Chapter 24

MULTIGRAPH DEPENDENCY MODELS FOR HETEROGENEOUS INFRASTRUCTURES

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Abstract The identification and mitigation of interdependencies among critical infrastructure elements such as telecommunications, energy and transportation are important steps in any protection strategy and are applicable in preventive and operative settings. This paper presents a graph-theoretical model and framework for analyzing dependencies based on a multigraph approach and discusses algorithms for automatically identifying critical dependencies. These algorithms are applied to dependency structures that simulate the scale-free structures found in many infrastructure networks as well as to networks augmented by random graphs.

Keywords: Infrastructure interdependencies, multigraph models, simulation

1. Introduction

One of the defining characteristics of critical infrastructures is the level of interdependence among individual infrastructure components such as energy, telecommunications and financial services. While the interdependencies act on different timescales and may exhibit buffering characteristics (e.g., in the case of emergency power supplies) or delays in the effects (e.g., an inability to schedule transportation services after a communication system failure), direct and transitive (often also circular interdependencies) can be identified in a large number of cases.

An area of particular interest in critical infrastructure protection research is the avoidance and analysis of widespread effects on large parts of the population and economies, which may, for example, result from cascading and circular effects among infrastructure components – as exemplified by the August 2003 power outages in the northeastern U.S. and Canada and the November 2006 power outages throughout much of continental Europe. While elaborate models, also incorporating physical characteristics and effects and with predictive capabilities exist for many of the individual critical infrastructure services (e.g., for electrical power grids at the national and transnational levels), it is desirable to also investigate larger-scale interactions among multiple infrastructure sectors. Specific questions include cascading effects that would occur if one infrastructure component becomes unavailable for an extended period, along with possible circular effects that might inhibit or at least severely impede the resumption of regular infrastructure services. This, however, requires the development of models that exhibit acceptable computational complexity and at the same time provide adequate modeling capabilities. The level of detail that can be incorporated in such models is of necessity a limited one compared to sector-specific models. However, in many cases the basic identification of the existence of interdependencies and critical dependency paths among infrastructure components already provides valuable information, which may be investigated further using more refined modeling processes.

This paper presents a model framework based on a simple graph-theoretic model that forms the basis of several models of increasing capabilities (and computational complexity) in which additional constraints are introduced and infrastructure characteristics such as the ability to buffer resources are added. Connectivity-based interdependency models, however, can provide important insights into the vulnerabilities introduced by interlinking infrastructure components, particularly if the interdependency characteristics differ significantly as in the case of power and telecommunication networks discussed in this paper.

The remainder of this paper is structured as follows: Section 2 summarizes the basic multigraph model, which forms the foundation for a family of models with increasing expressiveness and computational complexity. Section 3 provides several simplified case studies, which are intended to be illustrative and hence represent abstractions, not actual network structures. These model instances are further illustrated through simulation results described in Section 4. Section 5 briefly reviews related research. Section 6 provides conclusions and an outlook on current and future research.

2. Multigraph Model

Interactions among infrastructure components and infrastructure users are modeled in the form of directed multigraphs, which can be further augmented by response functions defining interactions between components. In the model, the vertices $\mathcal{V} = \{v_1, \ldots, v_k\}$ are interpreted as producers and consumers of m different types of services. A single vertex can act as a producer and consumer at the same time. If a node is not able to generate a needed type, the node is dependent on some other node delivering this service. Such a dependability has the dependability type d_j, which is chosen from the set $\mathcal{D} = \{d_1, \ldots, d_m\}$.

Pairwise dependencies between nodes are represented with directed edges, where the head node is dependent on the tail node. The edges of a given infrastructure are defined by a subset $\mathcal{E}$ of $\boldsymbol{\mathcal{E}} = \{e_1^1, e_2^1, \ldots, e_{n_1}^1, e_1^2, , \ldots, e_{n_m}^m\}$, where $n_1, \ldots, n_m$ are the numbers of dependencies of type $d_1, \ldots, d_m$, respectively, and e_i^j is the edge number i of dependency type j in the network. A dependency between nodes v_a and v_b is uniquely determined by $e_i^j(v_a, v_b)$. In addition to the type, two predicates $C_{\text{Max}}(e_i^j(v_a, v_b)) \in \mathbb{N}_0$ and $C_{\text{Min}}(e_i^j(v_a, v_b)) \in \mathbb{N}_0$ are defined for each edge. These values represent the maximum capacity of the edge $e_i^j(v_a, v_b)$ and the lower threshold for flow through the edge, respectively.

