LOGICS OF BECOMING IN SCHEDULING
Logical Movement behind Temporality

Junichi Yagi\textsuperscript{1}, Eiji Arai\textsuperscript{2}, and Keiichi Shirase\textsuperscript{3}
\textsuperscript{1}Shimizu Corporation, \textsuperscript{2}Osaka University, \textsuperscript{3}Kobe University
e-mail: jun@sit.shimz.co.jp

Abstract: Scheduling is for optimal allocation of time resource for activities. This paper reconsiders the concept of time as a dynamic framework for series of actions. Time resource is viewed as elastic bodies to incorporate fluctuation of becoming of events and their dynamic behaviour. Lastly, logic of becoming will be presented as foundation of scheduling.

Key words: internal time, external time, Fourier decomposition, action operator, dynamic scheduling, becoming, reflectors, origin of temporality

1 INTRODUCTION

It is the time resource that is dealt with directly under any scheduling scheme. The task is how can be achieved the optimal allocation of a given time resource over a set of assigned activities. The temporal order of the assigned activities is usually described in a Network Chart schematically. Then there are various existing methods how to allocate the activities over time, such as Gantt chart, Multi-Activity Chart \cite{3} called MAC, or else. Most of the existing methods use a parametric time, linear axis of time, on which the activities are orderly allocated in accordance with the order of events. They provide, however, a planner with only a means of static description of time resource distribution at his planning stage, or mere projection of his temporary thoughts in succession. Two basic problems are faced in scheduling, namely; (1) (planning stage) the provided data by which one allocates time resource to a particular construction activity is usually far from precise unlike machine operation in manufacturing. In another word, the beginning and ending nodes of each time interval allocated for a particular activity are both fluctuating to a notable extent. Most of the existing scheduling methods \cite{2}\cite{4}\cite{6} cannot incorporate uncertainty of this sort sufficiently enough or not at all. (2) (operating stage) The need to
reallocate the time resource as a construction project proceeds often rises. Although the final goal of the project is clearly defined, no existing scheduling method provides a dynamic adjustment mechanism in reference to the project’s goal.

This paper will develop a mathematical model of the dynamic scheduling to deal with both of these fundamental problems directly. The eventual fluctuation is shown given rise originally by the characteristic nature of *becoming* in the temporality. It will be also shown possible that an elastic model can be developed to incorporate the afore-mentioned temporality *per se*.

2 NON-LOCALLY DISTRIBUTED TEMPORALITY

The parametric time is used for the ordinary scheduling scheme in two ways; (1) to display an optimal allocation of the pre-assigned activities on the parametric time in sequence at the planning stage before the project starts, or that of the activities yet to be completed in the middle of the project, (2) to cast its advance of the progressive completion over the parametric time. In both cases, the parametric time is just an indicative tool like a one-dimensional screen over which the project shows its allocation in sequence and casts its progress.

However, a real project cannot be represented solely by the parametric time, for it consists of series of decisions for all sorts of “this or that”, this event or that event, this way or that way, this sequence or that sequence, this divergence or that divergence and so forth. At every moment of decision all the potentials are activated [7]. Two streams of activities in the form of potentials then flow into the current activity at present, one from the past and the other from the future. They convolute at present, bounce around their possible ways of settlement, come to a final settlement as to their optimal mixture, and, in the end, transform the integrated potentials into an event at present.

There exist therefore two different temporal orders; the implicit and explicit temporality [1], one for decisional act and the other for its consequential representation. They categorically differ in such a way that *potentials* are dealt with in the implicit order of time, but dealt with *events* in the explicit order of time. However they are distinct, they are dynamically integrated within the dynamics of the current action, two sides of the same action. The combinatory temporal order, Internal Time by External Time is depicted at Fig.1. Each internal time is non-locally distributed over the pre-assigned temporal whole, Ω with density function $f_j(t)$ such that $\sum_j f_j(t) = 1$
, $\forall t \in \Omega$ The event produced by the current activity is dichotomous whether it happens (1) or it does not happen (0). This eventual dichotomy however becomes itself explicitly through superposition of all the internal potentials implicated at present by the current action, $\sigma_{op} = \sum f_i(t)$.

