Directional Reception vs. Directional Transmission for Maximum Lifetime Multicast Delivery in Ad-Hoc Networks

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Abstract. In this paper, we present a mixed-integer linear program (MILP) designed to optimize max-min path lifetime for multicasts in directional antenna equipped networks in the presence of interference. We then employ the MILP to perform a head-to-head comparison between directional transmission and directional reception. We also propose and analyze a new directional reception heuristic. Our results indicate that directional reception can match directional transmission when extending path lifetime, with lower complexity and employing routes that use much less cumulative power.

Keywords: *directional antennas, optimization, maximum-lifetime, multicast*

Introduction

We investigate the construction of multicast trees that maximize path lifetime for ad-hoc networks where nodes are equipped with directional antennas. We compare the use of directional antennas for the *transmission* of signals (which we refer to as D-TX) versus their use for *reception* (which we refer to as D-RX). Specific contributions include:

1. To show that, while directional transmission can match the path lifetime obtained with directional reception, it does so at the expense of considerable increase in cumulative power and complexity;
2. To produce a mathematical formulation of the optimization problem that takes into account potential interference among links that are part of the same multicast tree; and
3. To propose a heuristic for forming a multicast tree using directional reception and to show that this heuristic outperforms previously proposed heuristic solutions that employ directional transmission.

We first introduce our mathematical program and associated communication model assumptions. Then, we utilize the model to compare the performance of directional transmission and reception optima, and to characterize the performance of heuristics designed to approximate both methods.

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Symbol	Definition
$\mathcal{N}$	Set of network nodes.
$\mathcal{R}$	Set of nodes that are receivers.
$\mathcal{B}_i$	Set of beams available at node i.
$\mathcal{B}_i(j)$	Set of beams at node i where j is within main lobe.
s	Source node.
P^t_i	Transmission power level at node i
$P^t_{i,b}$	Transmission power, node i using beam b.
$\boldsymbol{G_i}$	Gain / path loss vector at node i.
$F_{i,j,b}$	1 if flow possible from i to j when i using beam b.
$F_{i,j}$	total flow indicator from node i to node j ("super-flow").
$M_{i,j}$	Message (information) flow from i to j.
$B_{i,b}(k)$	Beam gain from node i to node k, using beam b ($b \in \mathcal{B}_i$)
$U_{i,j,b}$	Bound of interference at j when receiving from i using beam b.
Q	Large integer (Big-M).
S^i	SINR ratio required at node i.
N_t	Thermal / ambient noise.
R_i	Energy remaining (battery) available at node i.
P_{max}	Maximum transmit power setting for nodes.
θ_{min}	Minimum beamwidth for a node.
Pct^{inbeam}	Percentage of power in main beam lobe.
D-TX	Directional transmit MILP model.
D-RX	Directional receive MILP model.

Table 1: Notation.

Mathematical Program

The mathematical program presented in this work incorporates the effects of inter-node and side-lobe interference through a signal to interference and noise ratio (SINR) sufficiency requirement. We refer the reader to Table 1 for the notation we adopt throughout the paper. Antennas assume a common bulb-and-cone model shown in Figure 2(a), where Pct^{inbeam} represents the fraction of power in the main antenna lobe.

The basis for the model is the SINR sufficiency constraint, in contrast to previous MILP models that do *not* account for interference [1][2]. Put simply, a logical link from node i to node j is feasible whenever the ratio of the power received from the intended transmitter to that received from all other sources exceeds the receiver's SINR requirement, as represented in Inequality 1.

$$P^t_{i,b} \cdot B_{i,b}(j) - S^j \cdot \left[\sum_{\substack{k\in\mathcal{N}\\ k\neq i,j}} \sum_{l\in\mathcal{B}_k} P^t_{k,l} \cdot B_{k,l}(j) + N_t \right] \geq 0 \tag{1}$$

Inequality 1 represents SINR for a single beam configuration. The mathematical program requires that all possible configurations be enumerated, and consequently, only a small subset will be satisfied for any given multicast configuration. "Big-M" notation is introduced in Inequality 2, where $U_{i,j,b}$ relaxes the constraint unless the link is "active" as indicated by the binary variable $F_{i,j,b}$.

