Stability-Throughput Tradeoff and Routing in Multi-Hop Wireless Ad-Hoc Networks

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Abstract. We study the throughput of multi-hop routes and stability of forwarding queues in a wireless Ad-Hoc network with random access channel. We focuse on wireless with stationary nodes, such as community wireless networks. Our main result is charactrerization of stability condition and the end-to-end throughput using the balance. We also investigate the impact of routing on end-to-end throughput and stability of intermediate nodes. We find that i) as long as the intermediate queues in the network are stable, the end-to-end throughput of a connection does not depend on the load on the intermediate nodes, ii) we showed that if the weight of a link originating from a node is set to the number of neighbors of this node, then shortest path routing maximizes the minimum probability of end-to-end packet delivery in a network of weighted fair queues with coupled servers. Numerical results are given and support the results of the analysis.

1 Introduction

Consider a set of *static* devices spread over some region. Each of these devices is a wireless transceiver that transmits and receives at a single frequency band which is common to all the devices. Over time, some of these devices collect/generate information to be sent to some other device(s). Owing to the limited battery power that these devices are allowed to use, a device may not be able to directly communicate (transmit) with far away nodes. In such a scenario, one of the possibilities for the information transmission between two nodes that are not in position to have a direct communication is to use other nodes in the network. To be precise, the source device transmits its information to one of the devices which is within transmission range of the source device. This intermediate device then uses the same procedure so that the information finally reaches its destination[1].

Clearly, a judicious choice is required to decide on the set of devices to be used to assist in the communication between any two given pair of devices. This is the standard problem of routing in communication networks. The problem of optimal routing has been extensively studied in the context of wire-line networks where usually a shortest path routing algorithm is used: Each link in the network has a weight associated with it and the objective of the routing algorithm is to find a path that achieves the minimum weight between two given nodes. Clearly, the outcome of such an algorithm depends on the assignment of the *weights* associated to each link in the network. In the wire-line context, there are many well-studied criteria to select these weights for links, such as delays. In the context of wireless ad-hoc networks, however, not many attempts have been made to (i) identify the characteristics of the quantities that one would like to associate to a *link* as its weight, and in particular (ii) to understand the resulting network performance and resource utilization (in particular, the stability region and the achievable throughput regions). Some simple heuristics have been frequently reported to improve performance of applications in mobile ad-hoc networks (see [9] and reference therein).

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[1] We will see later that it is also possible that some of the information is lost before reaching the destination device.

To study this problem, we consider in this paper the framework of random access mechanism for the wireless channel where the nodes having packets to transmit in their transmit buffers attempt transmissions by delaying the transmission by a random amount of time. This mechanism acts as a way to avoid collisions of transmissions of nearby nodes in the case where nodes can not sense the channel while transmitting (hence, are not aware of other ongoing transmissions). We assume that time is slotted into fixed length time frames. In any slot, a node having a packet to be transmitted to one of its neighboring devices decides with some fixed (possibly node dependent) probability in favor of a transmission attempt. If there is no other transmission by the other devices whose transmission can interfere with the node under consideration, the transmission is successful. We assume throughout that there is some mechanism that notifies the sender of success or failure of its transmissions. For example, the sources get the feedback on whether there was zero, one or more transmissions (collision) during the time slot.

At any instant in time, a device may have two kinds of packets to be transmitted:

1. Packets generated by the device itself. This can be sensed data if we are considering a sensor network.
2. Packets from other neighboring devices that need to be *forwarded*.

Clearly, a device needs to have some scheduling policy to decide on which of these types it wants to transmit, given that it decided to transmit. Having a first come first served scheduling is one simple option. Yet another option is to have two separate queues for these two types and do a weighted fair queueing (WFQ) for these two queues. In this paper we consider the second option.

Working with the above mentioned system model, we study the impact of routing, channel access rates and weights of the weighted fair queueing on throughput, stability and fairness properties of the network.

It is worth mentioning that the above scenario may also be studied in the perspective of game theory in which case the nodes are assumed to be rational and need some incentive to forward data from other nodes. Typically in such scenario, a Nash equilibrium determines the operating point (routing, channel access rates and WFQ weights). Thus, the results of this paper may be helpful in comparing various operating points based on criteria of throughput, stability and fairness in the cases where Nash equilibrium is not unique.

Our main result is concerned with the stability of the forwarding queues at the devices. It states that whether or not the forwarding queues can be stabilized (by appropriate choice of WFQ weights) depends only on the routing and the channel access rates of the devices. Further, the weights of the WFQs play a role only in determining the tradeoff between the power allocated for forwarding and the delay of the forwarded traffic. The end-to-end throughput achieved by the nodes are independent of the choice of the WFQ weight.
Remark Most of the studies on random access in wireless networks assume that the sources always have data to send. This then is expected to give the *saturation performance*, which may be the throughput or probability of collision or some similar quantity of interest.