3 HOLOGRAPHIC MODEL

3.1 Action Operator

A holographic model can be developed based on the combinatory temporality to incorporate a dynamic aspect of scheduling and a fluctuation of work periods.

The set of density functions $\{f_\theta(\tau)\}_{\eta \in \Omega}$ represent collectively the collection of the internal times, each of which is non-locally distributed over the pre-assigned temporal whole $\Omega$. For the sake of convenience, the finite interval of $\Omega$ can be chosen as that which ranges from $-\pi$ to $+\pi$ without loss of generality. The underlined densities are assumed to satisfy one of the following conditions; (1) symmetric condition and (2) integrally symmetric condition. The latter is more generic condition in the form of $\{f_\theta : \int_{-\pi}^{\pi} f_\theta(\tau) d\theta = \int_{-\pi}^{\pi} f_\theta(\theta) d\theta\}$

By performing the Fourier harmonic decomposition for each density in the collection of internal times, the collection can be rearranged on a unit sphere surface to model the decision process in scheduling. At each moment, an operator of action integrates the whole surface of the sphere from specific angle and produces a temporal event (1 = an event happens, 0 = an event does not happen).
The coefficient $\hat{c}_s^*(s)$ on each slice of the sphere are obtained by rearrangement of the Fourier harmonics of the collection of internal time densities. Therefore, the sphere gives a geometric representation for the collection of the internal times, each of which is the probability of “the particular time event happens”. The action operator $\sigma_{op}$ is defined as follows.

$$\sigma_{op} \cdot \otimes T_r = 1 + \frac{1}{2\pi} \int_0^{4\pi} \hat{c}_r^*(s) d\sigma = 1 \quad (1)$$

where the operator acts on the total information of the collection of internal times distributed over the sphere surface at temporal angle $\zeta$.

### 3.2 Temporal Strain

The consecutive actions by the operator on the sphere produce the external temporal events out of the constellations of the internal time potentials. For the future is not realized event, it produces 0 and for the past was realized, it produces 1. Conversely, the internal time potentials are constellated so as to produce either 0 or 1 when integrated, provided that a scheduling is set up well balanced without any strains.

$$\hat{c}_s^*(s)$$

*Figure 3. Off-Balanced State of Constellation of Internal Time Potentials*

When there are strains in scheduling, overburden or waste of use of time resource, it produces more than 1 or less than 1 respectively. The off-balanced state of the internal time constellation is shown at Fig.3.
4 DYNAMICS

4.1 Restitution Force

Virtual force can be induced out of the potential space described above of the internal times as restitution force. The restitution force is a force required to restore a scheduling with positive or negative strains to an ideal scheduling with no strain. Activities are allocated in sequence on a critical path with some period as depicted at Fig.4. Each activity is the aggregate of the densities of the internal times within the period. All of the potential curves, obtained by aggregation, of the activities cross at the probability of 0.5 in a scheduling with no strain. Whereas, the intersections of the off-balanced sequence of the activities are away from the balancing point, where the restitution force acts on by the Hooke’s law.

\[ F_i = -k_i x_i \]  \hspace{1cm} (2)

The larger the standard deviation of the underlined density of an internal time is, the smaller the modulus coefficient is. The modulus is the inverse of the standard deviation, for the stronger non-locality of the density distribution allows the internal time more elastic. Suppose the density is normally distributed, that is the case for most of application, the relation of probability density and restitution force becomes explicit as shown at Eq. 3.

\[ F(x) = -\sigma^{-1} x \]  \hspace{1cm} (3)

\[ p(x) = \frac{1}{\sqrt{2\pi\sigma}} \exp\left[-\frac{1}{2} (\sigma^{-1} x)^2\right] = \frac{1}{\sqrt{2\pi\sigma}} \exp\left[-\frac{1}{2} F^2(x)\right] \]