The first studies of large complex networks evaluated the robustness of infrastructure attacks based on static failures [6, 9]. This is accomplished by removing a certain percentage of nodes in the network and estimating how the performance or connectivity of the network is affected by the induced failure. In dependency networks, such as the power distribution network and the telephony transport network, the breakdown of a node may cause cascading failures and have other time-dependent effects on the networks that are only detectable via a dynamic approach. We assume a discrete time model with the system in an initial state at time $t = 0$. Let $r_a^j(t) \in \mathbb{Z}$ be the amount of resource j produced in node v_a at time t. We define $D(t)$ to be a $k \times m$ matrix over $\mathbb{Z}$ describing the amount of resources of dependency type j available at the node v_a at time t. It follows that the initial state of D is given by

$$D_{aj}(0) = r_a^j(0). \tag{1}$$

For every edge in $\mathcal{E}$ a response function

$$\begin{aligned} R_i^j(v_a, v_b, t) \quad &= \quad f(D_{a1}(t-1), \ldots, D_{am}(t-1), \\ &\qquad C_{\text{Max}}(e_i^j(v_a, v_b)), C_{\text{Min}}(e_i^j)(v_a, v_b)) \end{aligned} \tag{2}$$

is defined, which determines the i-th flow of type j between the nodes v_a and v_b. The function f w.l.o.g. is defined as a linear function mapping $\mathbb{Z} \times \cdots \times \mathbb{Z} \times \mathbb{N}_0 \times \mathbb{N}_0$ to $\mathbb{N}_0$ (see below for a rationale for limiting f to linear functions), and may contain some prioritizing scheme over i and v_b. As seen from Equation 2, a single-step model with one state memory has been chosen, as we are currently not concerned with long-term feedback, although the model naturally extends to longer-term state retention.

Given the responses at time t, the available resources in a node v_a at time t in any node are given by

$$D_{aj}(t) = \sum_{i,s | e_i^j(v_s, v_a) \in \mathcal{E}} R_i^j(v_s, v_a, t). \tag{3}$$

A node v_a is said to be functional at time t if it receives or generates the resources needed to satisfy its internal needs, i.e., $D_{aj}(t) > 0$ for all dependency types j which are such that $e_i^j(v_b, v_a) \in \mathcal{E}$, where $b \in \{1, \ldots, a-1, a+1, \ldots k\}$. If this is the case for only some of the dependency types the node is said to be partially functional; if no requirements are satisfied the node is said to be dysfunctional.

The implemented model investigates how high-level network effects (functionality of nodes) and interrelations (connectivity of nodes) in interconnected infrastructures react to different attack scenarios. The presented model can be used to represent any topology given a set of infrastructures and their interconnections. The model cannot achieve the level of accuracy found in devoted network simulators (described in Section 5), but it has the advantage of being able to estimate the consequences of cascading failures in interconnected infrastructures.

By constraining the response function to a linear function and discrete values for both time steps and resources, linear programming approaches can be employed for optimization of the relevant parameters. Interior point methods such as [18] can achieve computational complexity on the order of $O(n^{3.5})$, making the analysis of large graphs feasible.

3. Dependency Analysis

This section explores how two interconnected networks influence each other. Two clearly interdependent networks are the electrical power distribution network and the telephony transport layer network. The analysis is based on several abstractions and represents an approximation to actual network topologies. The motivation for choosing these two infrastructure elements as the first subject of investigation is primarily due to their key enabling role in modern society. The BAS study [16], carried out by the Norwegian Defense Research Establishment in 1997, established the criticality of power supply and telecommunications to Norwegian society. In addition, the networks are interesting candidates for model verification as there is a fundamental difference in how service deliveries flow through the networks. In the power distribution network all the power originates from a small number of power plants or generators. A transportation network, which may well interconnect several power plants, delivers power to a large number of transformers that serve the low voltage distribution network. As a consequence, the resulting graph is a directed network where multiple edges of different orientation between two nodes rarely occur.

