Optimal Wavelength Converter Placement with Guaranteed Wavelength Usage

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Abstract. In this paper, we study the following problem. Given the network topology and traffic demand, determine how a minimum set of wavelength converters should be placed to ensure that the number of wavelengths needed will not exceed a given bound *L*+*u*, where *L* is the maximum link load in the network and u is a parameter defined by the network designer to reflect the overall availability of wavelength resources. This problem, however, is proved to be NP-hard. Hence we develop an efficient heuristic algorithm and extensive theoretical and experimental studies are carried out to verify the effectiveness and performance of the algorithm.

1 Introduction

Wavelength division multiplexing (WDM) [1][2] divides the bandwidth of an optical fibre into multiple wavelength channels so that multiple users can transmit data at distinct wavelengths through the same fibre concurrently. Since all-optical WDM networks can provide communication service with huge bandwidth and low latency, such networks are considered as candidates for the next generation wide-area networks which are required to meet the increasing traffic demand in the foreseeable future.

A *lightpath* is an optical communication path between a pair of source and destination nodes which may span multiple hops. In WDM networks, any pair of lightpaths (traffic demand) must be assigned with different wavelengths if they share the same link in any hops. Hence it is easy to see that the number of wavelengths required in a network is at least equal to the *natural congestion bound* or *maximum link load*, defined to be the maximum number of paths passing through any one link in the network.

Wavelength converter is an essential device in the multi-hop WDM networks that enhances the scalability of the network. In WDM networks without any wavelength conversion, the same wavelength must be assigned to all links in a lightpath (this is often referred to as the *wavelength continuity constraint*). If a node contains a wavelength converter bank, any lightpath that passes through this node may change its wavelength. Clearly wavelength assignments in networks with wavelength converters can be more efficient (uses less wavelengths) than wavelength assignments for the same set of paths where no wavelength converter is available. However, wavelength converters are expensive devices and it has been anticipated that they will continue to be so in the foreseeable future [3]. In addition, densely placed converters may cause the signal distortion [4]. Hence, it is not practical to equip every node with a converter bank.

Several wavelength converter placement schemes [5][6][7] have been proposed in the literature to reduce the overall wavelength requirements of a given network by employing a minimal set of converters nodes. However, we note that existing converter placement schemes do not take into account the availability of resources, such as the number of wavelengths and converters that are available for utilization, in a given network; hence they are not able to adapt to the availability of resources of different networks.

In this paper, we aim to take into account above-mentioned issues into consideration in the design of efficient wavelength converter placement schemes for WDM networks. Furthermore, we aim to design a scheme that is able to provide a flexible trade-off between the number of wavelength converters to be placed and the number of wavelengths required to support the communications of all lightpaths in a given network. In particular, the problem that we interested in is, given the traffic demand in a network with arbitrary topology, locate a minimal set of converters nodes in the network such that the number of required wavelengths does not exceed a given upper bound $L+u$, where L is the maximum link load in the network and u is a parameter that can be defined by the network designer to reflect the overall availability of wavelength resources.

The rest parts of this paper are organized as follows: Section 2 presents the problem assumptions, formulation and the methodology used in this work. Section 3 addresses the problem of determining the wavelength requirements for the networks with special topologies. The results we obtained in Section 3 are applied in Section 4. In Section 4, a two-step algorithm is proposed and analyzed. Experimental study was carried out in Section 5. Section 6 concludes the paper.

2 Theoretical Preliminaries

2.1 Network Model

We model the network as a undirected simple graph $G\ (V, E)$, where V is the vertex set and E is the edge set. The traffic demand is represented by a set of lightpaths $D=\{l_1, l_2, l_3 \dots l_k\}$. In this paper, we consider the case of static routing where all connections (lightpaths) are known in advance and stay for an infinite period of time in the network. The number of wavelengths needed to support all lightpaths in D is denoted by $W(G,D)$.

We assume that all communications support *duplex communication channels*, whereby data can transmit in both directions in the same fibre. The set of lightpaths that occupy the same link must be assigned with different wavelengths on this link regardless of their transmitting direction.

In this paper, we assume all converters have *full conversion capability* [8][9], this means the converter can translate an incoming wavelength into any outgoing wavelength. We adopted the *shared by node* model [9], that is the converters placed at a node can be shared by any lightpaths that pass through this node.