4.2 Coupled Elasticity

Each consecutive work activity on a critical path generally is distributed as normal distribution with its distinct values for the characteristic parameters, the mean and standard deviation. The path can be viewed therefore as a serial connection of multiple elastic bodies as shown at Fig.5.
The multiple coupling of elastic bodies can be formulated as shown at Fig.6. The internal forces $f_i$'s follow the Hooke's linear law, while their coupling behavior is generally non-linear. Their stable equilibrium solutions reveal the distribution of negative or positive strains in the scheduling under the external force $F$. Exertion of the external force here means controlling the whole period of a critical path in the project under various boundary conditions on its constituent activity elements whose moduli or inverse of standard deviations are allowed to vary due to various possible allocation of man x hour.

$$
\begin{align*}
\begin{bmatrix}
F_i \\
F_{i+1} \\
F_{i+2}
\end{bmatrix} &=
\begin{bmatrix}
f_m^i \\
 f_{i+1}^m \\
 f_{i+2}^m
\end{bmatrix} =
\begin{bmatrix}
k_m & -k_m & x_i \\
-k_m & k_m & x_{i+1} \\
-k_{m+1} & k_{m+1} & x_{i+2}
\end{bmatrix}
\end{align*}

Figure 6. Formulation of Coupled Elastic Time Resource

5 LOGICS OF BECOMING

The series of work activities are allocated on a critical path in scheduling. The elastic model for allocation of time resource over activities allows scheduling a dynamic analysis for planning and conducting schedule. Scheduling becomes analytic to examine stress distribution, where no stress indicates well-balanced schedule with neither waste nor over-burden of allocation of time resource and constant stress is the second best. Variety of appropriate boundary conditions can be imposed for examination of stress distribution and their effects are simulated. When over-stressed part is identified, reallocation of work load can be made before it happens.

Gantt chart is widely used for scheduling. It is however not an analytical tool, but only representational, for no internal structure is available. Non-locally distributed internal time made an analytical scheduling possible,
where the modulus coefficient of elasticity is simply the inverse of temporal deviation. Temporal order is now re-examined closely to reveal it is the source of dynamics, where potential probability and force are conjoined.

### 5.1 Extended Dichotomy

The collection of internal times is a temporal potential space. The external time of events is an event space. The action operator acts on the temporal potential space independently at each moment to produce an event by integrating the latent potentials in its own way. The whole potentials get into each moment with its unique settlement. Or two streams of potential activities flow into the current activity at present, one from the past and the other from the future, convolute at present, bounce around their possible ways of settlement, come to a final settlement, and, in the end, transform the final settlement into an event at present as current optimal mixture of all the temporal potentials.

The series of moments is time. Every moment is distinct. Its distinctness however differs from an ordinary sense. It is a **distinct mixture of the same whole of the potentials**. Here is the point of departure from standard dichotomous logics. Moment $A \times \bar{A}$ is now $A \times A$, the direct product of $A$ and $\bar{A}$, both of which are contained in $A$ as potency. Moment $\bar{A}$ is likewise $A \times A$.

\[
\begin{align*}
(A \times \bar{A}) \cup (\bar{A} \times A) &= \Omega^* \\
(A \times \bar{A}) \cap (\bar{A} \times A) &= \phi
\end{align*}
\]

The law of excluded middle is preserved. However it is the logics of extended dichotomy for **becoming** [5]. It contrasts markedly with the standard dichotomous logic, logics of being.

### 5.2 Duality in Logics

These contrasted logics are in fact dual sub-logics of the even higher logics. The logical space $\Omega \times \Omega$ can be expanded as follows.

\[
\Omega \times \Omega = [A \times A \cup \bar{A} \times \bar{A}] \cup [A \times \bar{A} \cup \bar{A} \times A] = \Omega \cup \Omega^* \tag{5}
\]

The self-referential logical space $\Omega \times \Omega$ contains both of the standard and extended dichotomous logics as shown at Fig.7 (a). These dual sub-logics are ceaselessly moving in and out each other at every moment via the action operator $\sigma^*$ as shown at Fig.7 (b). The self-referential logical space maintains itself by this ceaseless movement.
6 CONCLUSION

It has been demonstrated that the reformulation of temporality more fitted to an actual decision process makes it possible to provide us with an analytical tool for scheduling. The elastic model was derived from the temporal probability, where the dynamics of elasticity and probabilistic fluctuation are conjoined in the theory. The union is rooted in temporality of becoming.

7 REFERENCES