Traditionally, the telephony transport layer has been a hierarchical network (see, e.g., [14]). Although there has been a decided trend away from this due to progress in transmission and switching technology since the early 1990s, we have chosen to use this model because it is representative of much of the currently-deployed telecommunications infrastructure. The telephony transport layer may be idealized as an onion structure with a very low diameter. The signal always starts from the outer layer; depending on the range of the connection, it goes through the core of the network before retuning to a local switch in the outer layer of the network. All edges are bidirectional, thus all connected nodes are connected by an edge in each direction.

3.1 Electrical Power Distribution Network

One of the early studies of power distribution networks was the analysis of the Western (U.S.) States Power Grid carried out by Watts and Strogatz [27] in 1998. The degree distribution of the network was found to be exponential-like, but the clustering coefficients were too large for the network to be a classical random graph. The observed network consisted of approximately 3,500 nodes, a number which might be too small for being conclusive regarding the categorization of the network [12]. For the purposes of the present study, however, an exact representation of the power distribution grid is not necessary as we are primarily interested in topological characteristics. To this end, a network topology generator was implemented based on the assumptions that the number of source nodes is small compared to the number of transport and sink nodes, power generating nodes are not directly interconnected, the network is constructed to cover a topological area as efficiently as possible, and some links are forced on the network to interconnect distribution networks and create redundancy. Based on these assumptions, a tree-like model for the power distribution network is a reasonable approximation, although binary and k-trees are much too regular to represent the topology. The basic Barabási-Albert (BA) model [1] with some modifications provides a tree-like structure together with the level of irregularity found in real networks. The original BA model is initiated with a connected graph. In the power distribution network case, the source nodes are not interconnected. This is solved by simply providing the originating nodes with an initial degree $k_{\text{Init}} \geq 1$ that does not represent any real edges, just the centrality of the node in the network.

Given that one node is added at each time step in the BA model, as many disconnected trees as there are initial nodes in the network are generated. A sparse random graph is placed on top of the scale-free networks to connect lower-level nodes with each other. Since the network is very sparse, its statistical properties are not affected, but there is a major influence on network connectivity and the possible generation of feedback loops. The procedure used to generate the power distribution network topology involves network properties such as growth (a new node, defining the head of a new edge, is added to the network at every time step), preferential attachment (the tail of the edge is selected among the existing nodes with probability proportional to the degree of the node), and redundant connections (after the final time step a sparse random graph is placed on top of the network). As the network grows large, the influence of the sparse random graph becomes small and the probability of a node having k edges follows a power law with exponent $\gamma = 3$ [12].

Finally, a response function is defined for each edge. In the case of quantitative analysis of service delivery this function should be an implementation of Kirchhoff's first law, ensuring that the flow into a node along with the flow generated by the node equals the output and the consumption of the node. Such a detailed approach is not necessary to explore the model, as the model focuses on the functionality of the node. The principal issue is that electricity is consumed as it propagates through the networks and cannot be stored e.g., using subgraph cycles. Thus, the implemented response function only illustrates a resource which is being consumed as it flows through the network. Introducing a threshold function $T(x, c) = \delta(x - c)x$, where

$$\delta(x) = \begin{cases} 0, & x < 0 \\ 1, & x \geq 0. \end{cases} \tag{4}$$

The implemented response function is of the form

$$R_i(v_a, v_b, t)) = T(\frac{1}{2}D_a(t), C_{\mathrm{Min}}(e_i(v_a, v_b))), \tag{5}$$

where D_a is the current available in node a at time t. Equation 5 indicates that two units of input current to the node are required to produce one unit of output current along an outgoing edge. The dependency type is not specified because there is only one dependency in the network. We assume that only one power dependency exists between two nodes and no prioritization scheme is defined for the outgoing edges.

A node in the power distribution network is defined to be functional if it has incoming current or generates current. The given response function can provide information on whether a node is functional or not, but it does not provide any physical representation of the level of functionality of a node in the network, which provides a sufficient level of detail for the purpose of this study.

3.2 Telephony Transport Layer Network

Compared with the electric grid, Internet and autonomous system networks [21], the telephony transport layer network has received relatively little attention by the critical infrastructure modeling community. In this work, we assume the telephony transport layer has a traditional hierarchical network structure. The network is optimized locally for complete connectivity and globally to minimize the number of switches in an average connection circuit. In order to be functional, a switch must be connected to other switches and to a power supply, which is the focus of our analysis.