Now we formally define the problem addressed in this paper as follows: given the network G and a set of traffic demand $D=(l_1, l_2, l_3\dots l_k)$, locate a minimum set of nodes $S \subseteq V$ so that if we place wavelength converters at each node in S, the number of required wavelengths will not exceed the given bound $L+u$, where L is the maximum link load in the network and u is an integer parameter that can be defined by the network designer in the range of $[0, L/2]$. We refer this problem as *Optimal Wavelength Converter Placement with Bounded Wavelength Usage Problem* (*OPWB*).

2.2 The Computational Intractability of OPWB

Theorem 2.1 *OPWB* is NP-hard.

Proof. Suppose that *OPWB* is polynomial solvable, i.e., there is a polynomial algorithm $\mathcal{A}$ that can always yield an optimal solution $S \subseteq V$ for *OPWB*. Then it is apparent that $S \neq \phi$ if and only if $W(G,D)>L+u$. We therefore can determine whether $W(G,D)$ is larger than a given integer in polynomial time. Then by applying binary search, the exact value of $W(G,D)$ can also be calculated in polynomial time. However, to determine $W(G,D)$ for an arbitrary network G that with arbitrary traffic demand D is a NP-complete problem[10], so this algorithm $\mathcal{A}$ never exists and *OPWB* is NP-hard.

2.3 Graph Decomposition

Consider the case whereby a wavelength converter bank that is placed in a certain node v_i. All lightpaths that pass though v_i can convert their wavelength at v_i. The set of lightpaths that shared this converter are thus split into two parts, one from source node to the converter node v_i while another one from v_i to the destination node. The wavelength assignments for these two parts are independent from each other; thus placing a set of wavelength converters at a set of nodes S will result in the splitting of lightpaths that pass through the nodes in S into shorter lightpaths. This feature can be described by the *splitting operation* which is defined as follows:

Given a graph $G(V,E)$ and subset $S \subseteq V$, let $G_S(V',E')$ be a new graph derived from G by splitting each node $x \in S$ into $deg(x)$ one-degree nodes in V', where $deg(x)$ denote the degree of node x in G. Let $W_x \subseteq V'$ denote the set of vertices in G_S which are derived from node x in G. The process of decomposing node x in G into a new set of nodes W_x in G_S is referred to as the *splitting operation* (as in [6] & [7]). Fig 1 illustrates the decomposition of a given graph G into a new graph G_S by splitting nodes in the set S, where $S = \{3,4\}$. The process of having splitting operation on a graph $G(V, E)$ can also be stated as: $G_S(V', E) = split\,[G(V, E), S]$

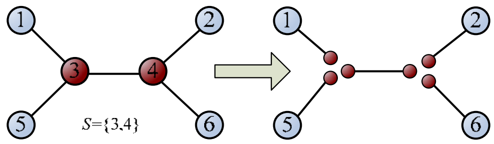


Fig. 1. Original graph G (V, E) and new graph G_S (V', E) obtained by splitting operation

Since the task of wavelength assignment in networks with special topologies, such as paths, stars and trees, can be done more easily than in network with arbitrary topologies, we adopt the approach of decomposing a given network into edge-disjoint subgraphs with special topologies which include paths, stars and trees. The decomposition process is carried out by using the splitting operation described above. We note that such an approach has also been used in [6] and [7]. However the objectives of our approach differ from those in [6] and [7] as follow: the objectives of the work in [6]& [7] are to select a set of converters for placement to satisfy L-assignability and $3/2L$-assignability, respectively, i.e. fixed bounds on wavelength usage; on the other hand, the objectives of our approach is to place a minimal set of wavelength converters to satisfy $L+u$-assignability, where u is a parameter that may be specified by the user. Hence the problem addressed in this paper is a generalization of those addressed in [6] and [7].

3 Networks with Special Topologies

3.1 Network with Path Topology

Theorem 3.1 [6]. Given a network with path topology (which is often referred as *linear network*), denoted by G_{path}, then $W(G_{path}, D) = L$ holds for arbitrary D, where L is the maximum link load of G_{path}.

