{max}}$ in one or more links. For the remainder of this proof, we assume the problem to be feasible.

The convex formulation of the problem allows us to solve it with Lagrangian duality theory [14]. We introduce a Lagrangian multiplier $\mu \geq 0$ corresponding to constraint (18a) and multipliers $\lambda_1, \ldots, \lambda_n \geq 0$ corresponding to the set of constraints given by (18b). With $\lambda = (\lambda_1, \ldots, \lambda_n)$, the Lagrangian of problem (18) is defined as

$$
\mathcal{L}(\{P_i\}, \lambda, \mu) = \sum_{i=1}^{n} P_i + \mu \left( \sum_{i=1}^{n} \frac{K}{h_i^2 P_i} + \ln[P] \right) + \sum_{i=1}^{n} \lambda_i (P_i - P_{\text{max}}).
$$

Differentiating Eq. (19) with respect to $P_j$, $j = 1, \ldots, n$, yields the stationarity conditions

$$
\frac{\partial}{\partial P_j} \mathcal{L}(\{P_i\}, \lambda, \mu) = (1 + \lambda_j) - \mu \frac{K}{h_j^2 P_j^2} = 0.
$$

For the variables $P_i$, $\lambda$, and $\mu$ to be primal and dual optimal, Eqs. (18a), (18b), and (20) have to hold. In addition, the complimentary slackness conditions

$$
\lambda_i (P_i - P_{\text{max}}) = 0 \quad \forall \ i \in \{1, \ldots, n\}
$$

have to be fulfilled. In conclusion, the optimal solution to the optimization problem (18) is given by

$$
P^*_i = \min \left\{ \sqrt{\frac{\mu K}{h_i^2}}, P_{\text{max}} \right\}
$$

for all $i \in \{1, \ldots, n\}$, with the Lagrangian multiplier $\mu$ chosen such that Eq. (18a) holds with equality

$$
\sum_{i=1}^{n} \frac{K}{h_i^2} \min \left\{ \sqrt{\frac{\mu K}{h_i^2}}, P_{\text{max}} \right\}^{-1} = \ln[P].
$$

Having found a value of $\mu$ that satisfies Eq. (23), we can directly derive the number of nodes $k$ transmitting with $P_{\text{max}}$

$$
k = \arg \max_i \left\{ \frac{K}{h_i^2} \sqrt{\frac{\mu K}{h_i^2}} \geq P_{\text{max}} \right\}.
$$

Once $k$ is computed, Eq. (14) immediately follows. In the next step, the success probabilities achieved on these $k$ links have to be deduced from $P$. Then for the $n - k$ remaining links, Eq. (23) can be simplified to

$$
\sum_{i=k+1}^{n} \frac{K}{h_i \sqrt{\mu}} = \ln[P] - \sum_{j=1}^{k} \frac{K}{h_j^2 P_{\text{max}}^2}.
$$

Solving Eq. (25) with respect to $\mu$ and applying it to the unconstrained part of Eq. (22), yields Eq. (15). Finally, Eq. (17) is obtained by computing the sum of the transmit powers $P_i^*$ in Eqs. (14) and (15). This concludes the proof.

The problem of minimizing the summed transmit power can also be thought of as finding the optimal distribution of success probabilities along the path. This allows for an interpretation of the per-link success probabilities as assignable resources.

**Lemma 1**: Given the optimal distribution of transmit power according to Eq. (15), the corresponding success probabilities compute to

$$
p_i = \exp \left[ -\frac{K}{h_i^2 P_i^*} \right].
$$

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