STUDY ON DYNAMIC CHARACTERISTICS OF A NEW TYPE OF TRANSMISSION

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Abstract: The new type of transmission is a kind of swaying mechanism specially developed for certain aircraft, having low rotational speed and great torque. Utilizing criterions of mass equivalent and bending stiffness equivalent, this paper firstly applies simplification to the transmission, getting a piece of continuous beam with varying circular sections. According to the mounting condition of transmission, then establishes the finite element dynamic analyzing model, using I-DEAS software, selecting method of synchronous vector iteration, finishes normal modal analyzing. Next, carries Hertzian contact resonance analyses to ball straight-rotary pair, ball spline pair and ball unload pair, calculates contact resonance frequencies. Considering the resistance torque being moving still concludes formulae of quantity of contact resonance frequencies. Lastly, based on load spectra, makes a conclusion that antiresonance behavior of the transmission is well, showing that the design plan is feasible. Experimentation of prototype indicates that the above theoretical analyses are correct.

Key words: Transmission; Hertzian contact; Dynamic analysis; Finite element method

1. INTRODUCTION

The new type of transmission is a new type of ball helical transmission with helical lift angle of 72° specially developed for certain aircraft, as shown in Fig.1. As shown in Fig.2, the new type of transmission is composed of two parts, one is high-pressure oil cylinder with piston style, another is swaying perform mechanism, including ball straight-rotary pair, ball spline pair and ball unload pair. It transforms linear movement of piston

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into sway of leading edge of vane using pure scroll of working balls. Thus, ball straight-rotary pair, ball spline pair and ball unload pair are important assemblies for transferring dynamic force and supporting loads.

There is no doubt that dynamic question is very important to aircraft. Hence, analyses on Dynamic characteristics of the new type of transmission are significative. This paper intends to compute natural characters of this transmission by its dynamic analytical simplified beam model, and apply Hertzian contact resonance analysis to ball straight-rotary pair, ball spline pair and ball unload pair.

![Configuration of the new type of transmission](image1)

*Figure 1. Configuration of the new type of transmission*

![Working principle of the new type of transmission](image2)

*Figure 2. Working principle of the new type of transmission*

2. CONFIGURATION AND WORKING PRINCIPLE OF THE NEW TYPE OF TRANSMISSION

2.1 Configuration of the new type of transmission

As shown in Fig.1, main body of the new type of transmission is cylinder; its top and bottom flanks are ears. During installation, five fixed ears of the
transmission, sited on bottom flank, were hinged to the base; two moving ears, sited on top flank, were hinged to leading edge of vane needing rotary motion, can sway.

2.2 Working principle of the new type of transmission

The high-pressure oil cylinder is power source; it transforms pressure of high-pressure oil into axial thrust. Ball straight-rotary pair transforms linear movement of driveshaft into sway of straight-rotary sleeve, at the same time, transforms axial force into driving torque, as showed in Fig.2.

3. COMPUTATION ON NATURAL CHARACTERS OF THE NEW TYPE OF TRANSMISSION

![Figure 3. Simplified model of the ball straight-rotary actuator](image-url)

According to the criterions of mass equivalent and bending stiffness equivalent, the new type of transmission is simplified to a piece of continuous beam with varying circular sections (Fig.3). Using 1-DEAS software, according to the mounting condition of transmission, select 2D-beam element, finite element dynamic analytical model of the new type of transmission can be got easily (Fig.4). In the abstract, weather synchronous vector iteration (i.e. SVI method) or Guyan reduction can be selected for dynamic calculating. In fact, according as load spectra, since only modes falling into 0~10000 Hz need to be computed, it is better to select SVI method.

Total eight modes are got; each order natural frequencies are listed in Table 1, their corresponding modal shape are shown in Fig.5. From Table 1 and Fig.5, it can be seen that natural frequencies and modal shapes of orders 1~2, 3~4, 5~6, 7~8 are equal respectively. This is because that modes of order 1, 3, 5 and 7 denote all orders of transversal vibration within XY plane of the new type of transmission, whereas modes of order 2, 4, 6 and 8 denote all orders of transversal vibration within XZ plane of the new type of transmission. Otherwise, since when applying boundary condition, the X, Y and Z degree of freedom of all nodes on inner cylindrical face of five bottom
ears of the transmission are restrained to death, there is no vibration along axis X of the transmission.