Theorem 3.2 [6]. Given a network G, if every connected component of G is a path, then $W(G, D) = L$ holds for arbitrary D, where L is the maximum link load of G.

It follows from Theorem 3.2 that if we split an arbitrary network into a set of linear networks, then L-assignability can always be achieved for the network. However, a major drawback of this approach is that a large number of nodes will have to be split in the process, thus resulting in high usage of wavelength converters.

3.2 Network with Star Topology

A *star* $G_{star}(V,E)$ is a graph whereby each vertex in G_{star} is of degree one except for one vertex whose degree is at least three. The vertex whose degree is three or above is referred to as *centre node* and all edges that adjacent to this vertex is called *legs*. We will show that the wavelength assignment problem for a network with star topology can be transformed to an *edge colouring problem* which is well studied.

Definition 3.1. Let G be a graph without loops, A *k-edge colouring* of G is an assignment of k colours to the edges of G in such a way that any two edges meeting at a common vertex are assigned with different colours. If G has a k-edge colouring, then G is said to be *k-edge colourable*. The chromatic index of G, denoted by $\chi'(G)$, is the smallest value of k for which G is k-edge colourable. The problem of finding a k-edge colouring of G whereby $k = \chi'(G)$ is called *edge colouring problem.*

Given a star network $G_{star}(V,E)$, we can construct a new graph $H^*(V^*,E^*)$ which we refer to as the *edge compatibility graph*, as follows.

Edge compatibility graph construction scheme (*EGCS*)

Input: A star network $G_{star}(V,E)$, $V=(v_1,v_2,\ldots v_n)$, $E=(e_1,e_2,\ldots e_m)$ and traffic demand $D=(l_1, l_2, l_3\ldots l_k)$.

Output: Edge compatibility graph $H^*(V^* \cup W^*, E^*)$.

1) $V^* = \phi$; $W^* = \phi$; $E^* = \phi$;
2) For each edge $e_i \in E$, create a vertex $v_i^* \in V^*$;
3) For each lightpath $l_i \in D$, we do the following:

   Case (*i*): l_i is a 2-hop lightpath.

   In this case, l_i will occupy two edges, say e_x and e_y in G_{star}. Insert an edge $e_i^* \in E^*$ in H^* that connects the two vertices $v_x^* \in V^*$ and $v_y^* \in V^*$ in H^* that correspond to the edges e_x and e_y.

   Case (*ii*): l_i is a 1-hop lightpath.

   In this case, l_i will occupy an edge, say e_x in G_{star}. Insert a new vertex $w_i^* \in W^*$ in H^* and insert an edge $e_i^* \in E^*$ in H^* that will connect the pair of vertices $v_x^* \in V^*$ and $w_i^* \in W^*$ in H^*.

Based on the construction scheme described above, it is easy to see that the *edge compatibility graph* H^* of a star network G_{star} satisfies the following properties:

- Each vertex $v_i^* \in V^*$ in H^* corresponds to an edge $e_i \in E$ in G_{star}.
- Each edge $e_i^* \in E^*$ in H^* corresponds to a lightpath $l_i \in D$ in G_{star}.
- Any two edges in H^* are adjacent if and only if their corresponding lightpaths occupy the same edge in G_{star}.

Since each pair of lightpaths in G_{star} must be assigned with different wavelengths if they occupy the same link, it is easy to see the task of assigning wavelengths to lightpaths in G_{star} is equivalent to that of assigning colours to the edges in H^* such that any two adjacent edges are assigned with different colours, i.e. solving the edge colouring problem on H^*. The edge colouring problem is known to be NP-hard [11][12] and various results have been proposed in the literature to provide upper bounds on the chromatic index of a given graph. Some of these results are listed as follow.

Bounds on the chromatic index:

König's Theorem [13]. If G is a bipartite multi graph whose maximum vertex degree is d, then its chromatic index $\chi'(G) = d$.

Shannon's Theorem [14]. If G is a multi graph whose maximum vertex degree is d, then $d \le \chi'(G) \le \frac{3}{2}d$ [14].

Vizing's Theorem (extended version) [15]. If G is a multi graph whose maximum vertex degree is d, and if h is the maximum number of edges joining a pair of vertices, then $d \le \chi'(G) \le d + h$.