Related literature: Wireless network stability has attracted much interest. Among the most studied stability problems are scheduling [11, 12] as well as for the Aloha protocol [1, 10, 14]. Tassiulas and Ephremides [11] obtain a scheduling policy for the nodes that maximises the stability region. Their approach inherently avoids collisions which allows to maximize the throughput. Radunovic and Le Boudec [3] suggest that considering the total throughput as a performance objective may not be a good objective. Moreover, most of the related studied do not consider the problem of forwarding and each flow is treated similarly (except for Radunovic and Le Boudec [3], Huang and Bensaou [7] or Tassiulas and Sarkar[13]). Our setting is different than the mentionned ones in the following: the number of retransnmitions, (which is one of the parameters that we optimize) is finite, and therefore in our setting, the output and the input need not be the same.

2 Network Model

In this section, we describe the working of the network in detail and introduce various quantities that determine the overall performance. We provide also the assumptions underlying this study and introduce appropriate notations.

2.1 Assumptions and definition

Consider a wireless ad-hoc network consisting of N nodes (we allow $N = \infty$ to study some simple symmetric cases without boundary effects). When N is finite, we number the nodes using integers $1, \ldots, N$. We assume a simple channel model:

- A node can decode a transmission successfully iff there is no other interfering transmission.
- Assume that all nodes share the frequency band, and time is assumed to be divided into fixed length slots. - Queues at Nodes , has two queues associated with it: one queue (denoted Q_i) contains the packets that originate at node i and the other queue (denoted F_i) contains packets that node i has received from one of its neighbors and has to be transmitted (forwarded) to another neighbor. If node i decides to transmit when both the queues Q_i and F_i are nonempty, it implements a weighted fair queue, i.e., node i sends a packet from queue F_i with probability f_i and sends a packet from Q_i with probability $1 - f_i$. If only one of these queues is non-empty, the node selects packet from this non-empty queue to transmit. When node i decides to transmit from the queue Q_i, it sends a packet destined for node d, $d \neq i$, with probability $P_{i,d}$. The packets in each of the queues Q_i and F_i are served in first come first served fashion.
- Arrival of data packets at a source node: We assume that the queue Q_i is always non-empty for nodes which are sources of data; this is the case, for example, when the nodes are sensors and they make new measurements as soon as the older ones are transmitted. This kind of models with assumption of *saturated* nodes are intended to provide insights into the performance of the system and also helps study effects of various parameters.

This model allows us to define a neighborhood relation between any two nodes: node i is neighbor of node j if node i can receive transmission from node j in absence of any other transmission. We use the function $A(\cdot, \cdot) : [1, N] \times [1, N] \to \{0, 1\}$ to denote the neighborhood relation: $A(i, j) = 1$ iff i is neighbor of j. We assume that the (binary) neighborhood relation is symmetric, i.e. $A(i, j) = A(j, i)$. Let $\mathcal{N}(i)$ denote the nodes which are neighbors of node i, i.e., $\mathcal{N}(i) = \{j : A(j, i) = 1\}$.

2.2 Channel Access Mechanism

As mentioned before, the time is assumed to be divided into fixed length slots. We assume that the packet length (or, transmission schedule length) is fixed throughout system operation. If node i has a packet waiting to be transmitted in either Q_i or F_i, then node i will attempt a transmission in a slot with some probability P_i, i.e., even when the node is ready to transmit, it may transmit or not in the slot, depending on the collision avoidance and resolution schemes being used, as well as the channel's current state. If the transmission is meant for some node $j \in \mathcal{N}(i)$, then the transmission from node i to j is successful iff none of the nodes in the set $j \cup \mathcal{N}(j) \backslash i$ transmits. This mechanism models the CSMA/CA random channel access mechanism which forms the basis of slotted ALOHA systems. Here we restrict ourselves to a fixed probability of channel access P_i for node i, i.e., the transmission probability does not account for the exponential backoff mechanism sometimes used in CSMA/CA channel access mechanisms in order to reduce the probability of successive collision of a packet. *To avoid pathological cases, in this paper we will assume that* $0 < P_i < 1, \ \forall i$.

2.3 Routing and packet loss

Routing is an essential task of transferring packets of information from the sources to the destination. We consider static source routing, i.e., when the source node sends a packet, it appends the information of route that the packet has to follow in the network. This information can be obtained, for example, by a proactive protocol as OLSR[4] and WRP[8]. These protocols contain routing table information by broadcasting control packet and attempt to maintain at all times up-to-date routing information from each node to every other node. By a route from node i to j we mean an ordered sequence of nodes which will forward packets that originate at i and have node j as their destination. By ordered set we mean here that the two successive elements in the set representing a route must be neighbors of each other. Also, the first element of this set is the source and the last element is the destination. We use the notation $R_{i,j}$ to denote the route from node i to j with the nodes i and j removed, i.e., $R_{i,j}$ denotes the ordered set of *intermediate* nodes on route from node i to j. Also, $R_{i,j,k}$ is used to denote the (ordered) subset of all nodes that occur not after node k in the set $R_{i,j}$. Note that we are assuming that all the packets from i to j follow the same route, i.e., there is no probabilistic routing at a packet level.

