THE DESIGNATION OF CAM’S CONTOUR FOR PISTON PUMP WITH STEADY FLUX

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Abstract: In order to get steady flux of the piston pump, an analytical method is given to push the driving cam’s contour forward. The contour can be expressed by patch function analytically, which can be used conveniently as well as the study of the pump’s performance is easy to be realized.

Key words: steady flux; analytical method; cam’s contour; acceleration’s continuity

1. ILLUSTRATION

Piston pump can provide high press and reliable work, so it is used in many fields such as mining oil, driving with hydraulic pressure and control system. In many cases, pumps’ exporting steady flux is very important. Many people meet this requirement through the control of the driving motor, but this method needs high performance of driving motor and another control system to control the motor. In application, the precision of the flux is usually not steady enough. An analytical method is used for the designation of cam’s contour, using which can drive pumps and give steady flux with high precision.

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2. CONSTRAIN OF PUMP’S STEADY FLUX

The export flux of the pump that contains a pair of pistons can be express by

\[ Q = \frac{v_1 + |v_1|}{2} S + \frac{v_2 + |v_2|}{2} S \]  \hspace{1cm} (1)

where \( S \) is the cross section area of the pistons, and \( v_1 \) \( v_2 \) is the velocity of the two pistons respectively, whose positive direction is the direction when they push liquid out.

In order to give constant flux, the velocity of the pistons should meet

\[ v_1 + |v_1| + v_2 + |v_2| = \text{constant} \]  \hspace{1cm} (2)

Constrain (2) is expressed with the pistons’ velocity, the designation of the cams should start from their velocity.

3. THE DESIGNATION OF THE PISTON’S VELOCITY

The phase difference between the two pistons usually is 180°, and one type of the simplest velocity curve is shown in figure 1. The two piston pump out liquid alternatively, and meet the constant flux.

![Figure 1. the Idealized velocity of the pistons](image)

The velocity (figure 1) is not continuous, and the acceleration will become infinite sometimes, which means serious collisions between cams and pistons. So, adding a transition domain to make the velocity chances continually is very important.

Suppose the width of the transition domain is 2\( \alpha \), and it’s very easy to design the velocities change linearly in the domain (figure 2). Obviously, the
pump can export liquid with steady flux. But the acceleration of this kind of cam is not continues, which means soft conflicts between the cam and the piston, and the cam can not run faster to avoid more vibrations.

![Figure 2 the velocity of the pistons with linear transition](image)

Figure 3 shows one cam’s velocity whose acceleration is continues, and the functions in the transition domain have to meet constrain expressed by equation (2).

![Figure 3 the piston’s velocity with continues acceleration (s is the rotation angle of the cam)](image)

The base function used in Hermite interpolations can just fit the former requirements, and the velocity of the cam can be expressed
\[
v = \begin{cases} 
  h_1 t^2 (3 - 2t), & \text{where, } t = \frac{s + a}{2a}, \quad s \in [-a, a] \\
  h_1 (1 - 3t^2 + 2t^3), & \text{where, } t = \frac{s - \pi + a}{2a}, \quad s \in [\pi - a, \pi + a] \\
  -h_2 t^2 (3 - 2t), & \text{where, } t = \frac{s - 3\pi/2}{\pi/2 - a}, \quad s \in \left[\frac{3}{2} \pi, 2\pi - a\right] \\
  -h_2 (1 - 3t^2 + 2t^3), & \text{where, } t = \frac{s - 3\pi/2}{\pi/2 - a}, \quad s \in \left[\frac{3}{2} \pi, 2\pi - a\right] 
\end{cases}
\] (3)

It is very easy to verify that the flux will be steady without any bound.