Bounds on the wavelength requirement of a given network:

Theorem 3.3. Given a star network G_{star} with traffic demand D, it's maximum link load is denoted by L, let H^* be its edge compatibility graph constructed using *EGCS*. If H^* is a bipartite graph, then $W(G_{star}, D) = L$.

Proof. We note the maximum link load L of G_{star} is equal to the maximum degree d of H^*, thus it follows from König's theorem the chromatic index of H^* is equal to L. This in turn implies that the wavelength requirement of G_{star} is L.

Theorem 3.4. If G_{star} is a star network with traffic demand D, let h denote the maximum number of lightpaths occupying the same pair of edges (links) in G_{star}, and let L denote the maximum link load of G_{star} , then $W(G_{star}, D) \le Min\ (3/2L, L+h)$ holds for arbitrary D.

Proof. Let H^* be the edge compatibility graph of G_{star} constructed by *EGCS*. The maximum link load L of G_{star} is equal to the maximum degree d of H^*. The maximum number of edges joining a pair of vertices in H^* is equal to the maximum number of lightpaths traversing the same pair of edges in G_{star}, i.e. h. Hence it follows from Shannon's Theorem and Vizing's Theorem that the chromatic index of H^* is bounded from above by $3/2L$ and $L+h$, respectively. This in turn implies that the wavelength requirement of G_{star} is bounded by $Min\ (3/2L, L+h)$.

3.3 Network with Bridges

Definition 3.2. Given a network $G\ (V, E)$, an edge $e \in E$ is called a *bridge* if $G - e$ is disconnected. Let C_1 and C_2 denote the two connected components of $G - e$, let $G_1 = C_1 \cup e$ and $G_2 = C_2 \cup e$, Then we say the two networks G_1 and G_2 are *singly connected* by bridge e.

Theorem 3.5. Given two networks G_1 and G_2, if G_1 and G_2 are singly connected by bridge e, then $W(G,D) = max[W(G_1,D_1), W(G_2,D_2)]$, where $G = G_1 \cup G_2$, $D = D_1 \cup D_2$; D_1 and D_2 is the set of lightpaths that traversing D_1 and D_2, separately.

Proof. Without lost the generality, we assume that $W\ (G_1, D_1) \ge W\ (G_2, D_2)$. We note that e is the only common edge of G_1 and G_2. Let T denote the set of lightpaths over e and $T=\{l_1, l_2, \ldots l_k\}$, $|T| = k$.

Consider the case whereby wavelengths have been assigned to all lighpaths in G_1 and G_2 using their respective assignment schemes, which we refer to as *Scheme 1* and *Scheme 2*.

We note that the wavelengths that have been assigned to G_1 and G_2 will form a *valid assignment* for G if the two schemes assign the same set of wavelengths to each lightpaths in T. The overall wavelength requirement of G in this case is $W(G,D) = W(G_1,D_1) = max[W(G_1,D_1), W(G_2,D_2)]$.

Next consider the case whereby Schemes 1 and 2 assign different set of wavelengths to the lightpaths in T. In this case conflict will arise between scheme 1 and scheme 2 in the assignment of the common lightpaths in T. In order to resolve this conflict, we can keep scheme 1 unchanged while reassigning the wavelengths in scheme 2 to satisfy:

- i) All lightpaths in T will be assigned with same wavelength as in scheme 1;
- ii) Any pair of lightpaths that are assigned with different wavelengths in scheme 2 before reassignment will still be assigned with different wavelengths.

This reassigning scheme is always possible to be carried out because scheme 1 uses no fewer wavelengths than scheme 2. Following the reassignment of wavelengths in G_2, the overall wavelength requirements of network G is again bounded by the $W(G_1,D_1) = max[W(G_1,D_1), W(G_2,D_2)]$.

Theorem 3.6. For a tree network G_{tree}, $W(G_{tree}, D) \le L+u$ if and only if for each star network $C_i \subseteq G_{tree}$, $W(C_i, D_i) \le L+u$, where D_i is the set of lightpaths that traversing C_i.

