

# Calculate BER Improvement due to Nonlinear Regenerators

F.G. Sun, Z.G. Lu, G.Z. Xiao, and C.P. Grover

Photonic Systems Group, Institute for National Measurement Standards  
National Research Council, M-50 1200 Montreal Road, Ottawa, Canada K1A 0R6  
Email: [fengguo.sun@nrc.ca](mailto:fengguo.sun@nrc.ca)

**Abstract** Use the method we developed recently we calculate the bit-error-rate (BER) improvement as a function of transmitter extinction ratio and the optical link noise parameter.

## Introduction

Various optical regeneration techniques have been proposed and demonstrated [1]-[3] in order to eliminate noise, crosstalk, and signal distortion. All-optical 2R regeneration based on polarization rotation induced by nonlinear birefringence in a semiconductor optical amplifier was recently demonstrated [4] with an improved extinction ratio of 15dB for an input extinction of 5dB. The operating principle of such regenerators relies on the nonlinear input-output transfer characteristic. Recently we proposed a new method to evaluate the performance of a regenerator [5]. With this method in this paper we calculate the bit-error-rate (BER) improvement as a function of transmitter extinction ratio and the optical link noise parameter.

## Calculation

We consider an optical transmission link of length  $L$ . The transmitter in the system is assumed to have a finite extinction ratio. A nonlinear regenerator is set at position  $l$  between the transmitter and the receiver. The system model is illustrated in Fig. 1.

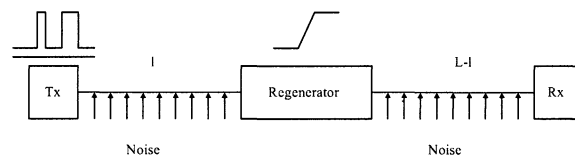


Fig. 1. System model.

The regenerator transforms the input signal  $x$  into an output  $f(x)$

$$\bar{x} = f(x).$$

(1)

Because of the noise accumulation, the probability that a signal and noise will

appear at a given level  $x$  is a function of the propagation length. Let  $P_0(x, l)$  ( $P_1(x, l)$ ) be the probability of getting a signal at a level  $x$  in the position  $l$  when the symbol ZERO (ONE) is sent from a transmitter, and let  $P^N(y, l)$  be the probability of finding additional noise at a level  $y$  after the signal has travelled over a distance  $l$ . Assuming that the ZERO and ONE symbols are equally probable, in the absence of a regenerator in the optical link, the BER can be represented by

$$BER_N = \frac{1}{2} \int_D^\infty P_0(x, L) dx + \frac{1}{2} \int_{-\infty}^D P_1(x, L) dx. \quad (2)$$

The first (second) term is the contribution of the ZERO (ONE) rail.  $D$  is the decision level. When a regenerator is used, from the probability theory, the BER contribution of the ZERO rail becomes

$$BER_{R0} = \frac{1}{2} \int_{-\infty}^\infty \tilde{P}_0[g(\bar{x}), l] |g'(\bar{x})| \times \int_{D-\bar{x}}^\infty P^N(y, L-l) dy d\bar{x}. \quad (3)$$

where  $\tilde{P}_0[g(\bar{x}), l] |g'(\bar{x})|$  is the probability of finding the output of the regenerator at a level  $\bar{x}$  when the transmitter sends out a ZERO symbol,  $P^N(y, L-l)$  is the probability of finding an additional noise in the second interval  $L-l$  at a level  $y$ , and  $g(\bar{x})$  is the inverse function of the nonlinear transfer function. Similarly, the BER contribution of the ONE rail can be written as follows

$$BER_{R1} = \frac{1}{2} \int_{-\infty}^\infty \tilde{P}_1[g(\bar{x}), l] |g'(\bar{x})| \times \int_{-\infty}^{D-\bar{x}} P^N(y, L-l) dy d\bar{x}, \quad (4)$$

where  $\tilde{P}_1[g(\bar{x}), l] |g'(\bar{x})|$  is the probability of finding the output of the regenerator at a level  $\bar{x}$  when the transmitter sends out a ONE symbol. Using equations (3) and (4), we obtain an optimal BER

$$\min_{0 \leq l \leq L} \{BER_{R0} + BER_{R1}\}, \quad (5)$$

where  $\min\{x\}$  is the minima of  $x$ . The optimal position of the regenerator is  $l_o$  and it provides the best BER value given by equation (5). The BER improvement attributable to the regenerator is

$$\log(BER_N) - \log(\min_{0 \leq l \leq L} \{BER_{R0} + BER_{R1}\}) \quad (6)$$

The method described here can be generalized to a situation in which more than one nonlinear regenerator is placed in the optical link. However, the calculation is much more complicated than if a single regenerator is used. In the following part of

the paper we will limit ourselves to a single regenerator.

We assume that these probability distribution functions have a Gaussian form, its standard deviation  $\sigma$  is a function of  $l$

$$\sigma = a\sqrt{l}, \quad (7)$$

and  $a$  is a constant. We also assume that the nonlinear transfer function has the form [4]

$$\bar{x} = f(x) = \begin{cases} \gamma \cdot x & x < 1/2 \\ \gamma \cdot (x-1) + 1, & x > 1/2 \end{cases}, \quad (8)$$

Using the method we proposed recently, we obtain

The BER improvement is defined by

$$\Delta \log(BER) = \log(BER_{N0} + BER_{N1}) - \log(BER_{R0} + BER_{R1}). \quad (9)$$

In Fig. 1 we show the BER improvement,  $\Delta \log(BER)$ , as a function of the extinction ratio when the standard deviation is set at 0.1. This noise level is set to give a BER of  $7.42 \times 10^{-7}$  without regenerator for an extinction ratio equals 20dB. When a regenerator is used, the BER is improved to  $5.18 \times 10^{-12}$ . Then, we set  $l/L = 0.5$  to calculate  $\Delta \log(BER)$  as a function of  $a\sqrt{L}$ . The result is shown in Fig. 2.

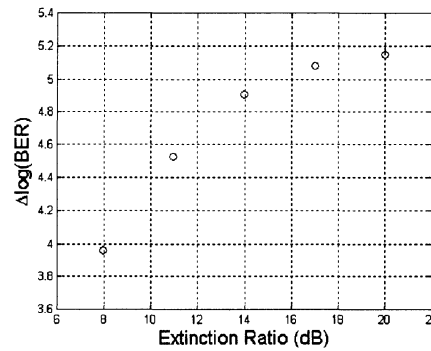


Fig. 1.  $\Delta \log(BER)$  versus transmitter extinction ratio when the regenerator is located in its optimal position;  $\sigma = 0.1$ .

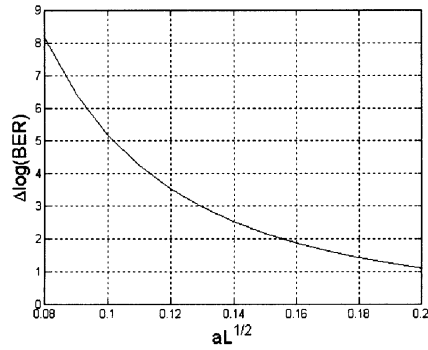


Fig. 2. Calculated  $\Delta \log(BER)$  as a function of  $a\sqrt{L}$  for  $l/L = 0.5$  and an extinction ratio of 20dB.

### Conclusions

In summary, we calculate the bit-error-rate (BER) improvement as a function of transmitter extinction ratio and the optical link noise parameter with a new method we proposed recently.

### References

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