4. THE COMPUTATION OF THE CAM'S CONTOUR

Integrate the velocity respect to time, and the displacement of the piston can be get:

\[
f(s) = \begin{cases} 
  f_1(s) & s \in [-a, a] \\
  f_2(s) & s \in [a, \pi - a] \\
  f_3(s) & s \in [\pi - a, \pi + a] \\
  f_4(s) & s \in [\pi + a, \frac{3}{2} \pi] \\
  f_5(s) & s \in \left[\frac{3}{2} \pi, 2\pi - a\right] 
\end{cases}
\] (4)

where

\[
f_1 = -\frac{h_1 s^4 + 6h_1 s^2 a^2 + 8h_1 s a^3 + 16C_1 a^3}{16a^3} \quad f_2 = h_1 s + C_2
\]

\[
f_3 = \frac{8h_1 s a^3 - 6h_1 s^2 a^2 + 12h_1 s \pi a^2 + h_1 s^4 - 4h_1 s^3 \pi + 6h_1 s^2 \pi^2 - 4h_1 s \pi^3 + 3}{16a^3} + C_3
\]

\[
f_4 = (-4h_2 s^4 + 20h_2 s^3 \pi + 8h_2 s^3 a - 36h_2 s^2 \pi^2 - 36h_2 s^2 \pi a + 28h_2 s \pi^3 + 48h_2 s \pi^2 a + 12h_2 s \pi a^2 - 8h_2 s a^3) / (-\pi + 2a)^3 + C_4
\]

\[
f_5 = -h_2 \left(s - \frac{(s - 3\pi/2)^3}{(\pi/2 - a)^2} + \frac{(s - 3\pi/2)^4}{2(\pi/2 - a)^3}\right) + C_5
\]

and \(C_1, C_2, C_3, C_4, C_5\) is the parameters to be determined. As the displacement is continue, and we will have
\[
\begin{align*}
  f_1(a) &= f_2(a) \\
  f_2(\pi - a) &= f_3(\pi - a) \\
  f_3(\pi + a) &= f_4(\pi + a) \\
  f_4\left(\frac{3}{2}\pi\right) &= f_5\left(\frac{3}{2}\pi\right) \\
  f_5(2\pi - a) &= f_1(-a)
\end{align*}
\]  

(5)

Solving equation (4), and \(C_2, C_3, C_4, C_5, h_2\) can be got as

\[
  h_2 = \frac{2h_1\pi}{\pi - 2a} \quad C_2 = -\frac{3}{16} h_1 a + C_1
\]

\[
  C_3 = -\frac{8h_1a^3\pi + 6h_1a^4 + 6h_1a^2\pi^2 - h_1\pi^4 - 16a^3C_1}{16a^3}
\]

\[
  C_4 = \frac{h_1\left(-509\pi^4 a - 272\pi^2 a^3 - 224\pi a^4 + 712\pi^2 a^3 + 792\pi^3 a^2 + 48a^5\right)}{16\left(-\pi^3 + 6\pi^2 a - 12\pi a^2 + 8a^3\right)(-\pi + 2a)} \\
  + \frac{C_1\left(512a^3\pi - 256a^4 - 384\pi^2 a^2 - 16\pi^4 + 128\pi^3 a\right)}{16\left(-\pi^3 + 6\pi^2 a - 12\pi a^2 + 8a^3\right)(-\pi + 2a)}
\]

\[
  C_5 = \frac{h_1\left(-19\pi a + 56\pi^2 + 6a^2\right) + C_1(16\pi - 32a)}{16(-\pi + 2a)}
\]

(6)

By plugging (6), formula (4) becomes expanded curve of the cam, and in which, \(h_1\) and \(C_1\) parameters to be valued.

According to equation (3), the journey of the pistons is

\[
  S = h_1\pi
\]

(7)

and the flux of the pimp is

\[
  Q = h_1 S \omega
\]

(8)

The minimum and maximum size in radial direction of the cam are

\[
  r_{\text{min}} = \frac{h_1(a^2 - 2C_1)}{4\alpha} \quad r_{\text{max}} = \frac{h_1(4\pi\alpha - a^2 + 2C_1)}{4\alpha}
\]

(9)
Application objects can be satisfied by giving $h_1$ and $C_1$ with reasonable values. Figure 4 shows an instance, the contour of the cam is calculated with expression (4).

Figure 4 the calculated displacement of piston and the contour curve of the cam. $a = \pi / 6.0$, $h_1 = 20$ and $C_1 = 50$.

5. CONCLUSIONS

This paper has given the analytical function for the cams, which can drive piston pump and give steady flux. People can realize desired pump through given reasonable values for the parameters and the study of their performances is very conveniently.

6. REFERENCES

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