![Figure 4. Finite element dynamic analytical model of the new type of transmission](image)

**Figure 4.** Finite element dynamic analytical model of the new type of transmission

![Figure 5. Preceding eight orders natural modal shapes of the new type of transmission](image)

**Figure 5.** Preceding eight orders natural modal shapes of the new type of transmission

| Table 1. Natural frequencies of the new type of transmission (Unit: Hz) |
|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| Order of mode | 1   | 2   | 3   | 4   | 5   | 6   | 7   | 8   |
| Natural frequency | 1892 | 1892 | 3461 | 3461 | 4738 | 473 | 7889 | 7889 |

4. **HERTZIAN CONTACT RESONANCE ANALYSES**

4.1 **Conception of Hertzian contact and basic calculating formulae**

When the new type of transmission is operating, balls are pressed closely to the inner and outer rolling ways. Material rounding the contact point
deforms under the normal pressure; incipient contact point is changed into a small plane. Profile of this contact plane is generally an ellipse. Stress and deformation acted on contact zone can be determined according to Hertzian method. Such contact is name as Hertzian contact.

Loaded under pressure \( P \), the two contacted elastomers are compressed along normal direction with elasticity, coming near with a quantity of \( \delta \). If materials of the elastomers are steel, taking \( E_1 = E_2 = 2.07 \times 10^{11} \text{Pa} \), \( \nu_1 = \nu_2 = 0.3 \), simplified formula of \( a, b \) and \( \delta \) are:

\[
a = 0.0236m_0 \left( \frac{P}{\sum \rho} \right)^{1/3} \quad (1)
\]

\[
b = 0.0236m_0 \left( \frac{P}{\sum \rho} \right)^{1/3} \quad (2)
\]

\[
\delta = 2.79 \times 10^{-4} \left( \frac{2K}{m_0} \right) \left( P^2 \sum \rho \right)^{1/3} \quad (3)
\]

where \( a \) is long radius of contact ellipse, mm; \( b \) is short radius of contact ellipse, mm; \( \delta \) is decrease of distance between the two contacted elastomers, mm; \( P \) is normal pressure, N; \( \Sigma \rho = \rho_1 + \rho_2 + \rho_{21} + \rho_{22} \), \( \rho_{ij} \) is master curvature, 1/mm, \( \rho_{ij} = 1/R_{ij} \); moreover, \( R_{ij} \) is master radius of curvature of elastomers surface at contact point, mm, \( i, j = 1, 2, i \) or \( j \) denotes one of the two elastomers; \( m_a, m_b \) and \( 2K/(m_1 m_a) \) are corresponding to function \( F(\rho) \) of master curvature, can be looked up from Table 5-1 [4]; for ball straight-rotary pair, ball spline pair and ball unload pair, \( F(\rho) \) is given by Eq. (4) as

\[
F(\rho) = \frac{(\rho_{11} - \rho_{12} + \rho_{21} - \rho_{22})}{(\rho_{11} + \rho_{12} + \rho_{21} + \rho_{22})} \quad (4)
\]

### 4.2 Conception of Hertzian contact resonance and basic calculating formulae

Hertzian contact can be regarded as a spring because of elastic, it relates to masses of two contacted elastomers, may bring vibration. If frequency of duty is the same as resonance frequency, resonance will create, which is named as Hertzian contact resonance.

For ball straight-rotary pair, ball spline pair and ball unload pair, contact of ball with inner or outer rolling ways or collars can be regarded as a spring, thus each contact pair can be considered as compound of two springs (Fig. 6a). Since mass of ball is smaller than inner and outer rolling ways or collars, ball is omitted, as shown in Fig. 6 (b). In Fig. 6, mo, mb mi denotes
masses of outer rolling way or collar, ball and inner rolling way or collar respectively. Ki and Ko denote stiffness of spring connecting mb with mi or mo, N/m.