Proof. If: We note a tree $G_{tree}(V, E)$ can be constructed by taking a union of some connected components $C_1, C_2, \ldots, C_r$, whereby the following conditions hold:

i). $G_{tree} = \bigcup_{i=1}^{r} C_i$;

ii). C_i is either a path or a star, for $i = 1, 2, \ldots, r$;

iii). Given two components: $C_a = \bigcup_{i=1}^{m} C_i$, $C_b = C_{m+1}$, C_a and C_b are singly connected for $m=1,2,3,..r-1$.

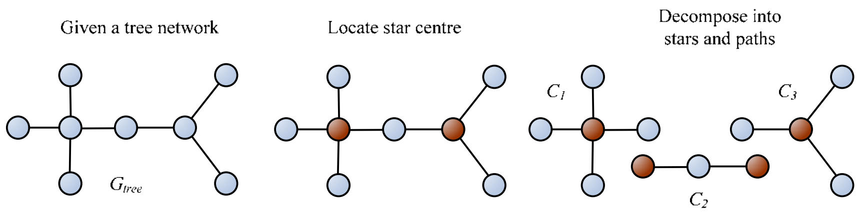


Fig. 2. Decompose a tree into a set of singly connected stars and paths.

The process of decompose a tree $G_{tree}(V, E)$ into $C_1, C_2, \ldots, C_r$ is shown in Fig 2. Then from theorem 3.5 we have $W(G_{tree}, D) = Max[W(C_1, D_1), W(C_2, D_2), ..., W(C_r, D_r)]$, where $(C_1, C_2, ..., C_r)$ is a set of single connected star networks or linear networks that satisfy conditions i-iii stated above and $D_1, D_2, ..., D_r$ are the lightpath sets that traversing $C_1, C_2, ..., C_r$, separately. For each linear network C_j, theorem 3.2 shows that $W(C_j) = \le L \le L+u$, thus $W(G_{tree}, D) \le L+u$ holds if the wavelength usage of each star networks, $W(C_i, D_i)$, is bounded by $L+u$.

Only if: C_i is a sub-network of G_{tree}, so $W(C_i, D_i) \le W(G_{tree}, D)$, if $W(C_i, D_i) > L+u$, then we will have: $W(G_{tree}, D) \ge W(C_i, D_i) > L+u$, so $W(G_{tree}, D) \le L+u$ holds only when $W(C_i, D_i) \le L+u$.

4 Proposed Algorithm and Analysis

4.1 Algorithm for OPWB

As proved in [10], in general to determine the wavelength usage of a network is a NP-complete problem. In fact, to the best of our knowledge, no upper bound has been proposed for the wavelength usage of a network with arbitrary topology. In [7] Jia et. al showed that even for a network with simple topology (a 4-nodes graph), its wavelength requirement may exceed *3/2L*. Furthermore, in [16] Wilfong et. al showed that for a single ring network, its wavelength requirement may also exceed *3/2L*. Fortunately, if *G* is a tree network then its wavelength usage can be bounded by *3/2L* regardless of the traffic demand [17]. Based on this fact, in the first step of our algorithm we aim to determine the minimum set S_1 so that $G_{S1}(V', E) = Split\,[G\,(V, E), S_1]$ will be a tree or a forest (a set of disconnected trees). This problem is often referred to as the minimum feedback set problem and is proved to be NP-complete [12]. However, as a well-studied problem, there exist many approximation algorithms with good performance guarantee. For example, in [18], a 2-approximate algorithm is proposed. Thus we can construct the vertex set S_1 which will be equipped with converters by applying these approximation algorithms.

After the converters are placed at each node in S_1, the wavelength usage of $G_{S1}(V', E)$ is bounded by *3/2L*. We can further tighten this bound by applying step 2. In this step, for each star $C_i \subseteq G_{S1}$, we examine the upper bound for its wavelength requirement determined by theorem 3.3 and theorem 3.4. For those star sub-networks whose upper bound exceed $L+u$, we include their centre nodes $v_i \in V'$ into set S_2. Converters will be placed at each node in S_2. After all these converters are placed, some stars are split into paths, the network G_{S1} is split into G_S and we can guarantee that for all remaining stars $C_i \subseteq G_S$, $W(C_i, D_i) \le L+u$. Thus from theorem 3.6 we have $W(G_S, D) \le L+u$, which implies that the total wavelength usage is bounded by $L+u$ and the total number of wavelength converters be placed is $|S_1|+|S_2|$.