$$P^t_{i,b} \cdot B_{i,b}(j) - S^i \cdot \left[\sum_{\substack{k\in\mathcal{N}\\ k\neq i,j}} \sum_{l\in\mathcal{B}_k} P^t_{k,l} \cdot B_{k,l}(j) + N_t \right] \geq F_{i,j,b} \cdot U_{i,j,b} - U_{i,j,b} \tag{2}$$

With D-RX, a node seeks to maximize its SINR by orienting its antenna toward a single transmitter (the node's parent in the multicast tree). With D-TX,

min T

s.t.

$$
\begin{aligned}
P_{i,b}^{t} \cdot B_{i,b}(j) - S^{i} \cdot \left[\sum_{\substack{k \in \mathcal{N} \\ k \neq i,j}} \sum_{l \in \mathcal{B}_k} P_{k,l}^{t} \cdot B_{k,l}(j) + N_t \right] - \frac{Q}{P_i^{max}} \cdot \sum_{\substack{k \in \mathcal{N} \\ k \neq i,j}} P_{i,m} \geq & \\
F_{i,j,b} \cdot [U_{i,j,b} + Q] - [U_{i,j,b} + Q] &: \forall i, j \in \mathcal{N}, \forall b \in \mathcal{B}_i, j \neq s, i \neq j \\
F_{i,j} = \sum_{\forall b \in \mathcal{B}_i} F_{i,j,b} &: \forall i, j \in \mathcal{N}, i \neq j \\
F_{i,j} + F_{j,i} \leq 1 &: \forall i, j \in \mathcal{N}, i \neq j \\
\sum_{\substack{\forall k \in \mathcal{N} \\ k \neq j}} F_{k,j} \geq 1 &: \forall j \in \mathcal{R} \\
M_{i,j} \leq |\mathcal{R}| \cdot F_{i,j} &: \forall i, j \in \mathcal{N}, i \neq j \\
M_{i,j} \leq \sum_{\substack{\forall k \in \mathcal{N} \\ k \neq i,j}} M_{k,i} - 1 \cdot (1 : i \in \mathcal{R}) &: \forall i, j \in \mathcal{N}, i \neq j \\
P_i^{t} \geq P_{i,b}^{t} &: \forall i \in \mathcal{N}, \forall b \in \mathcal{B}_i \\
T \geq \frac{P_i^{t}}{R_i} &: \forall i \in \mathcal{N} \\
P_i^{t} \leq P_{max} &: \forall i \in \mathcal{N} \\
F_{i,j,b} \in \{0, 1\} &: \forall i, j \in \mathcal{N}, i \neq j, j \neq s, b \in \mathcal{B}_i
\end{aligned}
$$

Fig. 1: D-TX: Formulation for Max-Min Network Lifetime with Inter-Node Interference

however, it may be desirable for a node in the tree to select an antenna beam that covers multiple receivers (the node's descendants in the multicast tree). Accordingly, the formulation of the D-TX optimization problem is more complex than that of D-RX, to account for the combinatorics of beam selection.

Our choice of max-min path lifetime ($\frac{1}{T}$) as our optimization metric reflects the metric used in previous work such as [3][4]. The time until death of the *first* node in a forwarding tree is defined as path lifetime. Wieselthier et al. showed that cumulative power (i.e. min-power) metrics do not correlate well to this metric [5]. Later, we show that our results are consistent with these findings.

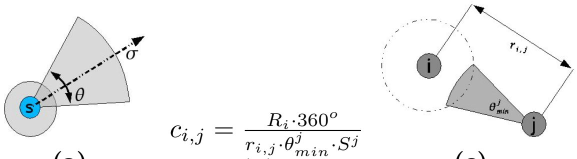
Figure 1 shows the complete D-TX model as translated into the mixed-integer linear program (MILP) notation in Table 1. We refer the reader to [6] for a complete discussion of the D-RX model. From top to bottom, represented are: SINR constraints, "super-flow" variables ($F_{i,j}$), flow consistency inequalities, receiver demand, SINR message throttling ($M_{i,j}$), message consistency, "super-power" ($P_{i,j}$), path lifetime ($\frac{1}{T}$), power cap, and binary constraints. MILP programs are well known to be NP-hard [7], and empirically, the difficulty required to find a solution is often related to the number of binary variables in the model. Bounds on number of constraints and variables for the D-TX and D-RX mathematical programs are shown in Table 2. This confirms our previous observation that the formulation of the D-TX problem is considerably more complex than that of D-RX.

