Calculate BER Improvement due to Nonlinear Regenerators
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Abstract Use the method we developed recently we calculate the bit-error-rate (BER) improvement as a function of transmitter extinction ratio and the optical link noise parameter.

Introduction
Various optical regeneration techniques have been proposed and demonstrated [1]-[3] in order to eliminate noise, crosstalk, and signal distortion. All-optical 2R regeneration based on polarization rotation induced by nonlinear birefringence in a semiconductor optical amplifier was recently demonstrated [4] with an improved extinction ratio of 15dB for an input extinction of 5dB. The operating principle of such regenerators relies on the nonlinear input-output transfer characteristic. Recently we proposed a new method to evaluate the performance of a regenerator [5]. With this method in this paper we calculate the bit-error-rate (BER) improvement as a function of transmitter extinction ratio and the optical link noise parameter.

Calculation
We consider an optical transmission link of length $L$. The transmitter in the system is assumed to have a finite extinction ratio. A nonlinear regenerator is set at position $I$ between the transmitter and the receiver. The system model is illustrated in Fig. 1.

![Fig. 1. System model.](image)

The regenerator transforms the input signal $x$ into an output $f(x)$

$$\tilde{x} = f(x) \quad (1)$$

Because of the noise accumulation, the probability that a signal and noise will
appear at a given level \( x \) is a function of the propagation length. Let \( P_0(x,l) \) (\( P_1(x,l) \)) be the probability of getting a signal at a level \( x \) in the position \( l \) when the symbol ZERO (ONE) is sent from a transmitter, and let \( P^N(y,l) \) be the probability of finding additional noise at a level \( y \) after the signal has travelled over a distance \( l \). Assuming that the ZERO and ONE symbols are equally probable, in the absence of a regenerator in the optical link, the BER can be represented by

\[
\text{BER}_N = \frac{1}{2} \int_{-\infty}^{\infty} P_0(x,L)dx + \frac{1}{2} \int_{-\infty}^{\infty} P_1(x,L)dx.
\]

(2)

The first (second) term is the contribution of the ZERO (ONE) rail. \( D \) is the decision level. When a regenerator is used, from the probability theory, the BER contribution of the ZERO rail becomes

\[
\text{BER}_{R0} = \frac{1}{2} \int_{-\infty}^{\infty} \tilde{P}_0[g(\tilde{x}),l]g'(\tilde{x}) \times \int_{D-\tilde{x}}^{\infty} P^N(y,L-l)dyd\tilde{x}.
\]

(3)

where \( \tilde{P}_0[g(\tilde{x}),l]g'(\tilde{x}) \) is the probability of finding the output of the regenerator at a level \( \tilde{x} \) when the transmitter sends out a ZERO symbol, \( P^N(y,L-l) \) is the probability of finding an additional noise in the second interval \( L-l \) at a level \( y \), and \( g(\tilde{x}) \) is the inverse function of the nonlinear transfer function. Similarly, the BER contribution of the ONE rail can be written as follows

\[
\text{BER}_{R1} = \frac{1}{2} \int_{-\infty}^{\infty} \tilde{P}_1[g(\tilde{x}),l]g'(\tilde{x}) \times \int_{D-\tilde{x}}^{\infty} P^N(y,L-l)dyd\tilde{x},
\]

(4)

where \( \tilde{P}_1[g(\tilde{x}),l]g'(\tilde{x}) \) is the probability of finding the output of the regenerator at a level \( \tilde{x} \) when the transmitter sends out a ONE symbol. Using equations (3) and (4), we obtain an optimal BER

\[
\min_{0\leq L} \left\{ \text{BER}_{R0} + \text{BER}_{R1} \right\},
\]

(5)

where \( \min \{ x \} \) is the minima of \( x \). The optimal position of the regenerator is \( L_o \) and it provides the best BER value given by equation (5). The BER improvement attributable to the regenerator is

\[
\log(\text{BER}_N) - \log(\min_{0\leq L} \left\{ \text{BER}_{R0} + \text{BER}_{R1} \right\})
\]

(6)

The method described here can be generalized to a situation in which more than one nonlinear regenerator is placed in the optical link. However, the calculation is much more complicated than if a single regenerator is used. In the following part of
the paper we will limit ourselves to a single regenerator.
We assume that these probability distribution functions have a Gaussian form, its
standard deviation $\sigma$ is a function of $l$

$$\sigma = a \sqrt{l} ,$$

and $a$ is a constant. We also assume that the nonlinear transfer function has the
form [4]

$$\bar{x} = f(x) = \begin{cases} 
    \gamma \cdot x & \text{if } x < 1/2 \\
    \gamma \cdot (x - 1) + 1 & \text{if } x > 1/2 
\end{cases} .$$

(8)

Using the method we proposed recently, we obtain
The BER improvement is defined by

$$\Delta \log(BER) = \log(BER_{R_0} + BER_{N_1}) - \log(BER_{R_0} + BER_{R_1}) .$$

(9)

In Fig. 1 we show the BER improvement, $\Delta \log(BER)$, as a function of the
extinction ratio when the standard deviation is set at 0.1. This noise level is set to
give a BER of $7.42 \times 10^{-7}$ without regenerator for an extinction ratio equals 20dB.
When a regenerator is used, the BER is improved to $5.18 \times 10^{-12}$.
Then, we set $l/L = 0.5$ to calculate $\Delta \log(BER)$ as a function of $a \sqrt{L}$.
The result is shown in Fig. 2.

![Fig. 1. $\Delta \log(BER)$ versus transmitter extinction ratio when the regenerator is located in its optimal position; $\sigma = 0.1$.](image-url)
Fig. 2. Calculated $\Delta \log(BER)$ as a function of $a\sqrt{L}$ for $l/L = 0.5$ and an extinction ratio of 20dB.

Conclusions
In summary, we calculate the bit-error-rate (BER) improvement as a function of transmitter extinction ratio and the optical link noise parameter with a new method we proposed recently.

References