We assume that all the queues in the network are large enough so that there is no packet *drop* due to buffer overflow. The only source of packet losses that we consider are those arising from excessive number of repeated collisions of a transmitted packet. Specifically, if node i is sending packet on route from node s to d, then if this packet has been attempted transmission $K_{i,s,d}$ number of times by node i and has suffered a collision every time, the packet is dropped. Note that here we allow for $s = i$.

3 Stability Properties of the Forwarding Queues: The Saturated Node Case

First objective of our analysis is to study the effect of the choice of the parameters of the schemes mentioned above (P_i's, $P_{i,j}$'s, routing and the parameter $K_{i,s,d}$'s) on the network performance, i.e., we derive the protocol's performances based on the heavy traffic, i.e., a node always has a packet in its buffer to be sent.

For a given routing, let π_i denote the probability that node i has packets to be forwarded, $\pi_{i,s,d}$ is the probability that queue F_i is nonempty and the packet in the first position in the queue F_i is from the route s to d and n_i is the number of neighboring nodes of node i.

3.1 The Rate Balance Equations

We fix a node i and look at its forwarding queue, F_i. It is clear that if this queue is stable then the output rate from this queue is equal to the input rate into the queue. Only issue to be resolved here is to properly define the term *output rate*. This is because, owing to a bound $K_{i,s,d}$ on the number of attempts for transmission of any packet, not all the packets arriving to F_i may be successfully transmitted. Hence, the output rate is defined as the rate at which packets from queue F_i are either successfully forwarded or are dropped owing to excessive number of collisions. Next we derive the expressions for input and output rates for queue F_i from first principles.

We start by obtaining the *detailed balance equations*, i.e., the fact that if the queue F_i is stable, then the input rate on any route using queue F_i is equal to the output rate from queue F_i on that route.

For any given nodes i, s and d, let $j_{i,s,d}$ be the entry in the set $R_{s,d}$ just after i. It is possible that there is no such entry, i.e., node i is the last entry in the set $R_{s,d}$. In that case $j_{i,s,d} = d$. Let $P_{i,s,d} = \Pi_{j \in j_{i,s,d} \cup \mathcal{N}(j_{i,s,d}) \setminus i}(1 - P_j)$ be the probability that a transmission from node i on route from node s to node d is successful. Also, let

$$L_{i,s,d} = \sum_{l=1}^{K_{i,s,d}} l(1 - P_{i,s,d})^{l-1} P_{i,s,d} + K_{i,s,d}(1 - P_{i,s,d})^{K_{i,s,d}} = \frac{1 - (1 - P_{i,s,d})^{K_{i,s,d}}}{P_{i,s,d}}$$

be the expected number of attempts till success or consecutive $K_{i,s,d}$ failures of a packet from node i on route $R_{s,d}$.

Lemma 1. *For any node i, s and d such that $P_{s,d} > 0$ and $i \in R_{s,d}$, the long term average rate of departure of packets from node i on route from node s to node d is* $\frac{\pi_{i,s,d} P_i f_i}{L_{i,s,d}}$.

Proof: see the full version of our paper [2]

Lemma 2. *For any fixed choice of nodes i, s and d such that $P_{s,d} > 0$ and $i \in R_{s,d}$, the long term average rate of arrival of packets into F_i for $R_{s,d}$ is*

$$P_s(1-\pi_s f_s)P_{s,d}P_{s,s,d}\Pi_{k\in R_{i,s,d}\setminus i}\sum_{l=1}^{K_{k,s,d}}(1-P_{k,s,d})^{l-1}P_{k,s,d}.$$

Proof: See the full version of our paper [2]

Proposition 1. *In the steady state, if all the queues in the network are stable, then for each i, s and d such that $i \in R_{s,d}$,*

$$\begin{aligned}\frac{\pi_{i,s,d}P_i f_i}{L_{i,s,d}} &= P_s P_{s,d}(1-\pi_s f_s)P_{s,s,d}\Pi_{k\in R_{i,s,d}\setminus i}\sum_{l=1}^{K_{k,s,d}}(1-P_{k,s,d})^{l-1}P_{k,s,d}\\ &= P_s P_{s,d}(1-\pi_s f_s)P_{s,s,d}\Pi_{k\in R_{i,s,d}\setminus i}(1-(1-P_{k,s,d})^{K_{k,s,d}})\end{aligned}$$

Proof: If the queue F_i is stable, then the rate of arrival of packets on route $R_{s,d}$ into the queue is same as the rate at which the packets are removed from the queue (either successfully forwarded or dropped because of excessive collisions). •

