Ensuring Two Routes Connectivity in Mobile Ad Hoc Networks with Random Waypoint Mobility

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Abstract—To increase mobile ad hoc network reliability, virtually decrease the packets loss to zero, and to support multimedia communications multi-route is required. In order to ensure the availability of two routes, node density must be above a certain value. To the best our knowledge, this paper is the first paper that mathematically determines the required node density to ensure the availability of two routes between any randomly chosen source and destination pair in mobile ad hoc networks with random waypoint mobility model. To this end, a complete probabilistic model is provided. The obtained results reveal that the increase in the node density exponentially increases the probability of having two routes. This exponential increase is limited by a certain threshold, after this threshold the increase is negligible. An interesting conclusion from this paper is that the required node densities to ensure two routes connectivity are the same for both mobile nodes moving according to the generalized random waypoint mobility model and static nodes that are uniformly distributed in the network area. This work can be used by mobile ad hoc network designers to study the network reliability and connectivity.

Keywords—mobile ad hoc networks; random waypoint mobility; path availability; multi-paths; greedy routing; path failures.

I. INTRODUCTION

Mobile ad hoc network is a multi-hop communication networks. It is a self organized and self healing network that can be formed on the fly without any kind of infrastructure. All nodes in this kind of network can freely move. The above mentioned characteristics make mobile ad hoc network an ideal candidate to extend the network coverage of cellular networks and recover from disasters.

Nodes mobility in mobile ad hoc networks makes their analysis more challengeable compared to other kinds of networks. Nodes mobility models are required to analyse the networks’ performance. Random WayPoint mobility model (RWP) is the most used mobility model [1]. In RWP, a random destination point is chosen uniformly by each network node. After that, the nodes move to the selected destination at a speed which is chosen uniformly from the interval $[v_{min}, v_{max}]$ [1]. Then, the network nodes pause for a predefined pause time, before move again and repeat the same steps. It is well known that the nodes moving according to the RWP have non-uniform spatial distribution regardless of their initial spatial distribution [1] and [2].

Least Remaining Distance (LRD) greedy forwarding strategy was proposed to estimate the hop count for mobile ad hoc networks where the nodes are uniformly distributed in a square area [11] and [12]. LRD forwarding node or source node forwards the packets to one neighbor that lies inside a half circular area with radius equals the transmission range (R) in the direction of destination and has the least remaining distance to the destination node (D).

O. Younes and N. Thomas [4] were the first authors who mathematically estimated the hop count in mobile ad hoc network where the nodes are moving according to RWP mobility model. Maximum Hop Distance (MHD) was proposed in [4] as a greedy forwarding strategy to calculate that hop count. MHD chooses a forwarder node in a half circle area with radius R centered in the forwarding node in the direction of D as in LRD. The difference between LRD and MHD lays in the optimization criterion to choose the forwarder node. LRD chooses a forwarder node which has the minimum distance to D; while MHD chooses a forwarder node which has the maximum per hop progress.

The contributions of this paper are twofold. On one hand, this paper relies on the analysis provided in [4] to mathematically estimate the required node density in terms of the number of neighbors to ensure the existence of two routes in mobile ad hoc networks where the nodes are moving according to the RWP mobility model. To this end, a complete probabilistic model is proposed. On the other hand, it compares MHD and LRD routing criterions potentials to choose the shortest routes.

The rest of this paper is organized as follows. Section II shows the related work, while Section III develops the mathematical analysis of multi-path availability in mobile ad hoc networks. The results are presented in section IV. Finally, section V concludes the paper.

II. RELATED WORK HERE

In mobile ad hoc networks, the hop count has direct impact on packet delivery ratio, per hop and end to end delay, flooding cost, network traffic load estimation and network connectivity and availability of [4]. Link failures in mobile ad hoc networks are frequent events, mainly due to the nodes mobility. Frequent link failures make mobile ad hoc networks
less reliable compared to other kinds of networks. To increase the network reliability and connectivity the routing protocol shall choose the most stable path among the available paths and catch more than one path [15] and [16].

An analytical model to evaluate mobile ad hoc network stability and availability based on the entropy concept and node mobility parameters was proposed by B. An and S. Papavassiliou[7]. The authors of [8-10] mathematically calculated the link duration. This link duration can be used as a routing metric to choose the most stable path.