The network, which is modeled as a number of disconnected trees, is connected to a fully-connected transportation network through their root nodes. The response function of the telephony network depends on whether or not the node has power as input. If no power is available, circuit switching cannot take place and no communication is possible. The response function for edges in the telephony transport layer is thus a threshold function given by

$$R_i(v_a, v_b, t) = \delta(D_a(t) - C_{\mathrm{Min}}(e_i(v_a, v_b))), \tag{6}$$

where D_a is the current available in node a at time t and δ is as defined in Equation 4. It follows from Equation 2 that a directed edge between nodes v_a and v_b is defined if power is available to node v_a. Again, no redundant links are defined between two nodes and no prioritization scheme is defined for the edges. As mentioned earlier, each connection in the telephony transport layer is bidirectional (one-way communications are of no interest). The functionality of a node thus depends on whether the node and the node it is connected to have power supply (i.e., the switch can deliver the two-way service it is meant to deliver).

3.3 Network Interconnections

The dependency between the electrical power distribution network and the telephony transport layer is assumed to be one-way. The power distribution network is fully functional when switches in the telephony transport layer are not functional. Conversely, the flow along an edge in the telecommunications network will halt if either the head node or the tail node lose power. The connection of the telephony transport layer to the power grid is randomized in the present model (i.e., it does not take into account geospatial proximity and other factors that result in functional clustering). However, this is deemed to be adequate for the purpose of our analysis. Readers are referred to [25] for an extension of the theoretical model with two-way dependencies between the electrical power distribution network and the telephony transport layer.

Telephony transportation layer nodes have two inputs (current and information) and produce one output (information). At every time step, the response functions for power distribution and telephony transportation edges can be computed given the network state in the previous time step. The functionality of the telephony transport layer follows directly from this. Since a one-way dependency is defined, failure can only propagate from the power distribution network into the telephony transportation layer.

3.4 Attack Scenarios

Studies of complex networks frequently conclude that many man-made and natural networks are scale-free in nature, and thus possess the well-known Achilles heel of robustness against random breakdown and vulnerability to targeted attacks [2]. The first item investigated in Section 4 is whether the introduction of a very sparse random graph on top of a scale-free infrastructure will reduce some of the vulnerability to targeted attacks.

Several possible scenarios may cause the failure of a node in an infrastructure. The cause may be an intentional or unintentional act by a human, or a change in the network environment (e.g., flooding), or a technical error. We consider three attack scenarios in our analysis: single node removal (consequence of a targeted terrorist attack or a single technical failure), removal of a small connected component (non-localized failure such as flooding or some other natural disaster), and removal of disconnected components (result of a coordinated terrorist attack).

4. Simulation Study

Small topologies were generated artificially to illustrate the properties of the model. A power distribution topology based on two power sources and 28 power distribution nodes was connected to a telephony transport network with 21 total switches, including three core switches. The switches were connected to randomly selected lower-level power distribution nodes (i.e., no power generating nodes were connected directly to the telephony transport layer). None of the nodes of the telephony transport layer were assumed to have an independent power supply.

The attacks involve the removal of one or two nodes with the following steps: (i) remove a node from the network, (ii) run the response function until the number of functional nodes in the network stabilizes, (iii) count the number of functional nodes in the network, and (iv) reinsert the node. This procedure is repeated for all nodes in the network. The pairwise removal of nodes follows a similar procedure, except that two nodes are removed at a time.

The results are presented as the fraction of functional nodes that remain after removing one or two nodes from the network. The results are presented as histograms in Figures 1, 2 and 3. The x-axis represents the fraction of functional nodes in a run, and the y-axis represents number of runs of the algorithm. The results are deduced from one topology generated as described in Section 3. A single topology is not sufficient to draw general conclusions about the properties of the proposed topologies, but it illustrates the ability and flexibility of the model.