Our algorithm can be described by the pseudocode:

Input: Network $G\,(V, E)$, $V=\{v_1, v_2 \ldots v_n\}$, $E=\{e_1, e_2 \ldots e_m\}$ with traffic demand set $D=(l_1, l_2, l_3 \ldots l_k)$, the upper bound for the wavelength usage $L+u$.

Output: Vertex set *S*.

1) Step one (place the converters at the feedback set nodes):

   $S = \phi, S_1 = \phi$;

   Find the minimum feedback set S_1 for G;

   $S = S \cup S_1$;

   $G_s(V,E)$ =split[$G(V,E)$,S];

2) Step two (place the converters at the centre nodes of stars):

```
S_2 = φ ;
for i=1 to n do:      /* for each vertex v_i */
    if deg(v_i) ≤ 2     /*check the degree of vertices */
        i++;
    else              /* v_i is a center node of a star */
    build the edge-compatibility graph H* of the star
    with centre v_i by the EGCS scheme
    check whether H* is a bipartite graph
    check the value of l and h, which denotes the maxi-
    mum linkload and the maximum number of edges join-
    ing a pair of vertices, separately.
    if H* is not a bipartite graph and Min(3/2 l, l+h) > L+u
    S_2 = S_2 ∪ v_i , i++;
    S = S ∪ S_2 ;
 End and output vertex set S
```

4.2 Performance Analysis

(a) Computational complexity

Theorem 4.1. The computational complexity of proposed two-step algorithm is $O(|E||V|+|D||V|)$

Proof. In the first step, finding the minimum feedback set by the approximate algorithm proposed in [18] can be done in $O(|E||V|)$ time; splitting operation can be done in $O(|E|)$ time in the worst case. In the second step, each legs of the stars should be checked to determine the maximum link load of stars, we note each edge can be included in two stars at most so the complexity of checking link load is $O(|E|)$. Next we note the number of lightpaths after step 1 is $|D||V|$ at most, thus building edge compatibility graph by *EGCS* can be done in $O(|E|+|D||V|)$; checking whether the edge compatibility graph is bipartite for all stars can be done in $O(|E|+|D||V|)$. Step 2 will cost $O(|E|+|D||V|)$ and the two-step algorithm we proposed will cost $O(|E||V|+|D||V|)$ in the worst case.

(b) The setting of *u*

As mentioned in section 1, the size of converter nodes set *S* is determined by network topology, traffic demand and given bound for the wavelength usage. In this section we will study the relationship between $|S|$ and the value of *u*:

1) $u=0$: In this case the wavelength usage is the minimum possible. Thus the size of *S* would be large. In the worst case, every star centre node of stars will be equipped with converter so the network will be split into a set of linear networks by *S*; this is the case that studied by [6].
2) $u=L/2$: It is proved in [17] and [7] that for the network with tree topology, this upper bound can always be meet for arbitrary traffic demand, thus we do not need place any converter in the second step. We can also note that in this case the *OPWB* is equal to the minimum feedback set problem, the wavelength converter placement problem under this case is studied by [7].

3) *0<u<L/2*, this is the general case that we are addressing in this paper, as shown our algorithm will generate a vertex set *S* with the size between case 1) and case2).

5 Experimental Study

In this section, we adopt experimental approach to study the relationship between the size of *S* and the value of *u*. The converter set *S* is constructed by the proposed algorithm. We assume the step 1 of the proposed algorithm has been done by the heuristics proposed in [18]. Three typical networks were studied which include NSFnet network, USA long haul network and mesh network. We also varied the size of mesh network from 4x4 to 7x7 to evaluate the effects of the network size. Some statistics of these networks are listed in Table 1.

Table 1. Statistics of some typical networks

Topology	Number of vertices	Feedback set size	Number of vertices whose degree larger than two
NSFnet	14	3	10
USA long haul	28	8	21
4×4 mesh	16	4	12
7×7 mesh	49	13	45

For each pair of vertices, they will generate a traffic demand at probability *p*, where *p* is a parameter controlling the total traffic load of the network. In this study we defined three types of traffic load condition:

i) Low traffic load, where *p* is set to 0.2;
ii) Moderate traffic load, where *p* is set to 0.5;
iii) High traffic load, where *p* is set to 0.8;

All traffic demands are routed by the shortest path algorithm. For each traffic load condition, we repeat the experiment by ten times to get the mean value of |*S*|, which denotes the size of wavelength converter nodes set. The results are shown in Fig 3-6.