Let

$$w_{s,i} = \sum_{d:i\in R_{s,d}}\frac{P_s P_{s,d}P_{s,s,d}L_{i,s,d}}{P_i}\Pi_{k\in R_{i,s,d}\setminus i}\sum_{l=1}^{K_{k,s,d}}(1-P_{k,s,d})^{l-1}P_{k,s,d},$$

and $y_i = 1-\pi_i f_i$. Note that $w_{s,i}$ *are independent of* $f_j, 1 \le j \le N$ *and depend only on the probabilities* P_j, $P_{s,d}$ *and the routing.*

Theorem 1. *In the steady state, if all the queues in the network are stable, then for each i, s and d such that $i \in R_{s,d}$,*

$$1-y_i = \sum_s y_s w_{s,i}.$$

Proof: Summing both the sides of the expression in Proposition 1 for all $s,d : i \in R_{s,d}$, we get the *global* rate balance equation for queue F_i. •

The system of equations in Theorem 1 can be written in matrix form as

$$\underline{y}(I+W) = \underline{1}, \tag{1}$$

where W is an $N \times N$ matrix whose $(s,i)^{th}$ entry is $w_{s,i}$ and $\underline{y}$ is an $N-$dimensional row vector.

The relation of Equation 1 has many interesting interpretations/implications. Some of these are:

– The Effect of f_i: At the heart of all the following points is the observation that the quantity $y_i = 1-\pi_i f_i$ is *independent* of the choice of f_j, $1 \le j \le N$. It only depends on the routing and the value of P_j.

- *Stability*: Since the values of y_i are independent of the values of f_j, $j = 1, \ldots, N$, and since we need $\pi_i < 1$ for the forwarding queue of node i to be stable, we see that for any value of $f_i \in (1 - y_i, 1)$, the forwarding queue of node i will be stable. Thus we obtain a lower bound on the weights given to the forwarding queues at each node in order to guarantee stability of these queues. To ensure that these lower bounds are all feasible, i.e., are less than 1, we need that $0 < y_i \leq 1$; $y_i = 0$ corresponds to the case where F_i is unstable. Hence, if the routing, $P_{s,d}$ and $P_j s$ are such that all the y_i are in the interval $(0, 1]$, then all the forwarding queues in the network *can be made stable by appropriate choice of* $f_i s$. Now, since y_i is determined only by routing and the probabilities $P_j s$ and $P_{s,d}$, we can then *choose* f_i (thereby also fixing π_i, hence the forwarding delay) to satisfy some further optimization criteria so that this extra degree of freedom can be exploited effectively.
- *Throughput*: We see that the long term rate at which node s can serve its own data meant for destination d is $P_{s,d}P(1 - \pi_s f_s) = P_{s,d}Py_s$ which is *independent of* f_s. Also, the throughput, i.e., the rate at which data from node s reaches their destination d. This quantity turns out to be independent of the choice of $f_j, 1 \leq j \leq N$. Similarly, the long term rate at which the packets from the forwarding queue at any node i are attempted transmission is $P_i \pi_i f_i = P_i(1 - y_i)$, which is also independent of the choice of $f_j, 1 \leq j \leq N$.
- *Choice of f_i*: Assume that we restrict ourselves to the case where $f_i = P_f$ for all the nodes. Then, for stability of all the nodes we need that

$$P_f > 1 - \min_i y_i.$$

  Since the length of the interval that f_i is allowed to take is equal to y_i, we will also refer to y_i as stability region.
- *Energy-Delay Tradeoff*: For a given set of P_js, $P_{s,d}$ and routing, the throughput obtained by any route $R_{l,m}$ is fixed, independent of the forwarding probabilities f_i. Hence there is no *throughput-delay* tradeoff that can be obtained by changing the forwarding probabilities. However, we do obtain an *energy-delay tradeoff* because now, for a given *stable routing*, we need to find value of f_i which will determine π_i. Clearly, f_i represents the forwarding energy and π_i gives a measure of the delay.
- *Throughput-Stability Tradeoff*: In the present case, we can tradeoff throughput with stability and not directly with the delay. This is achieved by controlling the routing. This point will be further dealt with in Section 4.
- *Per-route behavior*: Note that the above observations are based on the global rate balance equation for forwarding queue F_i of node i. Similar observations can be made when considering the detailed balance equation for queue F_i for some fixed source destination pair s, d such that $i \in R_{s,d}$.

3.2 Balance Equations under Unlimited attempts :$K_{i,s,d} \equiv \infty$

In this subsection, we consider an extreme case in which a node attempts forwarding of a packet until the transmission is successful. This case provides some further important observations while keeping the expressions simple. The detailed balance equation for queue F_i on route from node s to node d is

$$\pi_{i,s,d} f_i P_i P_{i,s,d} = P_{s,d} P_s (1 - \pi_s f_s) P_{s,s,d}.$$

By assuming that all nodes have same channel access rate $P_i = P$, $\forall i$, we have

$$\pi_i f_i = \sum_{s,d:i \in R_{s,d}} \frac{P_{s,d}(1 - \pi_s f_s) P_{s,s,d}}{P_{i,s,d}}.$$

Hence, introducing the transformation $y_i = 1-\pi_i f_i$, we see that the above set of rate balance equations can be written in matrix form as

$$\underline{y}(I + W_\infty) = \underline{1}$$

where W_∞ is a matrix with its $(s,i)^{th}$ entry being $w_{s,i} = \sum_{d:i\in R_{s,d}} \frac{P_{s,d}P_{s,s,d}}{P_{i,s,d}}$.