Figure 6. Vibrating systems with masses and springs of Hertzian contact

Assume one body be fixed, Hertzian contact resonance can be determined by Eq. (5) as

\[ m \ddot{x} + kx = 0 \quad (5) \]

Solving Eq. (5), we find that resonance frequency is

\[ f = \sqrt{\frac{k}{m}} / (2\pi) \quad (6) \]

where

\[ k = 1 / (1/k_i + 1/k_o) \quad (7) \]

4.3 Calculating formulae or methods of master radii of curvature and master curvatures

The key is to determine two master planes correctly. For ball spline pair, the first master plane is coinciding with the rolling way, i.e. longitudinal; the second plane is perpendicular to the first plane, i.e. transversal. For ball straight-rotary pair, the first master plane is the normal section of helical rolling way at contact point, being perpendicular to tangent line passing contact point, master radius of curvature is computed by program compiled according to differential geometry; the second plane is perpendicular to the first plane. For left or right ball unload pairs, refer to centripetal thrust ball
bearing, the first master plane is an axial section passing contact point; the second plane is perpendicular to the first plane passing contact point.

4.4 Computation of contact stiffness and resonance frequencies

4.4.1 Computing formulae of contact stiffness

In Eq. (7), $K$ is total stiffness; formulae of $K_i$ and $K_o$ are given by Eq. (8) as

$$k_i = \frac{P_i}{\delta_i}, k_o = \frac{P_o}{\delta_o}$$  (8)

where $P_i$ and $P_o$ is normal pressing force, N.

4.4.2 Computing method of resonance frequencies

For each contact pair, meanings of $m$ in Eq. (6) are different.
1. For ball straight-rotary pair, assume driveshaft be fixed, only straight-rotary sleeve is studied; there are six rows of balls, one of which is studied; $m$ equals to one-sixth of mass of straight-rotary sleeve.
2. For ball spline pair, fixed guiding sleeve is fixed, one-sixth of driveshaft is studied, $m$ equals to one-sixth of mass of driveshaft.
3. For left ball unload pair, left bearing base (i.e. outer collar) is fixed, left bearing housing (i.e. inner collar) is studied, $m$ is mass of left bearing housing.
4. For right ball unload pair, right bearing base is fixed, right bearing housing is studied, $m$ is mass of right bearing housing.

4.4.3 Result of computation

For ball straight-rotary pair, ball spline pair, left ball unload pair and right ball unload pair, Hertzian resonance frequencies are 2638, 3061, 1984 and 2731 Hz, respectively. Since the four Hertzian resonance frequencies approach upper limit of the frequency of load spectra, we can make a conclusion that antiresonance behavior of the transmission is well, and the design plan is feasible.
4.5 Calculating formulae of contact resonance frequencies

In fact, considering the resistance torque acted on transmission is moving, accordingly Hertzian contact pressure \( P \) is variable, thereby, above result is just corresponding to maximal control torque. Calculating formulae of contact resonance frequencies of four contact pairs are given by Eq. (9) as

\[
\begin{align*}
  f_Z &= 654.19P^{1/6}, \\
  f_H &= 758.98P^{1/6}, \\
  f_{ZX} &= 672.66P^{1/6}, \\
  f_{XY} &= 751.90P^{1/6}
\end{align*}
\]  

(9)

where \( f_Z, f_H, f_{ZX} \) and \( f_{XY} \) denote Hertzian contact resonance frequencies of straight-rotary pair, spline pair, left unload pair and right unload pair, Hz.

5. CONCLUSIONS

According to the structural feature of the new type of transmission, this paper has built the dynamic analytical simplified beam model, and finished the theoretical calculation of natural frequencies and normal modes using I-DEAS software. This paper has also completed the Hertzian contact resonance analysis of the four ball contact pairs. The above analyses and computations prove that dynamic characteristics of the new type of transmission satisfy the demands of certain advanced aircraft.

6. REFERENCES