A probabilistic model to find the node density which is required to ensure the existence of two routes between any randomly chosen source and destination pair in a mobile ad hoc network with uniformly distributed nodes was proposed in our previous work [5]. In that study, we mathematically found that the number of neighbors for each node must be above 18 in order to ensure two routes connectivity. The assumption in [5] that the network nodes are uniformly distributed does not apply for all scenarios in mobile ad hoc network, and this motivates us to make this study. For example, RWP mobility model has non-uniform node distribution. To the best of our knowledge, this paper is the first paper that mathematically investigates the multi-route connectivity issue in mobile ad hoc network where the nodes are moving according to RWP mobility model. Our main goal from this study is to mathematically calculate the required average number of neighbors to ensure two routes connectivity.

In order to calculate the probability that a connection exit between any randomly chosen Source node (S) and destination node (D) pair, we shall first calculate the Euclidean distance between S and D and the hop count. Usually, the hop count depends on the used greedy forwarding strategy. For more details on greedy forwarding strategies, the readers are referred to reference [14]. An approach to estimate the hop count in mobile ad hoc networks where the nodes are uniformly distributed in the network area was proposed in [11] and [12]. The main drawback of that approach is that it applies only for the uniform distribution case. Thus, we need another approach to calculate the hop count for non-uniform distribution cases, like the one presented in [4]. In this paper, we use that approach for the calculation of the hop count, and after that we mathematically estimate the required node density to ensure the network two routes connectivity. To the best of our knowledge, this paper is the first paper that investigates this issue.

III. MATHEMATICAL MODEL ANALYSIS

This study mathematically analyses the network connectivity and availability of mobile ad hoc networks where the nodes are moving according to the generalized RWP mobility model [1]. The network area is assumed to be a square area with side length equal to L. The transmission range is a circular transmission range with radius R, and it is the same for all nodes. MHD is the used forwarding strategy.

A. Euclidean Distance Between the Source node and the Destination node

To estimate the expected Euclidean distance between any randomly chosen S and D pair, we need a probabilistic model, since the nodes are mobile. The distance between any two randomly chosen nodes moving according to the generalized RWP mobility model from a line segment or a square area was calculated in [4]. The expected Euclidean distance \(d\) between S and D in a square area network is equal to [4]

\[
d = 0.4 \times L
\]

Where L is the side length of the square area network.

B. Per One Hop Progress

The number of hops (HC) between S and D depends on their Euclidean distance, per hop progress \(r\), and the used forwarding strategy. S node in MHD chooses a relay node that has the maximum \(r\). Since \(r\) is not known, the pdf of the distance between S and all neighbors is needed, and the node which has the maximum pdf of \(r\) is chosen as the relay node. The objective in MHD is to minimize the hop count by choosing the neighbor node which has the maximum distance \(\rho_{max}\) to S as the rely node. The pdf of \(\rho_{max}\) and the expected value of \(\rho_{max}\) \((r)\) are found by using equation (2) and (3), respectively [4]

\[
f_{\rho_{max}} = (2n) \times \frac{e^{2n-1}}{R^{2n}}
\]

Where:

- \(n\) : the number of neighbor nodes in a half circular area.
- \(R\) : the transmission range.

\[
r = \frac{2n}{2n+1} R
\]

C. The Remaining Distance to the Destination

Equation (4) taken from [4] finds the pdf of the remaining distance to D \((x)\).

\[
f_X(x) = \frac{2x}{\pi n d r^2 \sqrt{1 - \frac{(d^2 + r^2 - x^2)^2}{4d^2 r^2}}}
\]

Where:

- \(d\) : the Euclidean distance between S and D.
- \(r\) : per one hop progress.

By definition, the expected value of \(x\) \((\bar{x})\) is equal to [4]

\[
\bar{x} = \int_{d-r}^{\sqrt{d^2+r^2}} f_X(x)dx
\]
D. The Expected Hop Count

In [4] an iterative procedure is proposed to calculate the expected Hop Count (HC). This procedure can be summarized as follow: At the beginning, S needs to calculate the Euclidean distance to D. If the Euclidean distance to D is less than R, then D is one of S neighbors, and HC is equal to 1, otherwise S select a rely node to forwards the packets to it. After that, the selected rely node calculates r and \( \bar{x} \). In case \( \bar{x} \) is less than R, then D is one neighbor of that rely node, and HC is equal to 2. If \( \bar{x} \) is larger than R, the same steps are repeated till \( \bar{x} \) falls below R and each time HC is incremented by 1.

E. Network Availability

In this subsection, we find the node density which is required to ensure two paths connectivity between any randomly chosen source and destination pair. The Poisson distribution can be used to find the probability that a mobile node has \( n \) neighbors inside a specific area [17]. Equation (6) calculates the probability that a mobile node has \( n \) neighbors inside a half circular area.

\[
P(n) = \frac{(\rho C)^n}{n!} e^{-\rho C}
\]  

(6)

Where:
- \( P(n) \): the probability that a mobile node has \( n \) neighbors inside a half circular area.
- \( \rho \): node density.
- \( C \): half circular area.