Observe that if a source has at most one destination, i.e, $P_{s,d} \in \{0,1\}$, and if the number of neighbor is same for all the nodes so that $P_{i,s,d} = P_{s,s,d}$, then the rate balance equations become

$$y_i + \sum_{s:i\in R_s} y_s = 1.$$

The above relation has many interesting interpretations/implications. Some of these are:

Stability : if a node s' which is also a source for some destination d' does not forward packets of any other connection, i.e., if $\pi_{s'} = 0$ then for any $i \in R_{s',d'}$, the rate balance equation is

$$\pi_i f_i = \sum_{s,d:i\in R_{s,d},s\neq s'} (1 - \pi_s f_s) + 1,$$

implying that the forwarding queues of all the nodes in $R_{s',d'}$ are unstable since the above requirement requires $\pi_i \geq 1$ as f_i is bounded by 1. This implies that a *necessary condition for the forwarding queues in the network to be stable is that all the sources must also forward data.* This can have serious implications in case of ad-hoc networks. There is also an advantage of the above result as it reduces the allowed set of routes and thus makes the search for the optimal route easier. From the above rate balance equation it follows that, for a given P and P_f, the stability of the forwarding queue of node i depends in *an inverse manner* on the stability of the forwarding queues of the source nodes of the routes that pass through node i. Precisely, observe that the value of π_i increases with a decrease in value of π_s. This implies that if the routing is such that node i carries traffic of a source s which does not forward any route's packet, i.e., $\pi_s = 0$, then the value of π_i is more as compared to the case where, keeping everything else fixed, now node s forwards traffic from some route.

4 Stability of Forwarding Queues and Routing

In the following we will restrict ourselves to symmetric networks, i.e., we will assume that $P_i = P$, $\forall i$ and $f_i = P_f$, $\forall i$. However, we allow for general source-destination pair combinations and general routing. We will also assume that the number of neighbours of all the nodes are same, i.e., $n_i = n$, $\forall i$. Also, we will be assuming that $K_{i,s,d} \equiv 1$. Note that assuming a symmetric network need not imply that the number of nodes is infinite. *We mention that the restriction to symmetric case is only to simplify the presentation and all the following development will work for a general network as well.*

We give some necessary and some sufficient conditions for stability of the forwarding queues. These stability conditions can be grouped into two category: (i) stability conditions specific to a particular routing, and (ii) stability conditions independent of the routing.

Clearly, the stability conditions which account for routing will give tighter conditions. However, obtaining stability conditions that do not depend on the routing is in itself significant simplification in tuning the network parameters. For example, suppose that we are deploying a grid (or, mesh) network for which $n_i = 4$. In this case, if we can find a pair of values P an P_f such that *all the forwarding queues are guaranteed to be stable*, then one can decouple the problem of finding an optimal route and that of stability. We will use this decoupling later in the paper.

Let $r \stackrel{\Delta}{=} (1-P)^n$. Note that $P_{i,s,d} = r$. Also, for a given routing, let $d(i,s,d)$ be the number of elements in the set $Ri,s,d\backslash i$.

4.1 Stability Conditions

Proposition 2. *1- A* necessary *condition for stability of F_i for a given routing is that*

$$PP_f \geq \sum_{s,d:i\in R_{s,d}} (1-P_f)P_{s,d}Pr(1-r)^{d(i,s,d)}.$$

2- A sufficient *condition for stability of F_i,* irrespective of routing *is that*

$$PP_f \geq (1-P)^n.$$

Proof: 1- For a given routing, the input rate into the forwarding queue F_i is

$$\sum_{s,d:i\in R_{s,d}} y_s PrP_{s,d}(1-r)^{d(i,s,d)}.$$

Now, $y_s = 1 - \pi_s P_f \geq 1 - P_f$. Hence, the minimum rate at which packets can arrive to F_i is

$$\sum_{s,d:i\in R_{s,d}} (1-P_f)PrP_{s,d}(1-r)^{d(i,s,d)}.$$

The maximum rate at which F_i can be served is clearly PP_f. The proof is complete for 1.

2- The maximum arrival rate of packets into the queue F_i is $(1-P)^n = r$, because in any slot F_i can receive packet only if the node i and $(n-1)$ of its neighbours are not transmitting. Similarly, the maximum rate at which the queue F_i is served is PP_f. For stability we need the service rate to be at least the arrival rate. The proof is complete. •

4.2 Effect of Routing

Assume a symmetric network and assume that the condition of Proposition 2 is satisfied so that all the forwarding queues are always stable, irrespective of the routing of packets.