4.1 Coordinated Failures in a Single Domain

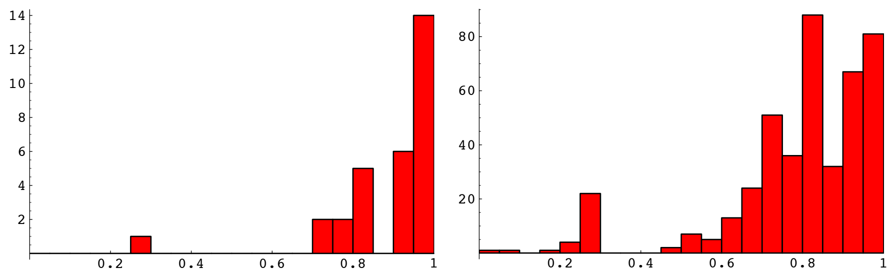
This scenario considers the single, non-buffered power distribution network. While atypical of the interdependencies existing between real-world critical infrastructures, the network permits the exposition of core elements of the model and the simulation environment.

4.1.1 Scale-Free Power Distribution Network. This scenario illustrates the well-known vulnerability of scale-free networks to targeted attacks. The electrical power distribution network is represented as a scale-free network and two scenarios are considered: (i) removal of one node, and (ii) removal of two random-selected power nodes.

Figure 1a shows that removing one node has limited influence on the network. Specifically, in almost 50% of the cases, more than 95% of the nodes are functional, which is very high as the simulated power distribution network has 28 nodes. We also note the high influence of removing one particular node – the generator in the largest sub-distribution network. As the distribution networks of the two generators are not interconnected due to the BA construction, removing the generator takes out the entire subgraph. The gap observed between 0.85% and 0.90% of functional nodes is most likely due to the small size of the network.

The results of the attacks are shown in Figure 1, which nicely illustrates the properties of a scale-free network.

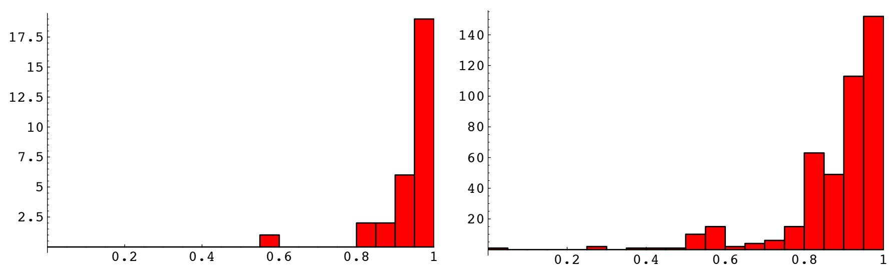
Figure 1b shows that removing two nodes from the network also has a limited effect on the network – more than 70% of the nodes are functional in the



(a) Fraction of functional nodes after random removal of one node (28 runs).

(b) Fraction of functional nodes after random removal of two nodes (378 runs).

Figure 1. Consequences of node removal on a scale-free topology.



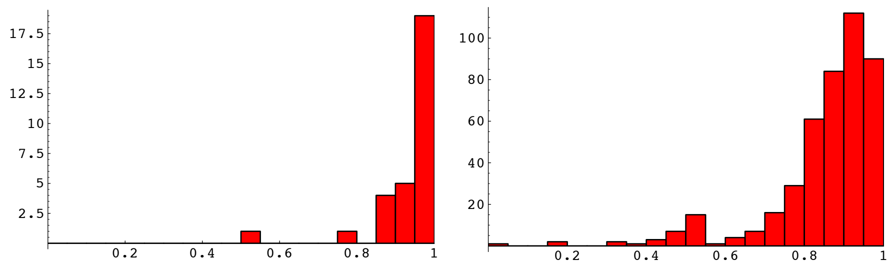
(a) Fraction of functional nodes after random removal of one node (51 runs).

(b) Fraction of functional nodes after random removal of two nodes (1,275 runs).

Figure 2. Consequences of node removal on a scale-free network with redundancy.

majority of the cases. Obviously, taking out both generators paralyzes the network. The peak observed around 30% is due to the removal of the largest generator plus a central node in the second power distribution network.

4.1.2 Scale-Free Network with Added Redundancy. In this scenario, a sparse random graph is placed on top of the scale-free graph to provide redundancy. The results of the simulation are presented in Figure 2. Figure 2a shows that the introduced redundancy improves the robustness of the network considerably. When one node is removed, the functionality of the network rarely drops below 90%, and never below 50%. Thus, the cost of adding redundant edges may pay off in terms of robustness. The same holds for the scenario involving the removal of two nodes (Figure 2b). The functionality rarely drops below 80% and the peak that was formerly located around 30%



(a) Fraction of functional telecommunication nodes after random removal of one power node (51 runs).