	D-TX:		D-RX:	
Type	Maximum Number	Contributor	Maximum Number	Contributor
Constraints	$O(\|\mathcal{N}\|^4)$	SINR	$O((2 \cdot \|\mathcal{N}\|)^2)$	SINR
Binary Var.	$O(\|\mathcal{N}\|^4)$	$F_{i,j,b}$	$O((2 \cdot \|\mathcal{N}\|)^2)$	$F_{i,j,b}$
Continuous Var.	$O(\|\mathcal{N}\|^3)$	$P_{i,j,b}$	$O(\|\mathcal{N}\|^2)$	$M_{i,j}$

Table 2: Size of MILP Formulations

D-RX Heuristic

In this section, we introduce the Directional Reception Incremental Protocol (DRIP), a low-computation heuristic designed to approximate the MILP optima. The network is modeled as a directed graph $\mathcal{G}$ where an $i \rightarrow j$ link $l_{i,j}$ has a weight $c_{i,j}$ assigned as in Figure 2(b). Link weight is proportional to the amount of battery power remaining at the transmitting node, denoted by R_i, and the power required for inter-node communication. In this case, higher weight indicates longer lifetime. All network links are represented by $\mathcal{L}$, where $\mathcal{L}(i)$ denotes links with node i as a *destination.* A forwarding tree $\mathcal{T}$ is built from source to all receivers incrementally, until all receivers are included in the tree. At each step, the highest weight link available is added. Once the algorithm terminates, post-processing eliminates any branches that do not contain receivers. The heuristic is described in pseudocode in Algorithm 1.



$$c_{i,j} = \frac{R_i \cdot 360^o}{r_{i,j} \cdot \theta^j_{min} \cdot S^j}$$

(a) **(b)** **(c)**

Fig. 2: DRIP Link Cost

Algorithm 1 DRIP Multicast Routing Algorithm

```
$\mathcal{T} \leftarrow s$
$\mathcal{N} \leftarrow \mathcal{N} \setminus \{s\}$
$\mathcal{L} \leftarrow \mathcal{L} \setminus \mathcal{L}(s)$
while ! $\mathcal{R} \subseteq \mathcal{T}$ do
    $i \leftarrow$ highestAvailableLinkWeight($\mathcal{L}$)
    $\mathcal{T} += i$
    $\mathcal{L} \leftarrow \mathcal{L} \setminus \mathcal{L}(i)$
end while
removeUnNeededBranches($\mathcal{T}$)
```

Results: D-TX vs. D-RX MILP Models

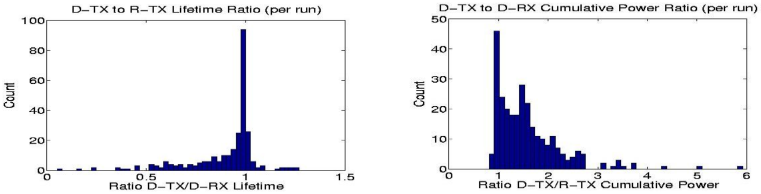
In this section, we illustrate the results of our head-to-head comparison of antenna use. The results are produced using our MILP models, and reflect the optimal path lifetime for the given network configuration and multicast group.

Experimental Setup:

To perform a fair comparison, both the D-RX and the D-TX models were applied to identical networks. Because of the complexity of the D-TX model, network size is restricted to 10 nodes ($|\mathcal{N}| = 10$). We use the following parameters for all nodes $i \in \mathcal{N}$: $Pct^{inbeam} = 0.7$ (30% of energy lost to sidelobes), $\theta_{min} = 45^o$, $S^i = N_{thermal} = 1$, $P_{max} = 100$, $\alpha = 2$, $R_i = 300$. Nodes are randomly placed in 5 × 5, 10 × 10 and 15 × 15 unit areas. Receiver set size varies among five values $|\mathcal{R}| \in \{1, 3, 5, 7, 9\}$. For each combination of network dimension and receiver set size, 20 individual runs were performed, for a total of 300 runs. Mixed-integer linear programs are solved with either GLPK [8], or CPLEX [9].