Under the present situation where stability is guaranteed irrespective of the routing used, we can change routing to obtain better throughput for the various routes while maintaining stability of the forwarding queues.

The probability that a packet on route $R_{s,d}$ reaches its destination is $r^{d(d,s,d)}$. Here, the quantity $d(d,s,d)$ depends on the routing used. We then have the following easy result

Lemma 3. *Shortest path routing maximizes the probability of success of a packet between a source-destination pair.*

Proof: From the expression of probability of success of a packet on a route, we need minimum value of $d(d,s,d)$ to maximize the probability. •

The above result was fairly straightforward to obtain and is also intuitive. It is similarly easily shown that

Corollary 1. *If number of neighbours is not same for all the nodes then a route with shortest number of interfering nodes achieves maximum probability of success of packet.*

Even though we are able to ensure that the forwarding queues are stable independent of the routing used, it is clear that maximizing the probability of success of a packet on any route does not necessarily maximize the *throughput* on that route. This is because the throughput on a route $R_{s,d}$ is $y_s PrP_{s,d}r^{d(d,s,d)}$, so that it is possible that the probability of success on a route increases but the forwarding queue of the source itself is loaded so much that the throughput that the source decreases.

However, we know that the minimum rate at which queue Q_s is served is $P_s(1-f_s) = P(1-P_f)$, independent of the load on queue F_s. Hence, by maximizing the probability of success for each source-destination pair by using shortest-path routing maximizes the minimum guaranteed throughput for the source-destination pair. This in itself is important consequence of Lemma 3.

Remark The results of this section deal with the effect of routing on the minimum guaranteed throughput. We assumed that the system is always stable, independent of the routing used (we also gave a sufficient condition for this to happen). However, we have not answered the question of maximizing the throughput itself. This is a hard problem in general as can be seen by the complex dependence of y_s on the routing. Moreover, assuming a shortest path routing does not always uniquely determine the routing in a network. This is because in a network there may be many paths between a given source-destination pair which qualify to be shortest path. A simple example is a Grid network. In our ongoing research work we are looking at the problem where we restrict ourselves to the space of shortest path routing and then aim at maximizing the throughput obtained by the routes. This amounts to maximizing y_s for each value of s. This also amounts to minimizing the value of π_s for each s. Clearly, this need not always be possible since two vectors need not always be componentwise comparable. Hence, we are looking at the problem of maximizing an overall utility function

$$\max_{\text{Shortest Path Routing}} \sum_s \frac{\left(y_s r^{d(d_s,s,d_s)}\right)^{1-\alpha}}{1-\alpha},$$

where we assume that a source s can have at most one destination, referred to as d_s. Above optimization problem is motivated by the concept of fairness in communication networks. When $\alpha \to \infty$, the above optimization problem aims at maximizing the minimum throughput obtained in the network. This also amounts, roughly, to minimizing the maximum value of π_s, so that all the forwarding queues in the network are uniformly well behaved.

4.3 Numerical Results: An Asymmetric Network

In this section, we study the observations made in the Section 4 by means of a simple asymmetric example network. In this example, we show that the results and observations made in Section 4 are also valid for a general network.

Consider the asymmetric wireless ad-hoc network consisting of 11 nodes as depicted in Figure 1. We assume that there are only five end-to-end connections defined as follows : $R_{1,11} = \{4,5,7\}$, $R_{3,6} = \{2,4\}$, $R_{11,6} = \{10,8\}$, $R_{9,3} = \{8,5\}$ and $R_{6,7} = \{4,5\}$ where $R_{s,d}$ is the set of intermediate node used by a connection from source s to destination d. The routing used in this example is based on hop-length in which each source selects a

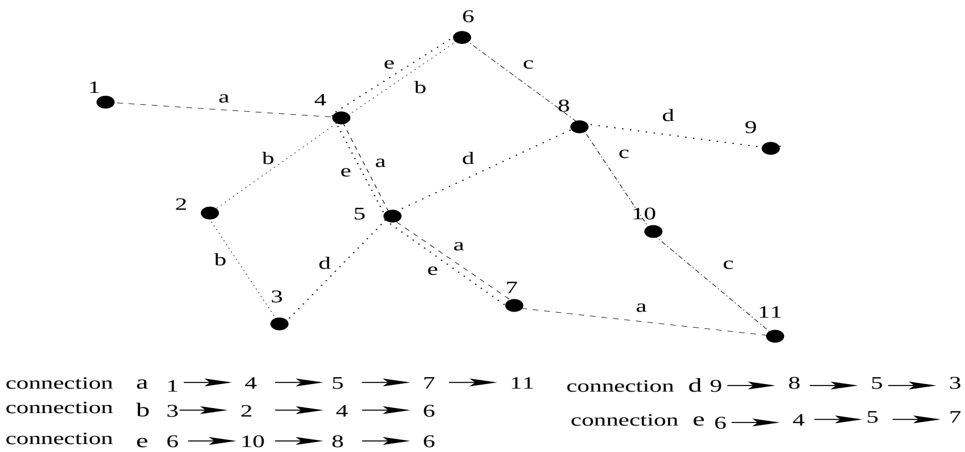

Fig. 1. The Asymmetric Network considered for studying effect of routing.