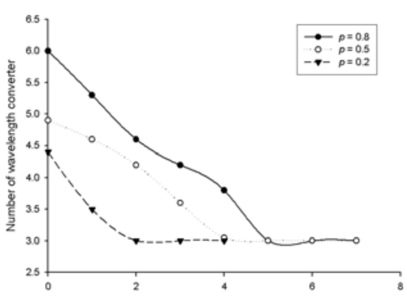


Fig. 3. Number of FWC in NSFnet

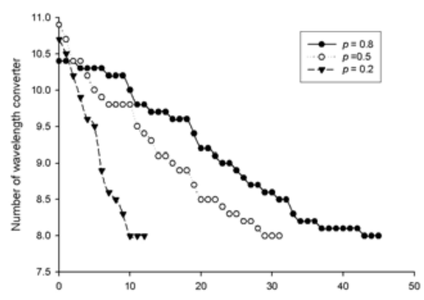
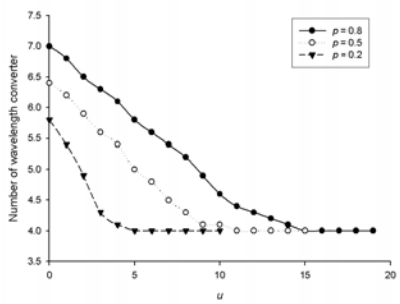


Fig. 4. Number of FWC in USA long haul network



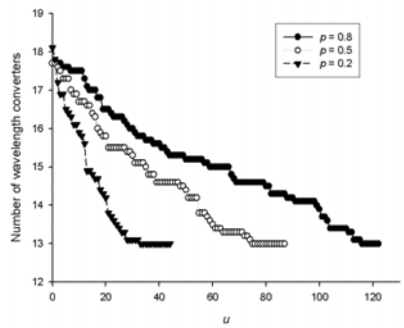


Fig. 5. Number of FWC in 4×4 Mesh network **Fig. 6.** Number of FWC in 7×7 Mesh network

The results clearly show that the size of *S* decrease about linearly as *u* increase from 0 to *L/2*, this relationship perfectly meet our expectation. With these |*S*|-*u* curves, the network designer can easily estimate the upper bound of wavelength usage when given the number of wavelength converters or estimate the number of required converters when given the upper bound for the wavelength usage.

Next we could note when the size of network increase, both the number of lightpaths and the maximum linkload increase. However, the |*S*|-*u* curve shows the same pattern, this means the size of network have little effect on the performance of the proposed algorithm.

We could also note when *u*=0, which means the wavelength usage is the minimum possible, the size of *S* constructed by the proposed algorithm is much smaller than the size of converter set constructed by the algorithm proposed in [6], which is equal to the number of vertices whose degree is larger than two (listed in the last column of Table 1). This result shows that taking the traffic demand into consideration will helps to reduce the redundant deployment of wavelength converters.

6 Conclusion

We have studied the problem of placing a minimal set of wavelength converters in WDM networks with arbitrary topology and the total wavelength usage is bounded. The traffic demand is also taken into consideration. In this work, the network designer can set the upper bound for wavelength usage in the range of [*L*, *3/2L*]. Thus the proposed algorithm is more flexible compared to existing work in this area. A two-step algorithm is proposed for this problem, its correctness is guaranteed by a set of theorems and its effectiveness is evaluated by theoretical and experimental studies.

This work can benefit WDM network design and development in several aspects. Firstly, by considering the traffic status, the number of converter can be further reduced compared to earlier works. Secondly, it can helps us to understand the relationship between the number of converters and the bound on wavelength usage, thus enable more efficient utilization of wavelength converters. Thirdly, by adopting our two-step algorithm and wavelength switching techniques, the wavelength assignment problem for a network with arbitrary topology can be reduced to a wavelength assignment problem in a set of independent stars and paths which in turn helps in reducing the overall computational complexity.

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