To have two routes between S and D, S and all relay nodes must have at least two neighbors inside a half circular area with radius R in the direction of D. Assume \( \xi \) is the probability that S or the relay node has at least two neighbors. Then based in equation (6), \( \xi \) is equal to \( P(n \geq 2) \). Then

\[
\xi = P(n \geq 2) = 1 - e^{-\rho C} - (\rho C)e^{-\rho C}
\]  

(7)

Since S and each relay node independently forward the packets to another relay node until D is reached, the probability that two routes exist between S and D (\( P_k \)) is equal to \((\xi)^{HC}\). Then

\[
P_k = \prod_{i=1}^{HC} \xi = \xi^{HC} = (1 - e^{-\rho C} - \rho C e^{-\rho C})^{HC}
\]  

(8)

IV. RESULTS

A square area mobile ad hoc network with side length (L) equal to 1000 m is considered. The network nodes are mobile nodes moving according to the generalized RWP mobility model. All nodes have the same transmission range (R) equal to 150m, and follow the MHD forwarding strategy. The objective here is to find the required number of neighbors (i.e. node density) to ensure two routes connectivity between any randomly chosen source and destination pair.

At first, we study the relationship between \( r \) and \( n \), and compare \( r \) in both MHD and LRD greedy forwarding strategies. Fig. 1 shows the relationship between \( r \) in MHD and LRD versus \( n \). We can clearly see from Fig. 1 that \( r \) in both MHD and LRD increases exponentially with \( n \) till it reaches the saturation region. It also shows that \( r \) approaches R when \( n \) is high enough in MHD, in contrast to LRD, where \( r \) does not approach R, no matter how much \( n \) is high. This is due to the different optimization criterion used in MHD and LRD, where MHD tries to optimize \( r \), while LRD tries to optimize the remaining distance to D.

To see which optimization criterion is better in terms of reducing the hop count, we plot the relationships between the hop count in both MHD and LRD and \( n \) in Fig. 2. Actually the one that has the potential to decrease the hop count is better, because this increases the network connectivity, the packet delivery ratio, and decreases the end to end delay and the network interference. Strangely, Fig. 2 shows that the hop
count in MHD does not depend on the node density \((n)\). Even though for low density, MHD potentially decrease the hop count compared to LRD, but at moderate and high node density LRD performs better than MHD. MHD increases the per hop progress \((r)\) for each step, but the optimization criterion to select the shortest path based on \(r\) optimization criterion fails to select the shortest path with moderate and high node density.

Finally, we investigate the effects of node density on the network connectivity. The relationship between the probability that two \(k\)-hop routes exist between \(S\) and \(D\) \((P_k)\) with node density \((n)\) is shown in equation (8), and it is plotted in Fig. 3. The increase in \(n\) leads to exponential increase in \(P_k\) till the saturation region is clearly seen in Fig. 3. In the saturation regions, node density increase brings negligible improvement in terms of network connectivity. Almost two paths connectivity is achieved when \(n \geq 9\). The same number was found in [5], where the nodes were static and uniformly distributed in the network area. This can highlight an important conclusion that the generalized RWP mobility model has negligible effect in the required node density to ensure two routes connectivity.

![Fig. 3. Probability that two \(k\)-hop routes exist between \(S\) and \(D\) versus the number of potential forwarding nodes \((n)\)](image)

**V. CONCLUSION**

A mathematical analysis of the two routes network connectivity in mobile ad hoc network with random waypoint mobility model was presented in this paper. It mathematically founds the node density which is required to ensure two routes connectivity between any source and destination pair. In addition to that, this paper compared the optimization criterions in MHD and LRD in terms of their potentials to choose the shortest paths. This paper showed that the optimization criterion in LRD is better than the optimization criterion in MHD at moderate and high node density. It also showed that at least 9 neighbors are needed in a half circular area with radius equal to the transmission range in the direction of destination to achieve two routes connectivity. One interesting conclusion we came up from this study was that the required node densities to achieve two paths connectivity in mobile ad hoc networks were the same for both static nodes that are uniformly distributed in the network area and mobile nodes moving according to the generalized random waypoint mobility model.

Some research directions for future works can be the effects of hop count in the packet delivery ratio, and the proposing of a novel optimization criterion to select the shortest path which outperforms the optimization criterion in both MHD and LRD. We can also investigate the effects of mobility with other mobility models, like Gauss-Markov mobility model, on the network connectivity.

**References**