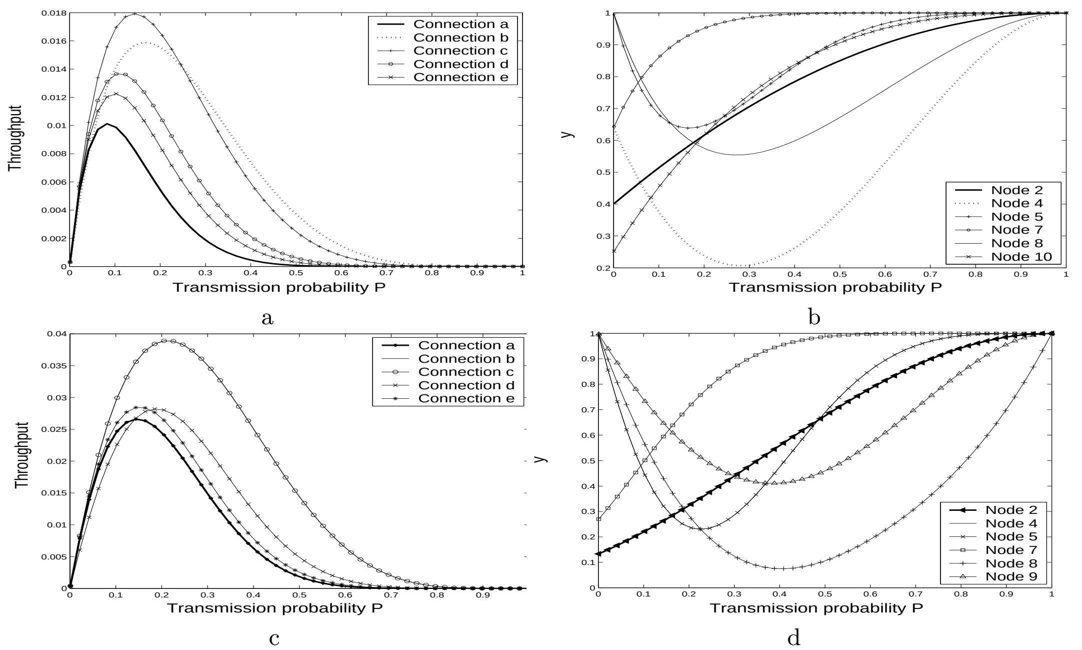

Fig. 2. (a) and (b) (resp. (c) and (d)) show he throughput of all sources and region of stability as function of the transmission probability P for $K = 1$ (resp. $K = 4$)

route with minimum number hop. To ensure the stability of the example network under consideration, we fix the channel access probabilities of nodes 4, 5, 8 and 10 to 0.3. The channel access probability of the other nodes are equal. In figures 2, we plot the throughput on various routes and the quantities $y_i,\ i \in \{2, 4, 5, 7, 8, 10\}$[2] against the channel access probability for the different values of limits on attempts (assuming $K_{i,s,d} \equiv K$ and $P_i = P$ for $i = 1, 2, 3, 6, 9, 11$). The existence of an optimal channel access rate (or, the transmission probability) is evident from the figures. Moreover, as expected, the optimal transmission probability increases with K. By comparing the throughput and the quantities y_i for different values of $K = 1, 4$. The existence of an optimal choice of the channel access probability is evident from the figure. The figure 2 shows that increasing the parameter K significantly improves the throughput but the region of stability decreases. It is therefore clear, there is a throughput-stability tradeoff which can be obtained by changing the limit on the number of attempts (K)

Now, using the same example network, we study the effect of routing on stability (as studied in section 3.2). In this example, we observe that the nodes 1, 3, 6, 9 and 11 don't forward packets from any of the connections, hence the forwarding queues of intermediate nodes that forward packets originating from these sources are less stable and become unstable when the limit on number of attempts K becomes large. To validate this observation, we added in the network (Figure 1) a connection f between node 5 and node 2 such that $R_{5,2} = \{3\}$. This implies that node 3 forwards packets originating at source node 5.

In figure 3, we compare the region of stability of node 2 and node 3 before and after adding the connection f. Clearly, the forwarding queue at node 2 becomes more stable when the node 3 starts forwarding packets of connection f. This confirms our observation of Section 3.2 that the stability of the network when all source forward data is more as compared to the case when some nodes are not source of packets. Thus nodes in a random access network have a natural incentive to forward data.