(b) Fraction of functional telecommunication nodes after random removal of two power nodes (1,275 runs).

Figure 3. Consequences of node removal from two networks.

has now moved to 60%. Of course, the removal of both generators still takes out the entire network.

An interesting observation can be made related to the second attack scenario described in Section 3.4. In each of the 15 most critical two-node removals, there was no pair of connected nodes. Consequently, removing any connected component of size two, still results in more than 50% of nodes remaining functional. This shows that well-targeted attacks on a critical infrastructure are likely to be more effective than an extensive attack against connected components.

4.2 Multi-Domain Dependencies

The final simulation illustrates how failures in the electrical power distribution network propagate into the telephony transport layer. Each node of the telephony transport layer is connected to a node in the power distribution network, and its functionality depends on the power supplied to itself and its neighbors. Figure 3 shows the fraction of functional telecommunication nodes as one or two nodes are removed from the power distribution network.

The results clearly illustrate the dependency between the two networks and validates the basics of the model. In our future work, we will explore more exiting features such as circular dependencies, multiple network interdependencies and metrics for identifying critical network components.

4.3 Discussion

In our opinion, pure scale-free topologies are not suitable for representing for real-world infrastructures. Unlike most man-made infrastructures, the pure BA topology contains very few redundant links. Imposing random connections on top of the BA structure makes the model more realistic; at the same time, the vulnerability of the network is reduced. As illustrated in our simulation study, when redundancy is introduced, the network is less sensitive to removing single nodes.

Future analyses should consider the removal of random nodes and random pairs of nodes from the network, and observe failure propagation throughout the network. This will identify the nodes that are central to network functionality and help determine where resources should be invested to increase operational reliability and infrastructure security.

5. Related Work

Research activities related to the monitoring and simulation of critical infrastructures are being conducted worldwide, although generally at a qualitative level. One of the earliest and most widespread methodologies involves the application of a control systems approach [24], including hybrid mechanisms [17]. Other approaches for modeling infrastructures include agent-based systems [4, 20, 26]. Such qualitative efforts also include the Critical Infrastructure Modeling and Assessment Program (CIMAP) and the European Project ACIP [23]. Additional approaches (e.g., [3, 22]) vary considerably in the level of detail considered, ranging from simple dependency analysis to elaborate models containing continuous physical submodels (e.g., for pipelines and electrical power grids) as well as behavioral models.

For the more constrained case of individual infrastructures such as pipelines and power grids, however, rich modeling and simulation environments already exist including the PSIcontrol system and proprietary mechanisms employed by operators. However, interconnections and interdependencies can only be modeled to a limited extent in such environments. Several properties are immediately derivable from interconnection characteristics alone as shown for power grid and Internet connectivity [5, 8, 13, 28]. Frequently, the underlying structure of the networks can be identified as being wholly or partially scale-free [7, 11, 15, 19]. This has significant implications for the vulnerability of interconnected and interdependent networks of critical infrastructure components to random failure [6, 9] as well as to targeted attacks [10].

6. Conclusions

This paper has presented the foundational elements of a family of models for investigating interdependencies among heterogeneous critical infrastructures in abstract topologies. To this end, we have provided an extensible graph-theoretical model, which incorporates a flexible response function for modeling vertex behavior, including activities internal to vertices and the provision of buffered and unbuffered infrastructure services.

With the help of simplified abstract models, we have demonstrated how the addition of random components to an otherwise scale-free network can influence the overall robustness of the network to vertex removal. The observations are verified by a simulation study involving a simple interconnection model for two unbuffered networks, an electrical power distribution grid and a fixed-line telephony network.

Our future research will focus on validating the model using simulations of large-scale power and telephony network topologies. We will also work on extensions to the model, including the ability to store dependency types within nodes, incorporating cyclic interdependencies between infrastructures, prioritizing resources within nodes, and introducing component failure as known from reliability theory. Furthermore, we will attempt to refine our analytic approach by using graph-theoretical and combinatorial optimization techniques to identify critical interdependencies and effective mechanisms for enhancing the robustness of critical infrastructures.

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