D-TX to D-RX Comparison:

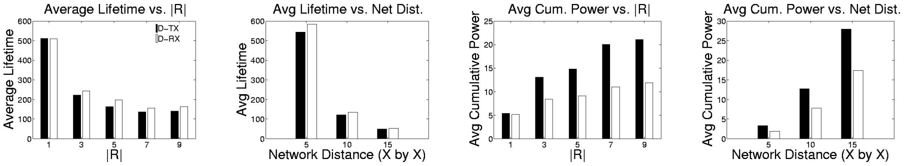
Figure 3 displays histograms of both max-min lifetime, and cumulative power use ratios over all runs. As illustrated in Figure 3(a), in most cases D-RX produces identical or superior max-min lifetime values to those of D-TX. More importantly, Figure 3(b) clearly indicates that D-RX achieves this with *much* less overall power. For a significant number of test networks, D-TX requires over 200% of the power of D-RX.



(a) $\frac{D-TX}{D-RX}$ Lifetime **(b)** $\frac{D-TX}{D-RX}$ Cumulative Pow.

Fig. 3: Performance Histograms (Optima)

Figure 4 dispenses with the histogram illustration, and shows actual lifetime ((a),(b)), and cumulative power values ((c),(d)) against a single network parameter. The weak correlation between path lifetime and cumulative power is evident. Figures 4(a) and (b) clearly show that for any breakout by $|\mathcal{R}|$ or network distance, D-RX and D-TX achieve comparable path lifetime. Figures 4(c) and (d), however, clearly show that D-RX requires significantly less cumulative power.



(a) **(b)** **(c)** **(d)**

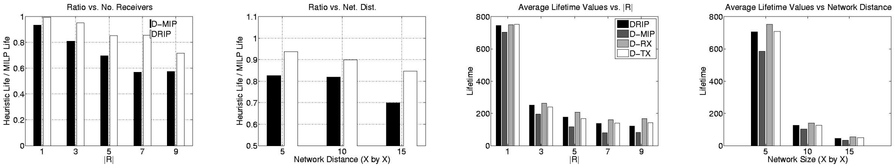
Fig. 4: Ratios vs. Single Network Parameter (Optima)

Analysis of Heuristic Methods

This section builds upon the analysis of the D-TX and D-RX MILP models. Here, we compare a previously defined directional *transmission* heuristic D-MIP [3] and our own directional *reception* heuristic DRIP. These heuristics are compared head-to-head, and also to their respective MILP optima.

Experimental Results:

Figure 5(a) and (b) show the average path lifetime obtained by DRIP and D-MIP heuristics, always normalized to the optimum path lifetime returned by the MILP. In all graphs, the ratio shown is the heuristic compared to the MILP optimal value. For example, for $|\mathcal{R}| = 9$, D-MIP only achieves approximately 55% of the MILP optimal, while DRIP achieves over 70%.



(a) **(b)** **(c)** **(d)**

Fig. 5: Heuristic Performance Ratios

Clearly, the MILP model provides an upper bound on path lifetime for either heuristic under the effects of interference. While both methods suffer with larger receiver sets and bigger network distances, D-MIP's performance deteriorates much faster with both. As the size of the receiver set increases, D-MIP's ability to approximate the optimal value declines. Recall, also, that D-RX can also have a higher lifetime value, meaning that the approximation, *and* the target value are larger. Figures 5(b) and (c) show the actual average lifetime values for the schemes in question.

Conclusion

This paper investigates the use of directional antennas for multicast delivery in ad-hoc networks. Our main contribution is to show that, while directional transmission can match directional reception in terms of max-min path lifetime, it does so at the expense of considerably higher power expenditure and complexity.

We present a mixed-integer linear program for finding the optimal antenna configuration, power settings, and logical topology for max-min path lifetime under the effects of *interference* when using directional antennas. The existing literature has focused on the use of antennas for directional *transmission*, and ignored interference. Our results (under more realistic assumptions) provide evidence that nodes are better served using the antennas for *reception*. Results from a simple heuristic for multicast tree selection employing directional reception further confirm these findings.

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