[2] Since the nodes 1, 3, 6, 9 and 11 don't forward packets of any connections, i.e., $y = 1$, we don't need to plot the quantity y for these nodes

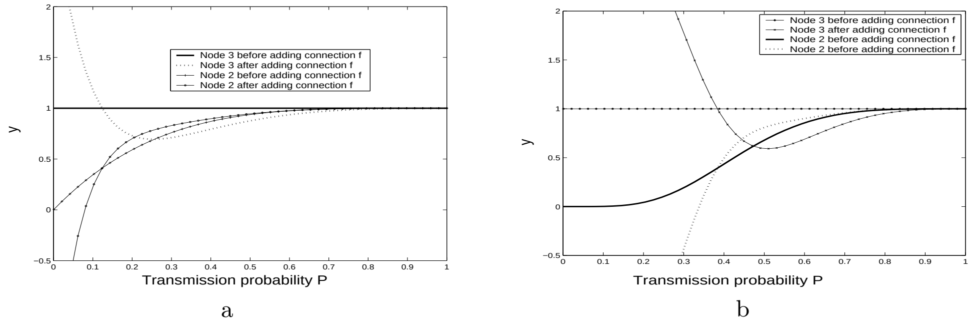

Fig. 3. (a) and (b) show the region of stability of node 2 and 3 as function of the transmission probability P for $K = 1, 4$

Now, we use the shortest path routing (based on the number of interferers on a path as defined in subsection 4) under the present situation where the stability of all the forwarding queues in the network is guaranteed. The routes for all connections under this shortest-path routing are $R_{1,11} = \{2, 3, 7\}$, $R_{9,3} = \{10, 7\}$ and $R_{6,7} = \{8, 10\}$.

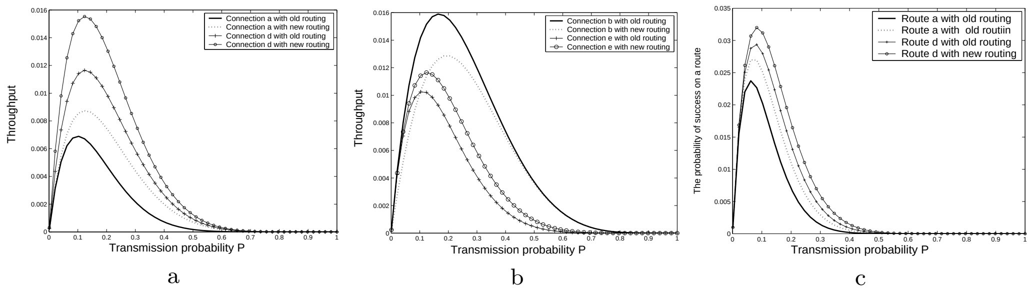

Fig. 4. (a) and (b) show the throughput of connections a, c, b and e as function of the transmission probability P for $K = 1$ and (c) shows the probability of success on a route a and d as function of the transmission probability P for $K = 1$

In figures 4 ((a) and (b)), we compare the throughput of all connections under the old and new routings. We observe that the throughput of all connections (except that of connection b), is better with new routing than those obtained under the old routing. The reason of decreasing the throughput of connection b is the change in quantity y_3. In old routing, $y_3 = 1$ (node 3 with old routing, does not forward packets of any connections). With the new routing, node 3 forwards the packets of connection a. However, the value of y_3 decreases with new routing, explaining the decrease of throughput of connection b (because now the source node of connection b, i.e., node 3 gives some of its resources to forwarding of packets on route a). In conclusion, the question of maximizing the throughput *uniformly for all nodes* is a hard problem. The complexity of this problem comes from the dependence of throughput and the quantity y. In figure 4 (c), we plot the probability of success of a packet on all connections versus the transmission probability P. We observe that, as predicted already in Section 4, the new routing improves the probability of success of *all* connections.

Remark 1. Studying an asymmetric network numerically requires one to consider all possible combinations of the network parameters. Since the degree of freedom (the parameters to choose) are usually very large in asymmetric networks, such a numerical study is not carried out generally.

In the full version of our paper [2], we also study some special cases as a symetric networks. In a symmetric network we have $n_j = n$ for all nodes; some examples are a grid network, a circular network or a linear network. Moreover, for the symmetric networks, we can simplify the expressions in the detailed balance equation (Proposition 1) while getting important insights into the working of the network.

5 Conclusion

Considering a simple random access wireless network we obtained important insights into various tradeoffs that can be achieved by varying certain network parameters.

Some of the important results are that

1. As long as the intermediate queues in the network are stable, the end-to-end throughput of a connection does not depend on the load on the intermediate nodes.
2. Routing can be crucial in determining the stability properties of the network nodes. We showed that if the weight of a link originating from a node is set to the number of neighbors of this node, then shortest path routing maximizes the minimum probability of end-to-end packet delivery.
3. The results of this paper extended in a straighforward manner to systems of weighted fair queues with coupled servers.

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